IT-enabled Management of Sharing Logistic Trucks

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Abstract: This paper proposes a management system for sharing trucks as cooperative logistics. To reduce fossil fuel consumption and carbon dioxide emissions resulting from transport, we should improve transport efficiency of trucks, which play an essential role as carriers in modern logistics services. We propose a language for specifying the routes of trucks and an order relation between the requirements of routes and the possible routes of trucks. The former is formulated as process calculus and the latter selects suitable trucks whose itineraries can satisfy the requirements of users and are more friendly the environment.

1 INTRODUCTION

Green logistics is important to minimize the ecological impact of logistics activities. Transportation accounts for about 23-percent of energy-related carbon dioxide (CO₂) emissions (Agency, 2009). IEA has expected, given current trends, energy demand and CO₂ emissions in transport nearly 80-percent higher by 2050 without any efficiency improvements. Trucks, which play an essential role as carriers in modern logistics services, emit a huge quantity of carbon dioxide (CO₂) into the atmosphere. There have been several approaches to reducing fossil-fuel consumption and CO₂ emissions from tracks.

- Low-emission tracks: the amount of CO₂ emission from tracks is reduce by using environmentally friendly trucks, which produces less harmful impacts to the environment than comparable conventional internal combustion engine vehicles running on gasoline or diesel, or one that uses certain alternative and sustainable fuels.

- Low-emission operation for individual tracks: since the amount of CO₂ emission from each track is basically in proportion to the distance traveled by the track. Shortening such a distance can reduce CO₂ emission. Environmentally friendly driving is important because driving of trucks, e.g., slowing down and up, depends on the amount of CO₂ emission.

- Low-emission management for trucks: the number of tracks seriously affect the amount of CO₂ emission. It is reduced by efficiently managing trucks in addition to reducing freights.

This paper addresses cooperative logistics as a solution to the third approach. Cooperative logistics has been expected as one of the most efficient and popular ways of improving truck-load ratios and reducing trucks. However, it tends to be complicated in comparison with existing (non-earth-friendly) logistics. In fact, several industries, e.g., food and automobile manufacturers, in addition to the dairy industry, have attempted to use cooperative logistics.

Nevertheless, most attempts to support cooperative logistics have been failed because of their management problems for several reasons. There are often conflicts between the requirements of suppliers/customers and the operations of shared trucks. Even if there are many shared trucks, it is difficult to find trucks that can satisfy the requirements of suppliers/customers, because the requirements are more complicated than those of moving people. Most cooperative logistics management tends to depend on humans, e.g., logistics managers in suppliers and truck drivers. This may be rational in a small scale cooperative logistics consisting of two or three suppliers, but seriously affect scalability. We need a systematic and scalable approach for managing cooperative logistics.

To solve these problems, we propose an e-logistics for sharing trucks between operators or customers to support cooperative logistics. This paper defines our e-logistics management system for sharing trucks. The system is constructed based on a theoretical framework consisting of a process-calculus-based language that describes truck routes and a mechanism for selecting suitable trucks according to the requirements of customers. This is because the selection
of trucks need to be exact in the sense that selected trucks must satisfy the requirements. The mechanism is defined based on an algebraic relation that determines whether a truck can visit various points, e.g., farmers and manufacturers, along its route to collect or deliver items. It enables collection/delivery points to select trucks according to the truck route because the route a truck takes is critical in determining its efficiency.

2 EXAMPLE SCENARIO

Milk-run operation is one of most typical cooperative logistics ones for improving truck-load ratios, refers to a means of transportation in which a single truck cycles around multiple suppliers to collect or deliver freight. The name is derived from the milk-runs carried out by farmers collecting milk from dairy cows spread out over pastures (Fig. 1). Using the milk-run approach, one truck calls at each of the suppliers on a daily basis before delivering the collected milk to the customer’s plant. On the other hand, in a traditional approach, e.g., the Just-In-Time approach, all suppliers have their own trucks and send one truckload per day to the customer (Fig. 1). The milk-run approach is effective in reducing the amount of CO$_2$ emitted by trucks.

We assume that a truck has sufficient carrying capacity. It starts at factory A and may visit factory A again. Figure 3 shows four trucks carrying out milk-runs on different routes. The first, second, and third trucks can satisfy the above requirements but the fourth cannot. The third is less efficient than the first on their rounds. The system proposed in this paper was inspired by our real experiences. Although the milk-run approach is effective in reducing the amount of CO$_2$ emitted by trucks, its management tends to be complicated, which is one of the most significant barriers preventing wider adoption of the approach in real logistics.
This paper assumes that one or more trucks involved in milk-run logistics operations call at multiple points along their routes. Customers and suppliers have to decide which truck and which route will best satisfy their requirements, and this decision is not an easy one.

3 REQUIREMENTS

The proposed e-logistics system was inspired from our discussions on real logistics companies. The system must therefore satisfy the following requirements:

- In cooperative logistics trucks may be shared by multiple customers, so that they collect products at one or more source points and deliver the products at one or more destination points on their way. The trucks need to visit the source points before they visit the destination points. Our system therefore needs to specify the order in which trucks call at various points.

- Some products may be collected/delivered at points by trucks without any need for a specific order of arrival at collection/delivery points. That is, the order of the movement of trucks between points does not affect the efficiency of the trucks’ operations. Suppliers or customers should select a truck according to the number of movements between the points that the trucks visit.

- One or more trucks are available in cooperative logistics, but their routes may be different. Most logistic trucks runs their driving plans, which were submitted to or assigned by truck operators’ offices, although they may be changed daily, weekly, or monthly. That is, trucks do not change their routes after starting their operations.

- Our e-logistics management system should receive the routes submitted by truck operators and maintains the routes in its database. When it receives queries about the trucks that can satisfy the requirements of customers, it returns such trucks to them. The system should provide truck operators and customers with web-based interfaces.

- Pallets or boxes that contain multiple products are considered as transport units in many current logistics systems, rather than as individual products. These types of containers may have multiple destinations required by their inner products and the receivers may take only some of the products in the container when it arrives at their point.

4 APPROACH

The selection of the routes of trucks for cooperative logistics, including milk-run operations, is critical for industrial efficiency and for minimizing carbon dioxide emissions. Careful consideration must be given to selecting suitable trucks with routes that satisfy the requirements of customers and suppliers. To select suitable trucks exactly, our e-logistics management system should be constructed based on a theoretical framework.

- It provides a specification language for describing and analyzing truck routes. The language is aimed at specifying only the routes of trucks formulated as an extended process calculus with the expressiveness of truck routes between collection/delivery points.

- It defines an algebraic order relation over the terms of the language. The relation is defined based on the notion of bisimulation and compares possible truck routes and the routes required by its specifications. This allows us to accurately determine whether the former satisfies the latter.

Note that the order relation is not intended to generate the most efficient route, because truck routes tend to be designed according to external factors. Thus, the computational complexity for this relation is not large. Some readers may think that simple executable languages, such as Lisp and Prolog, should be used to specify routes, but it is difficult to verify whether or not routes written in such languages will satisfy the requirements of customers and suppliers because these languages have many primitives that are not used in describing routes. We explain the reason why our e-logistics management system is constructed as a process calculus-based approach, because itinerary plans, which transporters are obligated to make for their trucks, can be treated as the sequences of destinations that the truck visit like expressions of process calculi. Therefore, we can easily transform itinerary plans for trucks into process calculus-based specification in comparison with other approaches, i.e., logic-based and graph-based specifications.

5 TRUCK SHARING MODEL

This section defines our model for sharing logistic trucks. The model provides a language for specifying about truck routes and a system for selecting the routes that can satisfy the requirements of suppliers/customers. The language consists of two classes. The first is designed to specify truck routes and the
second is designed to specify the routes required by products or customers.

**Definition 1.** The set \( E \) of expressions of the language, ranged over by \( E, E_1, E_2, \ldots \) is defined recursively by the following abstract syntax:

\[
E ::= 0 \mid \ell \mid E_1 \cdot E_2 \mid E_1 + E_2 \mid E_1 \# E_2 \mid E_1 \% E_2 \mid E_1 \& E_2 \mid E^* \]

where \( \ell \) is the set of location names ranged over by \( \ell, \ell_1, \ell_2, \ldots \), and where points correspond to the locations of suppliers and customers. We often omit \( 0 \).

We describe a subset language of \( E \) as \( S \), when eliminating \( E_1 \# E_2, E_1 \% E_2, E_1 \& E_2 \), and \( E^* \) from \( E \). Let \( S, S_1, S_2, \ldots \) be elements of \( S \).

Our e-logistics system assumes that each truck has its own route written in \( S \) and that its driver visits points along the route. Although its semantics is defined in the Appendix, we describe intuitive meaning of the terms is as follows:

- \( 0 \) represents a terminated route.
- \( \ell \) represents that a truck moves to a point called \( \ell \).
- \( E_1 \cdot E_2 \) denotes the sequential composition of two routes \( E_1 \) and \( E_2 \). If the route of \( E_1 \) terminates, then the route of \( E_2 \) follows that of \( E_1 \).
- \( E_1 + E_2 \) represents the route of a truck according to either \( E_1 \) or \( E_2 \), where the selection is done by the truck.
- \( E_1 \# E_2 \) means that a truck itself can go through either \( E_1 \) or \( E_2 \).
- \( E_1 \% E_2 \) means that a truck can follow either \( E_1 \) before \( E_2 \) or \( E_2 \) before \( E_1 \) on its route.
- \( E_1 \& E_2 \) means that two routes, \( E_1 \) and \( E_2 \), may be executed asynchronously.
- \( E^* \) is a transitive closure of \( E \) and means that a truck may move along \( E \) an arbitrary number of times.

where in \( E_1 + E_2 \) the truck can select the \( E_1 \) (or \( E_2 \)) route when the \( E_1 \) route is available. For example, if the \( E_1 \) route is available and the \( E_2 \) route is congested, the truck goes through the \( E_1 \) route. \( E_1 \# E_2 \) means that a truck can go through either \( E_1 \) or \( E_2 \). \( E_1 \% E_2 \) and \( E^* \) are used to specify possible routes.\(^1\)

For example, \( E_1 \# E_2 \) permits the truck to go through one of the \( E_1 \) or \( E_2 \) routes.

We show several basic examples of the language as shown in Fig. 4. The first diagram in Fig. 4 shows the transitions of \( a; b; c; e; d \), the second shows \( a; (b \# c); d; e \) the third shows \( a; (b \% c); d; e \), and the fourth shows \( a; ((b; c) \& d); e \).

We formally defines a system for selecting the routes of trucks that can satisfy the requirements in the Appendix. The system selects trucks according to their routes based on the concept of bisimulation (Milner, 1989). The relation is suitable for selecting a truck for a milk-run operation with a route that satisfies the requirements of suppliers and customers, as specified an inequality \( E \sqsubseteq n S \). The informal meaning of \( E \sqsubseteq n S \) is that \( S \) is included in one of the permissible routes specified in \( E \) and \( n \) corresponds to the number of movements of a truck that can satisfy \( E \). Since the language supports an external selection operator, i.e., \( + \), like other process calculi, our truck selection cannot be defined as simple algebraic relations, e.g., trace-based semantics. We show several basic properties of the order relation below. Let us look at some basic examples.

- \( (a \% b); c \sqsubseteq_3 a; b; c \) where the left-hand-side requires a truck to carry products to \( a \) and \( b \) in an indefinite order and then return to point \( c \); the right-hand-side requires a truck to carry products to three points, \( a, b, \) and \( c \), sequentially. When the right-hand-side is changed to \( b; a; c \), the relation is still preserved, but when the right-hand-side becomes \( c; a; b \) or \( a; c; b \), the relation is not preserved.

- \( (a; b; c) \# (a; c; b; c) \sqsubseteq_3 a; b; c \) where the left-hand-side means that a truck follows one of either \( a; b; c \) or \( b; c; a \). When the right-hand-side becomes \( a; c; b; c \), the relation

\(^1\) \( E^* \) specifies that the truck follows the \( E \) route more than zero times like the notion of Kleene closure. The operator is used to specify that the requirement of a truck’s route permits the truck to visit specified destinations if the truck wants to do this.

![Figure 4: Examples of specification.](image-url)
is not preserved, because \( \equiv_3 \) means that the truck can visit its destinations at most three times. Nevertheless, \( \equiv_4 \) is preserved with \( a; c; b; c \) at its right-hand-side.

- \( ((a; b; c) \# d^*) \); \( d \equiv_6 a; d; b; d; c; d \)

where the left-hand-side allows a truck to drop in at point \( d \) an arbitrary number of times on route \( a; b; c \) and then finish its movement at point \( d \). The right-hand-side is a star-shaped route between three destinations, \( a, b, c \), and point \( d \) satisfies the left-hand-side. When the right-hand-side becomes \( a; b; d; c; d, a; d; b; c; d, \) or \( a; b; c; d \), the relation is preserved, but \( a; b; c; d \) is the most efficient route.

6 E-LOGISTICS MANAGEMENT SYSTEM FOR SHARING TRUCKS

This section describes a prototype implementation of our e-logistics system. The experiment was constructed as a distributed logistics management system consisting of six supplier points in addition to a customer point with a route-selection server. Figure 5 shows the basic structure of the system. The server was responsible for receiving route requirements from suppliers and customers through a network and selecting suitable trucks with routes that satisfied these requirements.

![Figure 5: Basic structure of logistics management system.](image)

6.1 Route Selection Algorithm

Here, let us explain the selection algorithm used for the current implementation, which we tried to make as faithful to Definition 3 as possible. The server maintains its own repository database containing the routes of trucks. To reduce the cost of the selection algorithm, the possible routes written in \( E \) are transformed into tree structures before they are stored in the database. These are called transition trees or derivation trees in the literature on process calculus (Milner, 1989). Each tree is derived from a route in \( E \) according to Definition 2 and consists of arcs corresponding to \( \tau \)-transitions or \( \ell \)-transitions in the route. When a route selection server receives a required route from suppliers or customers, it extracts the required route written in \( S \) and then transforms the route into a transition tree. It next determines whether or not the trees derived from the routes stored in the database system can satisfy the tree derived from the required route by matching the two trees according to the definition of the order relation \( \equiv_\tau \subseteq E \times S \) as in the following:

1. If each node in one of the two trees has an arc corresponding to \( \ell \)-transitions, then the corresponding node in the other tree can have the same arcs, and the sub-nodes derived through the matching arcs of the two trees can still satisfy either (1) or (2).
2. If each node in the tree derived from the required route has one or more arcs corresponding to \( \tau \)-transitions, then at least one of the nodes derived through the arcs and the corresponding node in the tree derived from the truck’s route can still satisfy (1) or (2).
3. If neither (1) nor (2) is satisfied, the route selection server backtracks from the current nodes in the two trees and tries to apply (1) or (2) to their two backtracked nodes.

![Figure 6: Matching transition trees in route-order relation algorithm.](image)
routes in the database satisfy the required route, it selects the truck with the least number of truck movement between points, which is \( n \) of Definition 3. Although the cost of selecting a route is dependent on the number of trucks and the length of their routes, the system can handle each of the routes presented in this paper within a few milliseconds.

Non-deterministic operators, e.g., \( \# \) and \( \& \), tend to cause the exposition of a number of sub-trees in transition trees. Nevertheless, our algorithm can easily restrain the number of sub-trees resulting from non-deterministic operators because the expansion rules of expressions, i.e., the operational semantics of the language, distinguish between derivations resulting from deterministic operators and those resulting from non-deterministic operators. Readers may wonder why \( E^* \) operator creates an infinite number of sub-trees, but the current implementation interprets the operator in a lazy evaluation manner.

We have implemented the e-logistics system on a PaaS cloud computing infrastructure, called Google’ App Engine. Trucks’ routes are maintained in a key-value store, called Bigtable, provided by the infrastructure. When our system receives a truck route from a truck operator, it transforms the required route into a tree structure and matches between the structure and the structures corresponding to trucks’ routes. This means truck operators and suppliers do not need any special equipment to use the logistics management system. This is important because in cooperative logistics, most suppliers are small to medium enterprises that do not want to have to invest in additional equipment for cooperative logistics.

The current implementation assumes that the routes required for products or pallets are stored in

RFID tags attached to the products or pallets because they may have their own delivery requirements. It assumes that each client-side system at a supplier or customer point has more than one RFID tag reader, which periodically or explicitly tries to detect the presence of tags within its coverage area. It supported Phillips i-Code system (13.56 MHz), which provides each tag with 112 bytes. We were able to maintain each of the example routes presented in these papers in the first and second tag systems, where the length of the identifier for each point was 4 bytes. The current implementation of the algorithm was not optimized for performance. Nevertheless, we describe the basic performance of the implementation. By using an RFID reader embedded with a WiFi network interface, the cost of reading the route specification in a tag depends on the length of the specification, e.g., the cost of reading a specification with a length of less than 40 bytes is within 0.2 sec. When the routes of five trucks were registered in the server running on a computer (Intel Core 2 Duo 2 GHz and Windows XP), the cost of selecting a truck after the reader had detected a tag, including the cost of communication between the server and client via a TCP/IP session, was less than 1.2 sec. Client-side systems for suppliers and customers can be operated using only RFID readers, which connect to a server through either wired or wireless networks. This means they do not need any special equipment to use the logistics management system. This is important because in milk-run logistics, most suppliers are small to medium enterprises that do not want to have to invest in additional equipment for milk-run logistics.

7 RELATED WORK

There have been many attempts to use process calculi, e.g., as formal methods for various business enterprise processes. Several researchers have used process calculi, e.g., \( \pi \)-calculus, as business-process modeling languages, such as BPEL, (K. Xu, 2006; M. Mazzara, 2006; F. Puhlmann, 2005; Smith, 2003). \( \pi \)-calculus has been used as a formal composition language for software composition and Web service composition, e.g., Orc (J. Misra, 2004) and SCC (M. Boreale and Zavattaro, 2006). Process calculi are theoretically sound and support bisimulation analysis and model checking. They are also gaining increasing acceptance as a support tool in industry. However, there have been no process-calculus-based formal methods for logistics, in particular for improving the transport efficiency of trucks.
8 CONCLUSIONS

This paper presented an e-logistics system for managing for shared trucks to reduce the environmental impacts of transport operations. The system was formulated based on a process calculus-based language and an order relation over two terms corresponding to truck routes and the required routes in the language. The language could specify truck routes for milk-run operations and the required routes for shipping. The relation could be used to accurately determine whether a truck route satisfies the requirements of customers and suppliers. A prototype implementation system based on the framework was constructed using Java language and RFID tag systems and applied to our experimental distributed logistics management system.

REFERENCES


APPENDIX

To accurately express such routes, we need to define a specification language based on a process calculus approach such as CCS (Milner, 1989). The semantics of the language are defined by the following labeled transition rules:

Definition 2. The language is a labeled transition system \( \{ E, L \cup \{ \tau \} \{ \frac{\alpha}{\cdot} \subseteq E \times E | \alpha \in E \cup \{ \tau \} \} \). The transition relation \( \rightarrow \) is defined by two kinds of axioms or induction rules as given below:

\[
\begin{align*}
E_1 \tau &\rightarrow E_1' \\
E_1 + E_2 &\rightarrow E_1' + E_2' \\
E_1 \& E_2 &\rightarrow E_1' \& E_2' \\
E_1 \# E_2 &\rightarrow E_1' \\
E_1 \% E_2 &\rightarrow E_1' \\
E_1 \& E_2 \tau &\rightarrow E_1' + E_2' \\
E_1 \# E_2 \tau &\rightarrow E_1' + E_2' \\
E_1 \% E_2 \tau &\rightarrow E_1' + E_2' \\
E_1 + E_2 \tau &\rightarrow E_1' + E_2' \\
E_1 \& E_2 \tau &\rightarrow E_1' \& E_2' \\
E_1 \# E_2 \tau &\rightarrow E_1' \# E_2' \\
E_1 \% E_2 \tau &\rightarrow E_1' \% E_2' \\
E_1 + E_2 \tau &\rightarrow E_1' + E_2' \\
E_1 \& E_2 \tau &\rightarrow E_1' \& E_2' \\
E_1 \# E_2 \tau &\rightarrow E_1' \# E_2' \\
E_1 \% E_2 \tau &\rightarrow E_1' \% E_2'
\end{align*}
\]

where \( 0 ; E, E \& 0, \) and \( 0 \% E \) are treated to be syntactically equal to \( E \tau \) and \( E \) is recursively defined as \( 0 \# (E ; E) \). We often abbreviate \( E_0 \tau \rightarrow E_1 \rightarrow \cdots \rightarrow E_{n-1} \rightarrow E_n \) to \( E_0 (\tau)^n E_n \).

In Definition 2, the \( \tau \)-transition defines the semantics of a truck’s movement. For example \( E \rightarrow E' \) means that the truck moves to a point named \( \ell \) and then behaves as \( E' \). Also, if there are two possible transitions \( E \ell_1 \rightarrow E_1 \) and \( E \ell_2 \rightarrow E_2 \) for a truck, the processing by the truck chooses one of the destinations, \( \ell_1 \) or \( \ell_2 \). In contrast, the \( \tau \)-transition corresponds to a non-deterministic choice of a truck’s routes.

Readers may think that the above operational semantics could be more compact. However, the aim is to design a system that can be easily implemented because the purpose of our e-logistics system is not to provide just a theoretical foundation for determining truck-route logistics, but a practical mechanism for selecting suitable trucks for milk-run operations. The language does not needs recursive or loop nota-
tions, because each truck does not continue to run for 24 hours everyday.

- Route specification, \(a; b; c; d\), in \(S\) is interpreted as follows:
  \[
  a; b; c; d \xrightarrow{a} b; c; d \\
  \xrightarrow{b} c; d \\
  \xrightarrow{c} d \\
  \xrightarrow{d}
  \]

The first diagram in Fig. 4 illustrates the above derivation.

- Next, we show an example of a specification in \(E\). This is a route requirement.
  \[
  a; (b \#c); d; e \xrightarrow{a} (b \#c); d; e \\
  \xrightarrow{\tau} b; d; e \text{ or } c; d; e
  \]

where \# corresponds to a combination of two required routes so that trucks are required to follow both routes as shown in the third diagram in Fig. 4. That is, a truck needs to call at point \(a\) and then at either \(b\) or \(c\). Next, it calls at \(d\) and then \(e\).

- We show another route requirement specification, \(a; (b \#c); d; e\), in \(E\). It has two derivations as follows:
  \[
  a; (b \#c); d; e \xrightarrow{a} (b \#c); d; e \\
  \xrightarrow{\tau} b; d; e \text{ or } c; d; e
  \]

where \# means that trucks can take either one of the two routes before they take the other. The second diagram in Fig. 4 shows possible routes that could satisfy this requirement specification.

- The first requirement presented in the previous section is described as specification \((a; (b \#c)) \triangleleft d^* \triangleleft e^*\). We show one of the possible derivations from the specification as follows:
  \[
  (a; (b \#c)) \triangleleft d^* \triangleleft e^* \xrightarrow{a} (b \#c) \triangleleft d^* \triangleleft e^* \\
  \xrightarrow{b} c \triangleleft d^* \triangleleft e^*
  \]

We can also have another derivation from the specification as follows:
  \[
  (a; (b \#c)) \triangleleft d^* \triangleleft e^* \xrightarrow{a} (b \#c) \triangleleft d^* \triangleleft e^* \\
  \xrightarrow{c} b \triangleleft d^* \triangleleft e^*
  \]

where \(E \triangleleft d^*\) means that the truck can visit \(d\) more than zero times while it moves along \(E\). Further, \((a; (b \#c)) \triangleleft d^* \triangleleft e^*\) means that the truck can visit \(d\) more than zero times while it moves along \(E\).

Next we show an algebraic order relation as a theoretical foundation of our selection of trucks.

**Definition 3.** A binary relation \(\mathcal{R}^n (\mathcal{R} \subseteq (E \times S) \times \mathcal{N})\) is an \(n\)-route prebisimulation, where \(\mathcal{N}\) is the set of natural numbers, if whenever \((E, S) \in \mathcal{R}^k\) where \(k \geq 0\), then, the following holds for all \(\ell \in L\) or \(\tau\).

i) if \(E \xrightarrow{\ell} E'\) then there is an \(S '\) such that \(S \xrightarrow{\ell} S '\) and \((E', S') \in \mathcal{R}^{k-1}\)

ii) There is an \(E'\) such that \(E (\xrightarrow{\tau})^{*} E'\) and \((E', S) \in \mathcal{R}^k\)

iii) if \(S \xrightarrow{\ell} S'\) then there exist \(E', E''\) such that \(E (\xrightarrow{\tau})^{*} E'\) and \((E'', S') \in \mathcal{R}^{k-1}\)

where \(E \sqsupseteq_n S\) if there exist some \(n\)-route prebisimulations such that \((E, S) \in \mathcal{R}^n\). We call the \(\sqsupseteq_n \) \(n\)-route order. We often abbreviate \(\sqsupseteq_n \) to \(\sqsupseteq\) where \(n\) is infinite.

\(\square\)