Experimental/FEM Optimization of Medium Voltage Rubber Insulated Electric Cables Vulcanized with Steam Water  
Numerical Simulations and Inverse Analyses

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Abstract: A comprehensive combined numerical model based on Genetic Algorithm (GA) optimization and heat transfer Finite Element computations is presented. The numerical analyses are carried out to evaluate the final crosslinking degree of a medium voltage electric cable subjected to industrial peroxide reticulation. The final task is to minimize the difference between numerically predicted and experimentally determined crosslinking degree along the thickness of the insulator, when a variable steam temperature profile along the pipe length is assumed to explain the unexpected under-vulcanization of the cable in the internal layers. To minimize the gap between experimentally determined curing degree and numerical predictions, a Genetic Algorithm (GA) optimization is used.

1 INTRODUCTION

In a companying paper (Milani and Milani 2015), a real production line 103 meters long has been analyzed from an experimental point of view. Experimental Differential Scanning Calorimetry (DSC) results shown that, at four different vulcanization conditions with steam, the resulting reticulation degree at the end of the production process is sensibly lower than that expected from simplified evaluations based on the knowledge of the half time life of the peroxides used, especially when the steam temperature is low and the exposure time is reduced. The situation is critical in the internal layers.

The present paper is aimed at analyzing the vulcanization process from a numerical point of view, trying to identify the reasons at the base of such an unexpected under-vulcanization of the cable.

The Finite Element analysis of the heat transfer process during rubber vulcanization has long tradition and has been successfully applied by many authors to predict the reticulation degree at the end of several different production processes Lenir (1984) and Kosar and Gomzi (2007).

In this framework, the combined approach proposed in the present paper may be regarded as innovative and beneficial for producers interested in a quantitative evaluation of the level of cure obtained in the production process.

The input parameters optimization is performed by means of an inverse analysis with a non standard meta-heuristic approach based on Genetic Algorithm concepts. The procedure appears particularly appealing instead of the utilization of gradient based routines into a standard least squares minimization on experimental data, because the crosslinking density function at the end of the industrial process is not analytically known.

At fixed input parameters, the determination of the curing level is typically obtained by means of Finite Element simulations.

When FE are used to determine the crosslinking degree of the cable with constant vulcanization temperature, a partial match between numeric predictions and experimental evidences from DSC is found, indicating a clear discrepancy between set parameters and real values. Such result suggests that the steam curing temperature along the pipe length probably decreases, at the same time addressing that one of the key input parameters to optimize in the GA is the variable steam temperature.

Individuals of the GA are represented by the temperatures at different positions of the pipe,
whereas the objective (fitness) function is represent by the sum of the squared difference between numeric prediction and experimental determination of the crosslinking degree. The proposed GA is robust and non-standard, based on a specifically developed zooming strategy which consists in the subdivision of the population at each iteration into two sub-groups, depending on individuals grade of fitness (elitist strategy). Different genetic procedures are applied to the sub-groups, namely both two typologies of admissible mutations for the elite sub-population and mutation and reproduction for the remaining individuals. In order to improve algorithm convergence, a user-defined population percentage, depending on individuals fitness, is replaced with new phenotypes at the end of each iteration, enforcing in this way the chromosomes renewal.

The aim of the numeric approach proposed is not only to fit experimental data through least squares best fitting, but also to suggest a simple and efficient computational tool able to determine the expected level of crosslinking.

2 GOVERNING PARTIAL DIFFERENTIAL EQUATIONS

The real production plant has to be idealized before applying any mathematical model. At this aim, the vulcanization process can be schematically subdivided into two simply phases (Figure 1): (1) heating zone and (2) a cooling zone.

Figure 1: Schematic representation of the vulcanization process of a wire. -a: heating phase. –b: cooling phase.

Index \( j \) indicates the metallic conductor (semi-diameter \( R_j \)) and index \( p \) indicates the insulation width (ray \( R_p \)).

The axial symmetry of cable leads to a two independent variables system: the distance \( r \) of an insulation layer respect to cable axis and exposure time \( t \). At constant cable speed, a cable section at a distance \( z \) with respect to the starting point of the production line, is characterized by an exposure time equal to \( t = z / u_c \). This means \( z \) is a variable dependent from \( t \).

During the heating phase pressurized steam at \( T_n \) temperature is used, exchanging heat with Ethylene-Propylene Diene Monomer Rubber (hereafter abbreviated as EPDM for the sake of clearness) surface mainly by convection. Fourier’s heat equation law in cylindrical coordinates is used to numerically determine temperature profiles along cable thickness Milani and Milani (2008). For the insulation layer the heat balance field equation is the following:

\[
\rho_p c_p \left( \frac{\partial T}{\partial t} \right) - \lambda_p \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + r_p \Delta H_r = 0
\]  

(1)

Where

- \( \rho_p \), \( c_p \) and \( \lambda_p \) are EPDM density, specific heat capacity and heat conductivity respectively;
- \( \Delta H_r \) is the insulation specific heat of reaction;
- \( r_p \) is the rate of crosslinking;

The term \( r_p \Delta H_r \) in equation (1) is the heat produced by the decomposition of the peroxide. \( \Delta H_r \) depends both on type of peroxide used and on type of hydrogen to extract (allylic, vinylic, etc.). For simplicity, we assume a linear behavior for \( r_p \) with respect to concentration, i.e. \( r_p = \frac{dC}{dt} \).

Similar considerations can be repeated for the conductor, obviously assuming \( \Delta H_r = 0 \):
\[ \rho c_p \frac{\partial T}{\partial t} - \lambda_j \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right) = 0 \]  

\[ \text{Where the index } j \text{ refers to the conductor layer.} \]

Since the heat equation is of second order in space, two boundary conditions must be specified. In particular, for the problem at hand, at \( r = 0 \) a symmetry condition on the temperature field is imposed in the well-known form \( \frac{\partial T}{\partial r} = 0 \), whereas at \( r = R_p \) we impose:

\[ \lambda_p \frac{\partial T}{\partial r} + h \left( T(R_p, t) - T_a \right) + q_{\text{rad}} = 0 \]

Where \( h \) is the heat transfer coefficient between EPDM and steam, \( T_a \) is steam temperature and \( q_{\text{rad}} \) is the heat flux transferred by radiation. Here it is worth noting that, in the present case (i.e. for vulcanization with steam), the evaluation of \( q_{\text{rad}} \) would be a rather difficult task. As a matter of fact, the well-known radiation formulas in polar coordinates cannot be applied rigorously since the water vapor participates in the radiation exchange between the tube wall and insulation surface. Since typical values of the convection coefficient are used for the steam condensation, radiation is not included in the model due to the complexity of accurately including the radiation effects.

Finally, at the interface between conductor and insulation, an equilibrium equation on the heat flux exchanged is imposed in the form:

\[ \lambda_j \frac{\partial T}{\partial r}(R_j, t) = \lambda_p \frac{\partial T}{\partial r}(R_p, t) \]

For transient conduction, heat equation is of first order in time, requiring the assumption of an initial temperature distribution:

\[ T(r, 0) = T^0 \quad 0 \leq r < R_j \]
\[ T(r, 0) = T^0 \quad R_j \leq r \leq R_p \]

No differences occur in the cooling zone, except that boundary equation (3) is replaced by a pure convection equation:

\[ \lambda_p \frac{\partial T}{\partial r} + h_w (T(R_p, t) - T_w) = 0 \]

where \( h_w \) is the water heat transfer coefficient and \( T_w \) is the water cooling temperature.

Initial temperature conditions are obtained from the profile evaluated at the last step of the cooling zone, i.e. at \( T(r, t_c) \), where \( t_c = L_c / u_c \) with \( L_c \) curing zone length.

\[ \text{Figure 2: Schematic representation of the numeric procedure adopted to determine final tensile strength of each layer.} \]

### 3 NUMERIC PREDICTIONS UNDER CONSTANT TEMPERATURE PROFILES CONDITIONS

The determination of temperature profiles across cable section does not require inverse analysis optimization, but simply the Finite Elements solution of problem (1)-(5), as illustrated schematically in Figure 2 when a temperature profile for the vulcanization agent (stem), i.e. an individual within a Genetic Algorithm scheme (GA, see after), is considered.

Numerically estimated temperature profiles and residual unreacted peroxy concentrations in one vulcanization conditions experimentally tested is represented in Figure 3.

In subfigure –a the temperature profiles at constant steam temperature are represented. In subfigure –b the reacted peroxy concentration of internal middle and external layers are depicted, whereas a comparison between simulated and experimental data is provided in subfigure -c.

The data show a great diversion between the deduced percentage of unreacted peroxides and the experimental evidences.
Figure 3: Test #1. –a: temperature profiles at water steam constant temperature. -b: evolution of peroxide reaction in three positions of the cable. –c: comparison with experimental data.

It can therefore be concluded that an inverse analysis would be extremely useful to optimize and control the final level of crosslinking of the cable.

4 THE GA APPROACH PROPOSED FOR THE OPTIMIZATION

When the temperature profile of the steam along the line is not known, inverse analyses are needed to determine the profile that allows the best fitting of the final crosslinking level across the cable section. Meta-heuristic approaches are particularly indicated because the analytical function representing the steam temperature variation along the length is unknown.

The meta-heuristic approach utilized is a non-standard and robust GA that has been already used in the same or different contexts by the authors in Milani and Milani (2007, 2008, 2009, 2011).

The core of the GA proposed is a set of standard (reproduction, crossover and mutation) and non-standard (zooming and elitist strategy) genetic procedures. The iterative optimization strategy is schematically shown in Figure 2. Each individual population belonging is represented by admissible temperature $T_{ni}$ of the steam on a series of control nodes $i$ along the length of the vulcanization pipe. The main novel characteristic of the proposed GA consists in the subdivision of the population into two subgroups with improvement of the best fitness individuals with zooming , Milani (2013) ; Kang and Zong (2004) and Haupt and Haupt (2004). An admissible initial population $\mathbf{x} = \{x_i : i = 1,..,N_{ind} \}$ is randomly generated In the in Step 0 at the first iteration. In Step 1, $x_i$ fitness $F(x_i)$ is evaluated solving for each layer a PDEs system with fixed $x_i$. In Step 2, two sub groups are created, namely $\mathbf{x} = \{x_i : i = 1,..,N_{ind} \}$ admissible $\}$ and $\mathbf{y} = \mathbf{x} - \mathbf{\bar{x}} = \{y_i : i = 1,..,N_{ind} - N_{\text{elite}} \}$. $\mathbf{\bar{x}}$ is the group of all the individuals with the $N_{\text{elite}}$ (user defined) higher fitness values. This step represents the zooming strategy. In Step 3a for each $x_i$, a random improvement of the individual (in terms of fitness) is tried using a mutation operator. The recursive double operation applied randomly $N_{mut}$ times, leads to a new individual generation $\mathbf{x}_{IM}$, which overwrites the original $\mathbf{\bar{x}}_i$ only if its fitness $F(x_{IM})$ is greater than $F(x_i)$. At the end of the double loop, a new sub-group $\mathbf{\bar{x}}_{II} = \{x_{II} : i = 1,..,N_{\text{elite}} \}$ $\mathbf{\bar{x}}_{II}$ admissible $\}$ is obtained.

In Step 3b a mutation loop is applied randomly $N_{mut}$ times for each individuals $y_i$ with low fitness, leading to an improvement of $y_i$ fitness. The new individuals $y_{IM}$ overwrite the original $y_i$ only if their fitness is greater than $y_i$ one (elitist approach).
At the end of the double loop, a new sub-group \( \mathbf{y}_M = \{ y_{i \text{ind}} : i = 1, \ldots, N_{\text{ind}} - N_{\text{elit}} \mid y_{i \text{admissible}} \} \) is obtained. A classic reproduction operator is applied only for individuals of \( \mathbf{y}_M \) with high fitness (i.e. on \( (N_{\text{ind}} - N_{\text{elit}})/\psi \) parents with user defined parameter \( \psi > 1 \)) in order to create a new offspring group \( \mathbf{c} \). The remaining \( (1 - \psi)(N_{\text{ind}} - N_{\text{elit}})/\psi \) individuals are generated \textit{ex-novo} using Step 0 procedure and are catalogued into \( \mathbf{c}_N = \{ c_{Nj} : j = 1, \ldots, (N_{\text{ind}} - N_{\text{elit}})/\psi \mid c_{Nj} \text{ admissible} \} \) vector. 

Finally, the last population at the \( i \)-th iteration is collected into \( \mathbf{x} = [\mathbf{x}_M \ \mathbf{c} \ \mathbf{c}_N] \) and the procedure is repeated from the beginning.

5 THE NEW SIMPLIFIED MECHANISTIC MODEL PROPOSED

The same electric cable subjected to the aforementioned experimental tests is supposed to be subjected to a variable steam pressure along the vulcanization pipe to determine. Results obtained for Tests from #1 to #4 are shown from Figure 4 (#1) to Figure 6 (#4).

Results for the best fitness individual at the last GA iteration are reported assuming that the individual best fits experimental DSC data. The temperature profiles with variable steam temperature are represented in subfigure –a (core, middle, skin layers).

In subfigures –b the unreacted peroxide concentration along cable thickness is shown (with a comparison with DSC results), whereas in subfigures –c the GA temperature profile along the length is depicted.

Comparing simulations results it is possible to note that:

1) there is a visible drop of temperature in Test #1, which fully justifies the unsatisfactory crosslinking level obtained during the experimentation. Comparing the numerical results with DSC experimental predictions, it is clear that without this drop the crosslinking should be close to optimal;

2) the minimization fitting function is non convex and may provide multiple solutions. A similar result may be obtained assuming a constant steam temperature along the line. This remark justifies also the utilization of a meta-heuristic approach to deal with the problem at hand. Standard minimization algorithms based on first derivative evaluations may potentially fail in finding the optimal solution.

The application of the GA combined with FEs allows concluding that a drop of steam temperature along the pipe length may be an important factor to justify the unexpected under-vulcanization for certain cure conditions, but such results do not rigorously show conclusively that this is likely the main cause.
Before placing all of the cause for discrepancy between the computed and experimental state of cure profiles on an axial temperature decrease, it is therefore interesting to have an insight into the effect linked to a variation of the different coefficients assumed constant in the FE computations.

In particular, it may be worth exploring the effect of the values of the presumed heat transfer coefficients and the initial temperature of the insulation on the computed solutions on the agreement with the experimental results.

In addition, the state of the steam in the apparatus seems to indicate that the steam injected into the tube is superheated. If this is the case, then the expression for the relation between the temperature and pressure for saturated vapor does not apply. However, if the surface of the insulation is below the saturation temperature of the steam at the inlet pressure of the tube, then steam will immediately condense on the surface of the insulation at that temperature. Since the heat transfer coefficient will be fairly high under these conditions it might be reasonable to assume a constant surface temperature equal to the saturation temperature, that in Test #1 is 202°C for a pressure equal to 16.5 bar. If the surface of the insulation is above this temperature then the mode of heat transfer is simply forced convection with the steam temperature decreasing until the saturation temperature is reached.

In order to take into account in a simplified manner a complex problem of heat exchange, which is unsteady, three additional sets of simulations are performed assuming the following heat transfer coefficients for the steam: $h=30 \text{ W/m}^2 \text{ K}$, $h=300 \text{ W/m}^2 \text{ K}$ and $h$ variable along the length of the pipe, with and optimization of the heat transfer coefficient.
on experimental data by means of the same GA approach previously used with variable steam temperature. Such lower and upper bounds values for $h$ are assumed in agreement with indications provided in Milani et al. (2008).

We assume steam temperature constantly equal to $T_0 = 202^\circ C$ and the total curing time $t_c$ equal to 5.6 minutes, i.e. design conditions of Test #1 are investigated, being the vulcanization level in such case critical and unexpectedly low. Whilst authors are aware that a realistic numerical simulation should take into account the variability of both $T_0$ and $h$, such simulations cannot be performed with the GA approach proposed if a relation between $h$ and $T_0$ is not provided.

However, it is worth underlining that the evaluation of $h$ (especially as a function of $T_0$) is a very difficult task, especially when steam condenses and there is a passage between vapor and liquid phase, due to unknown heat unsteady transfer processes. In addition, the common Newton’s law of heat exchange by convection is probably too simplistic and holds only for forced convection, whereas probably in this case –as already pointed out- there is an unknown dependence of $h$ with the temperature difference between steam and rubber surface.

It is finally worth emphasizing that the values adopted for $h$ in the two sets of simulations with constant $h$ represent large bounds indicated for steam in forced convection in many handbooks and therefore such numerical analyses may well approximate upper and lower bounds.

Temperature profiles obtained assuming lower and upper bound constant values for $h$ are depicted in Figure-a and –b respectively. Furthermore, in Figure-c the same results are represented with a variable $h$ values. The numerical prediction of the unreacted peroxide along the thickness of the cables, with a comparison with experimentally determined values is finally reported in Fig. 9.

As can be noted, when a large value for $h$ is assumed (upper bound) the heat exchange is intuitively favored and the % unreacted peroxide found numerically sensibly deviates from experimental values.

The same applies for an excessively reduced value of $h$ (lower bound). In such a condition, the heat exchange between rubber surface and steam becomes slow and the peroxide reacts with lower velocity. The resultant % of unreacted peroxide is therefore higher than that experimentally determined.

Conversely, result obtained assuming $h$ as variable are in quite good agreement with experimental evidences, see Fig. 9-c. $h$ profile along the tube length determined by means of the GA proposed is represented in Figure 10, with a 3D representation of the unreacted peroxide % (along the thickness and length of the cable). As can be noted, there is a monotonic decrease of $h$, which assumes very high values at the beginning (superheating condition) and then decreases along the line to typical values for steam water convection. While the present simulations are obviously affected by errors induced by the strong simplifications assumed, the results obtained give interesting information on the physical processes occurring to the steam along the line.

It is finally interesting to notice that, when dealing with the initial inlet temperature, GA simulations are performed assuming a value equal to 25°C.
Such value is certainly a lower bound and does not take into account the initial heating phase inside the extruder. Here it is only worth noting that to provide realistic numerical simulations inside an extruder is a very difficult task, involving 3D FE modelling with coupled thermo-mechanical approaches. Authors repeated some numerical simulations adopting a simplified procedure, with an increase of the curing temperature from 25 to 90°C in the first 20 meters (length of the extruder) and then starting the simulations. Results are reported in Figure 10. As authors experienced, however, the concentration of unreacted peroxide at the end of the simulations (Figure 10-b) is very similar to that found with an inlet temperature equal to 25°C.

6 CONCLUSIONS
A GA approach has been proposed to check and to predict the behavior of a real production plant. The general methodology has been validated analyzing experimental evidences at the end of the production process.

The mathematical approach proposed couples the solution of the heat transmission law in cylindrical coordinates with variable steam temperature and the application of a Genetic Algorithm with inverse least squares data fitting to determine the vulcanization conditions of the samples.

Figure 8: Test #1. Temperature profiles obtained with different values of \( h \). –a: lower bound for \( h \) –b: upper bound for \( h \) –c: variable \( h \) (cont.).

Figure 9: Test #1. Unreacted peroxide % along the cable thickness obtained with different values of \( h \). –a: lower bound for \( h \) –b: upper bound for \( h \) –c: variable \( h \).

The drop of the steam temperature depicted by this approach could lead to a suboptimal degree of crosslinking especially near the core of the insulation, where the heat diffusion is lower.
The optimal degree of crosslinking for a well-defined compound could be achieved varying mainly the following production parameters: (1) rate of extrusion, (2) temperature of crosslinking, (3) ratio between heating and cooling area.

Avoid the formation of parts where the steam passes to liquid water.

In order to support and drive the plant manager through such a difficult task, a numerical tool has been developed and it is now available to be used to set up industrial CV lines.

Figure 10: Test #1. Results obtained with \( h \) kept as variable to optimize with the GA approach. –a: normalized peroxide concentration along the thickness and the length of the cable. –b: drop of \( h \) along the length of the cable.

As it has been shown by the numerical simulations provided in a companying paper (Milani and Milani 2015), an accurate experimental estimation of thermo-physical data (pressure and temperature measures) of the steam provided point by point along the pipe length is a key issue to precisely predict the level of vulcanization of the resulting cured item. When dealing with steam vulcanization, indeed, it can occur that pressure measures along the pipe are not strictly sufficient to determine the steam temperature, because it could be superheated. If this is the case, then the expression for the relation between the temperature and pressure for saturated vapor does not apply. This is the reason why pressure measures should always be coupled with temperature evaluations. If such a monitoring system is at disposal to the producers, Finite Elements can accurately predict the level of vulcanization at the end of the production, without the need to provide experimental a-posteriori DSC analyses on selected samples. Conversely, the monitoring system coupled with Finite Elements and GA could be used to optimize the curing apparatus, maximizing the output mechanical properties of the insulator, especially in presence thick items.

REFERENCES