The Symmetry of Oligonucleotide Distance Distributions in the Human Genome

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Abstract: The inter-oligonucleotide distance is defined as the distance to the next occurrence of the same oligonucleotide. In this work, using the inter-oligonucleotide distance concept, we develop new methods to evaluate the lack of homogeneity in symmetric word pairs (pairs of reversed complement oligonucleotides), in equivalent composition groups. We apply the developed methods to the human genome and we conclude that a strong similarity exists between the distance distributions of symmetric oligonucleotides. We also conclude that exceptional distance symmetry is present in several equivalent composition groups, that is, there is a strong lack of homogeneity in the group and a strong homogeneity in the included symmetric word pairs. This suggests a stronger parity rule than Chargaff’s: in the human genome, symmetric oligonucleotides have equivalent occurrence frequency and, additionally, they present similar distance distributions.

1 INTRODUCTION

Chargaff’s first parity rule states that, in any sequence of double-stranded DNA molecules, the total number of complementary nucleotides is exactly equal (%A=|%T and %C=|%G). Clearly, this is an inevitable consequence of the complementarity of opposing nucleotides in the two strands of the DNA molecule. Chargaff’s second parity rule states that %A=|%T and %C=|%G in a single strand of DNA (Forsdyke and Mortimer, 2000). The extensions to second parity rule state that, in each DNA strand, the proportion of an oligonucleotide (a subsequence of adjacent nucleotides) should be similar to that of its reversed complement (the oligonucleotide obtained reversing its letters and interchanging complementary nucleotides). Unlike the first rule, there is no single accepted reason that justifies this single strand symmetry. However, the relative ubiquity of this phenomenon suggests a relationship with genomic evolution (Forsdyke and Mortimer, 2000).

Several works discuss the prevalence of Chargaff’s second parity rules for several oligonucleotide lengths, and in different organisms (Afreixo et al., 2013b; Albrecht-Buehler, 2006; Baisnêe, Hampson and Baldi, 2002). However, the universality of Chargaff’s second parity rule has been questioned for organellar DNA and some viral genomes (Mitchell and Bridge, 2006). Powdel and others (2009) studied the symmetry phenomenon from an interesting new perspective, by defining and analysing the frequency distributions of the local abundance of mono/oligonucleotides along a single strand of DNA. They found that the frequency distributions of reverse complementary mono/oligonucleotides tend to be statistically similar. Afreixo et al. (2014) introduced a new symmetry measure, which emphasizes that the frequency of an oligonucleotide is more similar to the frequency of its reversed complement than to the frequencies of other equivalent composition oligonucleotides.

The inter-nucleotide distances introduced by Nair and Mahalakshmi (2005) convert any DNA sequence into a unique numerical sequence, where each number represents the distance of a symbol to the next occurrence of the same symbol. Afreixo et al. (2009) explored the global inter-nucleotide representation and proposed the extraction of four
sequences, one for each nucleotide, to represent the inter-nucleotide distances. This methodology allows to perform comparative analysis between the behaviour of the four nucleotides. Bastos et al (2011) proposed an inter-dinucleotide distance distribution and compared the distance distributions of all dinucleotides in the human genome. Moreover, evolutionary patterns have been recognized from information contained in the distance distributions of the genomes of different organisms (Afreixo et al., 2009).

In this work we explore the symmetry of distance distributions by comparing each inter-oligonucleotide distance distribution to the distance distribution of its reversed complement, using homogeneity discrepancy measures. We also characterize the discrepancy in equivalent composition groups (ECGs), and compare ECGs results for different oligonucleotide lengths.

We focus our study in the human genome as an example of a typical genome exhibiting single strand symmetry.

2 MATERIALS AND METHODS

2.1 Materials

We analyse the whole human genome, reference assembly build 37.3, available from the website of the National Center for Biotechnology Information (http://www.ncbi.nlm.nih.gov/), discarding all ambiguous or non-sequenced nucleotides from the analysis, that is, all non-ACGT symbols.

In our data processing, the chromosomes of the human genome were processed as separate sequences, words were counted with overlap and non-ACGT symbols were considered as sequence separators.

2.2 Methods

Let \( A \) be the alphabet formed by the four nucleotides \{A, C, G, T\} and let \( s = s_1, s_2, ..., s_N \) be a symbolic sequence defined in \( A \). A genomic word, or oligonucleotide, \( w \), is a sequence of length \( k \).

Assuming that the sequence is read through a sliding window of length \( k \), we can define the inter-oligonucleotide (inter-\( w \)) distance sequence, \( d^w \), as the sequence of differences between the positions of the first symbol of consecutive occurrences of that oligonucleotide. For instance, in the DNA segment \( s = AAAAGTCGATTCGTCCGGG \), the inter-CG distance sequence is \( d^{CG} = (3, 5, 4, 2) \).

The inter-\( w \) distance distribution (or word distance distribution), denoted as \( f_w \), gives the relative frequency of each inter-\( w \) distance. For each \( k \), there are \( 4^k \) distance distributions.

The reversed complement of a genomic word is a sequence obtained by reversing the order of the letters in the word, interchanging A and T and interchanging C and G. For instance, the reversed complement of ACTGG is CCAGT. A symmetric word pair is defined as the set composed by one word, \( w \), and the corresponding reversed complement word, \( w' \), with \( w'' = w \) (Afreixo, Garcia and Rodrigues, 2013a; Afreixo et al., 2014).

In this work we compare the inter-\( w \) distance distribution, \( f_w \), of symmetric word pairs. To the set formed by the distance distributions of symmetric word pairs we will call complementary distributions.

An equivalent composition group (ECG), of words with length \( k \), is a set composed by all the words with the same total number of As or Ts. For instance, the four dinucleotides AA, AT, TA and TT form an ECG. For words of length \( k \) there are \( k + 1 \) equivalent composition groups and the group formed by words comprising \( m \) As or Ts is denoted as \( G_m \), with \( 0 \leq m \leq k \). The number of words of length \( k \) in \( G_m \) is given by

\[
\#G_m = \binom{k}{m} 2^k
\]

Every symmetric word pair is a subset of an ECG, which contains several distinct symmetric word pairs (Afreixo, Bastos and Rodrigues, 2014).

We will call equivalent composition distributions to the distance distributions of words in the same ECG.

Under the second parity rule, and under a scenario of nucleotide independence, it is expected that reversed complements have similar frequency and similar inter-distance distribution (homogeneous distributions), but so do all words in the same ECG.

The similarity between the frequencies of occurrence of reversed complements and of other equivalent composition words is described by Afreixo et al. (2014).

We assess homogeneity in symmetric word pairs and in ECGs, using the distance distributions of words of length up to five, in the complete human genome.

Using empirical data from the contingency table, whose columns are filled with the absolute
frequency of inter-\(w\) distances of a set, \(S\), of words, we find the expected frequency of each distance for each word (dividing the product of the row total and the column total by the total sum). The chi-squared statistic is defined as

\[
X^2 = \sum_{i,j} \frac{(n_{ij} - e_{ij})^2}{e_{ij}},
\]

where \(n_{ij}\) is the observed frequency count in word \(i\) for distance \(j\), and \(e_{ij}\) is the corresponding expected frequency, in the homogeneity context.

### 2.2.1 Symmetric Word Pair Measures

To evaluate the dissimilarity between the inter-words distributions of symmetric word pairs (\(w\) and \(w'\)) we use an effect size measure based on a chi-square statistic to measure the discrepancy between the distance distributions of reversed complement words: the phi coefficient given by

\[
\phi_{w,w'} = \sqrt{\frac{X_{w,w'}^2}{N^w + N^{w'}}},
\]

where \(N^w\) and \(N^{w'}\) are the number of occurrences of \(w\) and \(w'\) in the sequence, respectively. Equal distributions will result in \(\phi_{w,w'} = 0\) and an increase in dissimilarity will be reflected in an increased \(\phi_{w,w'}\).

For interpreting the phi coefficient, we consider a value above 0.10 as a descriptor for small effect size, above 0.30 for medium effect size, above 0.50 for large effect size (Cohen, 1988), above 0.60 for strong effect size and above 0.80 for a very strong effect size (Rea and Parker, 1992).

We define the weighted distribution of the complementary distributions \((w,w')\), denoted as \(f_{w,w'}\), the following distribution

\[
f_{w,w'} = \frac{f_w \times N^w + f_{w'} \times N^{w'}}{N^w + N^{w'}},
\]

and the distance corresponding to the 99\(^{th}\) percentile of the weighted distribution of the symmetric pair is denoted as \(d_{0.99}^{w,w'}\).

### 2.2.2 ECG Measures

We define an ECG distribution profile as the weighted distribution of the equivalent composition distributions. The \(G_m\) distribution, denoted as \(f_{G_m}\), is given by

\[
f_{G_m} = \sum_{w \in G_m} \frac{f_w \times N^w}{n_{G_m}},
\]

where \(n_{G_m}\) is the total number of occurrences of words that belong to \(G_m\), in the sequence. The 99\(^{th}\) percentile of this weighted distribution is denoted as \(d_{0.99}^{G_m}\).

Since different ECGs may contain distinct numbers of elements, to evaluate the dissimilarity between the inter-word distance distributions of each ECG we use the Cramér’s \(V\) coefficient given by

\[
V_{G_m} = \frac{\Phi_{G_m}}{\sqrt{\#G_m - 1}},
\]

which takes into account the degrees of freedom of the chi-square distribution (under the homogeneity hypothesis) to normalise the phi coefficient.
distributions of the elements in $S$ because that leads to a low mean squared error unbiased estimate of the cutoff point under the null hypothesis assumption (homogeneity in $S$).

Since the structure of words with overlap (words with a suffix that matches with one proper prefix of the word) prevents some short distances from occurring, we also expected a large variability, in the first $k$ distances, between distributions of the same ECG. Therefore, we also explore the similarity between the equivalent composition distributions, excluding the first $k$ distances of the empirical distributions in the calculations of $X_{gm}^2$ and $V_{gm}$.

3.1 Inter-word Distance Analysis for Symmetric Pairs

For each word length (from 1 to 5), we use the inter-$w$ distance distributions to build dendrograms that show hierarchical clusters. The inter-word distance distributions in the same cluster are more similar to each other than to those in other groups.

We use the complete linkage clustering and the average linkage clustering to build the dendrograms, and compute the similarity matrix with the Euclidean distance. We performed several cluster analysis varying the dimension of the similarity matrix.

![Figure 1: Box plots of the: maximum recorded distance of each distribution, $d_{w_{max}}^w$ (top); the 99th percentile of each distribution, $d_{99_{w99}}^w$ (bottom). Organized by word length.](image)

To compute the cluster analysis of all the distance distributions, of words of the same length, we had to define a cut point in the distributions. We use the maximum 99th percentile of the ECG distributions and consider a residual class containing the remaining distances.

Since some distances from 1 to $k$ may be absent due to the structure of the words, we also perform the cluster analysis removing the first $k$ distances and normalizing the distributions.

In all the obtained dendrograms, we observe that the first similarity levels are formed by complementary distributions. This indicates that inter-word distance distributions of symmetric word pairs are the most similar, over all the words of the same length. Figure 3 shows one dendrogram of distance distributions of trinucleotides using distances from 1 to the maximum of the 99th percentile of the ECG distributions and a residual class.

These results motivated us to compare and evaluate the similarity between the inter-word distance distributions of symmetric word pairs. Thus, we compute the phi coefficient, $\varphi_{w,w'}$, and sort the symmetric pairs according to that value.

In general, we obtained very low values of phi. Table 1 presents the maximum recorded phi for each word length. We found that, for $1 \leq k \leq 4$, all the symmetric pairs have low values of $\varphi_{w,w}$, meaning similarity between the complementary distributions. However, for $k = 5$, we detected 16 symmetric pairs with medium effect size ($0.3 \leq \varphi_{w,w'} < 0.5$), 2 symmetric pairs with large effect size ($0.5 \leq \varphi_{w,w'} < 0.6$) and 1 pair with strong effect size ($0.6 \leq \varphi_{w,w'} < 0.8$). All of these distance distributions belong to oligonucleotides comprising one or more CGs.

Another result that stands out for $k \geq 3$, is that the distributions that reach the highest values of phi coefficient are always distributions of CG-rich
words (i.e., oligonucleotides comprising one or more CG). On the other hand, the distributions that reach the lowest values of phi coefficient are distributions of words rich in Ts or As. Table 2 displays the symmetric word pairs whose distance distributions have the 6 highest and the 6 lowest \( \varphi_{w, w'} \), organized by word length.

Table 1: Maximum and 90th percentile of phi coefficient.

<table>
<thead>
<tr>
<th>k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>max(( \varphi_{w, w'} ))</td>
<td>0.001</td>
<td>0.001</td>
<td>0.016</td>
<td>0.094</td>
<td>0.662</td>
</tr>
<tr>
<td>90th percentile of ( \varphi_{w, w'} )</td>
<td>0.001</td>
<td>0.001</td>
<td>0.008</td>
<td>0.055</td>
<td>0.019</td>
</tr>
</tbody>
</table>

Table 2: Symmetric word pairs with the 6 highest and the 6 lowest effect size.

<table>
<thead>
<tr>
<th>( \varphi_{w, w'} )</th>
<th>Word length</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>max1</td>
<td>A/T</td>
<td>AC/GT</td>
<td>CGC/GCG</td>
<td>GTAC/GTGC</td>
<td>CGACG/GTCG</td>
<td></td>
</tr>
<tr>
<td>max2</td>
<td>AA/TT</td>
<td>CGA/TCG</td>
<td>GCTA/TAGC</td>
<td>AGCG/CCGG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>max3</td>
<td>CA/TG</td>
<td>CCG/GCG</td>
<td>ACCG/CGGT</td>
<td>CGCGA/TCGCG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>max4</td>
<td>ACG/CGT</td>
<td>GC/GCA</td>
<td>CGC/GCG</td>
<td>CGCG/GTCG</td>
<td></td>
<td></td>
</tr>
<tr>
<td>max5</td>
<td>GAC/GTC</td>
<td>GCTG/GACG</td>
<td>GTAC/GTACG</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>max6</td>
<td>GCC/GGC</td>
<td>CGCA/TGCG</td>
<td>CGCG/CGCG</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min6</td>
<td>CAG/CTG</td>
<td>AAAT/ATT</td>
<td>AAAAT/ATTT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min5</td>
<td>TAA/TTA</td>
<td>GAAA/TTTC</td>
<td>AGAAA/TTTCT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min4</td>
<td>AGAT/CTCT</td>
<td>TAAA/TTTA</td>
<td>AAATA/ATT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min3</td>
<td>CC/GG</td>
<td>TCA/TGA</td>
<td>AGAA/TTCT</td>
<td>TAAAA/TTTT</td>
<td></td>
<td></td>
</tr>
<tr>
<td>min2</td>
<td>GA/TC</td>
<td>AAA/TTT</td>
<td>AAT/ATT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>min1</td>
<td>C/G</td>
<td>AG/CT</td>
<td>AAT/ATT</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The similarity between the complementary distributions is clearly observable in histograms. An extraordinary observation that comes out of this study is the conservation of the similarity in the unexpected spikes of the symmetric distributions, for the generality of the symmetric pairs. Figure 4 displays three word distance distributions of symmetric pairs (the first 150 distances). The similarity between the distributions of symmetric pairs is remarkable even when the distributions are so irregular as those of GCTA/TAGC or ATCAC/GTGAT. All of these cases present negligible effect sizes.

Figure 4: Inter-word distance distributions of the first 150 distances for symmetric pairs, in log-scale: AAAA vs TTTT, \( \varphi_{AAAATTTT} \approx 0.003 \) (top); GCTA vs TAGC, \( \varphi_{GCTATTAGC} \approx 0.011 \) (middle); ATCAC vs GTGAT, \( \varphi_{ATCAGTGTAG} \approx 0.03 \) (bottom).

3.2 Inter-word Distance Analysis for ECG

To find the ECG groups with stronger exceptional symmetry, we compute the Cramér’s V values obtained for each group of equivalent composition distributions.

As already mentioned, to compute \( V_{gm} \), we set a distance cut point and create a residual class with the remaining distances. The \( V_{gm} \) is calculated for distances from 1 to \( d_{0.99}^{gm} \). Since the word structure of some words prevents some distances from 1 to \( k \) from occurring, because of the word overlap, we
also explore the similarity between the equivalent composition distributions, excluding the first $k$ distances of the empirical distributions.

For nucleotides, $k = 1$, we conclude that there is no significant dissimilarity between the distance distributions in each ECG. In fact, those equivalent composition distributions get effect size values much less than 0.1.

Considering all distances up to the cut point and a residual class, we observe that, for $k > 1$, the minimum effect size is associated to $G_{k-1}$. Moreover, $G_k$ tends to reach one of the highest effect sizes (see Table 3a).

With the removal of the first $k$ distances, and due to the non-existence of some distances in a few distributions in the same ECG, we expect a decrease in the effect sizes. In fact, for $k > 1$, this decrease occurs and the existence of homogeneity in $G_{k-1}$ holds true. Moreover, the most homogeneous ECG is $G_k$, that is, the group of words comprising only As and Ts.

The lack of exceptional symmetry in $G_k$, with the removal of the first distances, may be related to the extraordinary spikes that poly-A and poly-T distributions reach at distance one (see Figure 4, top). For $k \geq 4$, some groups present strong $\varphi_G$

Table 3: Cramér’s V effect size of each ECG, organized by word length. (a) distances from 1 to $d_{0.99}$, with a residual class. (b) distances from $k+1$ to $d_{0.99}$, with a residual class.

(a)

<table>
<thead>
<tr>
<th>ECG</th>
<th>Word length</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_0$</td>
<td>0.0003</td>
<td>0.31</td>
<td>0.23</td>
<td>0.17</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>$G_1$</td>
<td>0.0003</td>
<td>0.31</td>
<td>0.23</td>
<td>0.17</td>
<td>0.14</td>
<td></td>
</tr>
<tr>
<td>$G_2$</td>
<td>--</td>
<td>0.29</td>
<td>0.07</td>
<td>0.07</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>$G_3$</td>
<td>--</td>
<td>--</td>
<td>0.24</td>
<td>0.04</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$G_4$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.20</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>$G_5$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.16</td>
<td></td>
</tr>
</tbody>
</table>

(b)

<table>
<thead>
<tr>
<th>ECG</th>
<th>Word length</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_0$</td>
<td>0.0004</td>
<td>0.24</td>
<td>0.17</td>
<td>0.12</td>
<td>0.10</td>
<td></td>
</tr>
<tr>
<td>$G_1$</td>
<td>0.0007</td>
<td>0.06</td>
<td>0.12</td>
<td>0.08</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>$G_2$</td>
<td>--</td>
<td>0.06</td>
<td>0.04</td>
<td>0.06</td>
<td>0.04</td>
<td></td>
</tr>
<tr>
<td>$G_3$</td>
<td>--</td>
<td>--</td>
<td>0.05</td>
<td>0.03</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>$G_4$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.05</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>$G_5$</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>0.05</td>
<td></td>
</tr>
</tbody>
</table>

In general, ECG discrepancies (Table 3a) are higher than symmetric pair discrepancies (Table 2), suggesting the existence of an exceptional symmetry of distance distributions.

Figure 5 displays the equivalent composition distributions of trinucleotides in $G_0$ and in $G_3$. In $G_0$ the irregularity of the distributions is clearly visible in the first 100 distances. Furthermore, for distances higher than 500, we also observe a huge divergence between two groups of distributions. The combination of these behaviours results in a dissimilarity between the distance distributions related to this ECG. In Figure 5 (bottom) we observe that the distributions have a more homogeneous behaviour, which results in a smaller Cramér’s V effect size (Table 3).

To evaluate the variability inside each ECG we use the standard deviation of the Euclidean distance between the word distribution and its ECG profile. The Euclidian distance was computed considering distances from $k+1$ to $d_{0.99}$ and a residual class. We conclude that, in general, $G_0$ is the ECG with one of the highest variations (Table 4). The only exception is verified for $k = 3$, in which $G_0$ presents the lowest dispersion. We also observe that $G_1$ reaches one of the highest variations.

We extend our study to the evaluation of the ECG weighted distribution as a profile of the inter-
word distance distributions. For each word \( w \), we want to analyse if the most similar ECG distribution, in relation to \( f_w \), is the \( G_w \) distribution.

Let \( G_w \) denote the ECG of the word \( w \) and \( \bar{G}_w \) denote any of the other ECGs. To assess the similarity between the word distribution, \( f_w \), and each of the ECG weighted distributions, we compute the Euclidean distance between \( f_w \) and \( f_{\bar{G}_w} \). For \( i = 0, \ldots, k \), considering word distances from \( k + 1 \) to \( d_{0.99}^w \) and a residual class. Then, we sort the Euclidean distances and extract the ECG distribution most similar to \( f_w \).

We found that the lowest Euclidean distance is not always associated to the \( G_w \) distribution, meaning that the most similar ECG weighted distribution in relation to \( f_w \) is not always \( G_w \). For example, only 38% of tetranucleotides have distance distributions closer to \( G_w \) distribution than to any of the other ECGs, and all the distance distributions of tetranucleotides in \( G_1 \) are closer to some \( G_i \) distribution than to the \( G_{10} \) distribution. Table 5 summarizes the percentage of distance distributions, \( f_w \), that are closest to \( G_w \) distribution (than to \( G_{\bar{w}} \)), over all \( k \)-mer distributions. It also presents this percentage, over all the equivalent composition distributions.

These results may give evidence that, even inside an ECG group, the words could not follow the same profile, which is agreement with the exceptional distance symmetry of some ECG. As an example, recall the distance distributions of trinucleotides inside \( G_0 \) (Figure 5, top), which suggest the existence of two distinct distribution profiles. These results are in agreement with previously related exceptional distance symmetry of some ECG, and with the hierarchical clustering performed in subsection 3.1, where distributions of words in the same ECG were not grouped in the same cluster.

To assess similarities between the ECG weighted distributions (the ECG profiles), we build dendrograms for each word length. We used the complete linkage clustering and the average linkage clustering to build the dendrograms, and we computed the similarity matrix with the Euclidian distance. To perform the hierarchical clustering we set a cutoff in all the distributions and create a residual class. To ensure an adequate representation of all the ECG we define the distance cut point at the maximum of the 99th percentile of the ECG profiles, that is, \( \max \{ d_{0.99}^{G_i} : i = 0, \ldots, k \} \).

We observe that, for \( k \leq 4 \), the \( G_k \) and the \( G_0 \) profile distributions are grouped in the same cluster. Moreover, for \( k > 2 \), the \( G_{k-1} \) and the \( G_1 \) profile distributions are also grouped in the same cluster. Figure 6 display some of the obtained dendrograms.

Table 4: Standard deviation of the Euclidean distance between the word distance distribution and its ECG distribution.

<table>
<thead>
<tr>
<th>ECG</th>
<th>( k = 1 )</th>
<th>( k = 2 )</th>
<th>( k = 3 )</th>
<th>( k = 4 )</th>
<th>( k = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( G_0 )</td>
<td>3.70E-08</td>
<td>0.029</td>
<td>0.004</td>
<td>0.017</td>
<td>0.017</td>
</tr>
<tr>
<td>( G_1 )</td>
<td>1.25E-07</td>
<td>0.011</td>
<td>0.017</td>
<td>0.010</td>
<td>0.023</td>
</tr>
<tr>
<td>( G_2 )</td>
<td>0.001</td>
<td>0.006</td>
<td>0.010</td>
<td>0.014</td>
<td></td>
</tr>
<tr>
<td>( G_3 )</td>
<td>0.006</td>
<td>0.007</td>
<td>0.012</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( G_4 )</td>
<td>0.008</td>
<td>0.010</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( G_5 )</td>
<td>0.008</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 5: Percentage of words of length \( k \), %\( G_i \), whose distance distribution is closest to the \( G_i \) distribution than to any of the other ECGs. Percentage of words in each ECG, %\( G_i \), whose distance distribution is closest to the \( G_i \) distribution than to \( G_w \), over all \( k \)-mer distributions.

<table>
<thead>
<tr>
<th>( G_i )</th>
<th>( k = 1 )</th>
<th>( k = 2 )</th>
<th>( k = 3 )</th>
<th>( k = 4 )</th>
<th>( k = 5 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>%( G_0 )</td>
<td>100</td>
<td>100</td>
<td>50</td>
<td>69</td>
<td>65</td>
</tr>
<tr>
<td>%( G_1 )</td>
<td>100</td>
<td>75</td>
<td>42</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>%( G_2 )</td>
<td>100</td>
<td>75</td>
<td>43</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>%( G_3 )</td>
<td>100</td>
<td>50</td>
<td>42</td>
<td></td>
<td></td>
</tr>
<tr>
<td>%( G_4 )</td>
<td>88</td>
<td>53</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>%( G_5 )</td>
<td>75</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4 CONCLUSIONS

In this work, we contribute with a new method to evaluate one refinement of Chargaff’s second parity rules: symmetry of word distance distributions. For each word length, we propose measures of symmetry in symmetric word pairs based on the comparison of the inter word distance distributions. We also compare the homogeneity of symmetric words with the homogeneity inside an ECG. In general, we conclude that the lack of homogeneity between symmetric words is negligible. In some ECGs the discrepancy in word distance distributions is negligible but in other ECGs it is very strong. These results led us to identify the exceptional words in the context of the symmetry of distance distributions: mostly CG-rich words.
Figure 6: Dendrogram using Euclidean distance and complete linkage clustering for ECG weighted distributions, distances from 1 to the maximum of the 99th percentile of the ECG distributions and a residual class.

(a) $k = 2$; (b) $k = 3$; (c) $k = 4$; (b) $k = 5$.

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