Coherence Net

A New Model of Generative Cognition

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Abstract: We propose a new algorithm and formal description of generative cognition in terms of the multi-label bag-of-words paradigm. The algorithm, Coherence Net, takes its inspiration from evolutionary strategies, genetic programming, and neural networks. We approach generative cognition in spatial reasoning as the decompression of images that were compressed into lossy feature sets, namely, conditional probabilities of labels. We show that the globally parallel and locally serial optimization technique described by Coherence Net is better at accurately generating contextually coherent subsections of the original compressed images than a competitive, purely serial model from the literature: Coherencer.

1 INTRODUCTION

Generative cognition has been implicated in a broad range of cognitive faculties, including but not limited to imagination, episodic memory, and spatial navigation (Vertolli & Davies, 2013; 2014). By generative cognition, we mean the production of new output from a given data set that is not explicitly stored in the data set. As a cognitively salient example, the hippocampus’s ability to anticipate new objects and spatial relations from those that are remembered would fall under the class of generative cognition (Mulally & Maguire, 2013). When an individual imagines a new scene on the basis of an environmental trigger with elements that were not explicitly encoded in memory, this would also qualify. It can be viewed as a form of decompression where data lost in the compression phase is deduced from implicit relations present in the compressed data. It is also distinct from creativity in that the result can be mundane.

We chose to approach this problem using a multi-label, bag-of-words approach (for review of the multi-label literature, see Zhang & Zhou, 2013). In place of documents and words, we modeled a visual task using images and their associated pixels. Each image has labels associated with pixel clusters that indicate objects in the image. Since a given image can have many objects, each image is given a collection of labels instead of one, hence multi-label.

Unlike the standard bag-of-words approach, we are not interested in the labeling process. We assume images and their corresponding pixel clusters have been correctly labeled. We derive the images and associated labels from the Peekaboom database of labeled images (Von Ahn, Liu, & Blum, 2006). This database is one of the most extensive in the literature, with over 50,000 manually labeled images. Instead of the standard classification or labeling task, we are interested in using associations between the labels to select a collection that could be used to generate a plausible new image instance.

According to Zhang and Zhou’s (2013) formal description of the multi-label task, generative cognition and classifier tasks are the inverse of one another.

2 FORMAL DEFINITION

Zhang and Zhou (2013) describe the multi-label task in the following way. Let $X$ denote the input space (e.g., images, documents) and $Y$ denote the label space of all possible labels. The standard task is to learn a function $h(\cdot)$ that takes as input some member $x_i$ from the input space and returns some combination of labels from the label space as output (i.e., $h : X \rightarrow 2^Y$). We learn this function from the multi-label training set $D$, where all $\epsilon$ training examples are described in terms of their input features and corresponding label sets (i.e., $D = \{(x_i, y_i) | 1 \leq i \leq \epsilon\}$). Note that $x_i$ is a
\(d\)-dimensional feature vector \((x_{11}, x_{12}, \ldots, x_{1d})\), where each feature corresponds to a single dimension in the input space. In the standard classification task we want to find the function \(h(\cdot)\), called the classifier, in order to predict the correct labels that go with an as yet unseen input \(x\).

By contrast, the generative cognition task is to find a function \(g(\cdot)\) that is the inverse of \(h(\cdot)\) (i.e., \(g : 2^Y \rightarrow X\)). This means that it takes as input a label or set of labels and outputs one of the original input instances (e.g., image, document). We propose to preliminarily achieve this through the parallel task of finding a set of labels \(Y_n\) \((g_1 : 2^Y \rightarrow 2^n)\) where \(n\) indexes the current iteration of the algorithm—that extend the input \(y\) and together indicate some instance \(x_1\). We can think of the generation task as finding a function \(g_1(\cdot)\), called the generator, that finds a subset of labels that would be picked by an accurate classifier \(h(\cdot)\) for some instance \(x_1\) in the input space \(X\). Formally, this means

\[
3x_1, (h(x_1) \supseteq g_1(y_1)) \tag{1}
\]

We can take the manual labeling of the Peekaboom database as another function \((g_2 : 2^Y \rightarrow X)\) that maps a set of labels \(Y_n\) to the subsets of features that indicated them (e.g., for the label ‘dog,’ a given collection of pixels that look like a dog). Thus, the composition of \(g_1\) and \(g_2\) meets the requirements of the given task (i.e., \(g = g_2 \circ g_1\)).

Models of both the generative and classifier tasks return a real-valued function \(f(\cdot, \cdot)\) that takes an instance-label pair \((x, y)\) as input and outputs a number denoting the confidence that the label is accurate for that instance (i.e., \(f : X \times Y \rightarrow \mathbb{R}\)). However, in the generative case, we assess the confidence of a label \(y\) relative to the current potential set of labels \(Y_n\). If we think of \(Y_n\) as a hypothetical instance \(x\), then we get a modified version of \(f(\cdot, \cdot)\)’s input, or \((x \equiv Y_n, y)\), for the model of the generator. For both the generator and the classifier tasks, \(f(\cdot, \cdot)\) should output a larger confidence value on a relevant label \(y'\) than an irrelevant label \(y''\) for a given instance or hypothetical instance \(x\), or \(f(x, y') > f(x, y'')\) (Zhang and Zhou, 2013). The multi-label classifier \(h(\cdot)\) and generator \(g_1(\cdot)\) can then be derived from \(f(\cdot, \cdot)\) by incorporating a thresholding function that determines how large the confidence needs to be for a label to be considered accurate for a given instance (i.e., \(t : X \rightarrow \mathbb{R}\)). We then get the output, \(h(x)\) or \(g_1(y)\), by assessing for a given instance or hypothetical instance \(x\) whether each possible label \(y\) passes the threshold given by \(t(x)\). Formally, this means

\[
h(x) = \{y | f(x, y) > t(x), y \in Y\} \tag{2}
\]
\[
g_1(y) = \{y | f(Y_n, y) > t(Y_n), y \in Y\} \tag{3}
\]

In effect, both functions use \(t(\cdot)\) to dichotomize \(Y\) into relevant and irrelevant label sets (Zhang and Zhou, 2013). However, the generative \(t(\cdot)\) is slightly more complex as the hypothetical \(x \equiv Y_n\) changes with each iteration of the algorithm.

Though standard machine learning techniques can accurately resolve the standard multi-label problem, the generative problem suggests a different approach. At minimum, since there are often many possible \(x_1\) that might satisfy equation 1 and we are not interested in one \(x_1\) over any other, many of the standard techniques are more thorough than is really required by the task. Thus, the fact that \(g \sim h^{-1}\) should not necessarily suggest that simply reversing the directionality of one of the standard techniques is a good solution; though, some researchers have effectively taken this approach in similar domains (see, for example, Hinton, Osindero, & Teh, 2006).

Vertolli and Davies (2013; 2014), by contrast, showed that generative cognition is amenable to heuristic optimization techniques and, thus, we turn to this approach in order to address this task.

We will describe a new, more cognitively plausible heuristic optimization algorithm called CoherenceNet. We will then test how this algorithm performs against a competitive, serial, local hill searching algorithm called Coherent that has been shown to be competitive in the literature (Vertolli & Davies, 2013; 2014).

3 TASK DESCRIPTION

In Vertolli and Davies (2013; 2014), the task is to imagine a fleshed-out scene from a single word query. The model is given a query label (e.g., “car”) with which to generate a collection of other labels (e.g., “road” and “sky”). This collection needs to be semantically coherent: the retrieved labels must belong together. For example, a scene containing “bow,” “violin,” and “arrow” would be incoherent because it mixes two senses of the meaning “bow.”

Supported by cognitive limitations in working memory, Vertolli and Davies restrict these collections to 5 labels, including the query. We continue in this tradition as it provides a simpler, preliminary evaluation than dealing with larger label sets or sets of mixed sizes.

The algorithms select the four other labels by their conditional probability \(P : Y \times Y \rightarrow \mathbb{R}\), which they approximate from the conditional relative
frequency of pairs of labels. Mainly, for labels $a$ and $b$, the conditional relative frequency is the total number of image instances that contain both labels $S = \{(x_i, Y_i) \in D | a, b \in Y_i\}$ divided by the total number of image instances that contain the given label $B = \{(x_i, Y_i) \in D | b \in Y_i\}$. Formally, this means

$$P(a|b) \equiv \frac{|S|}{|B|} \quad (4)$$

One important property of this formalization is that it is non-commutative (i.e., $P$ yields a different value for $a-b$ than it does for $b-a$). Parallel research on co-occurrence in the machine learning literature suggests that this is more realistic and that most models do not account for it (see Huang, Yu & Zhou, 2012; Zhang & Zhou, 2013).

We describe the cognitive generation task as a decomposition step in a compression-decompression sequence (see Vertolli, Kelly, & Davies, 2014), with the classification task as the related compression task: That is, we assume that, after the initial processing required to derive the labels $Y$, the original instances $X$ are no longer explicitly accessible. They have been reduced to the triples $CP = \{(a, b, P(a|b)) | a, b \in Y, a \neq b\}$ in the memory of the agent or model. Thus, the condition from equation 1, with the modifier that $|g_1(y)| = 5$, is not trivial: $g_1(\cdot) must, from a single input label, output a potential instance on the basis of $f(\cdot), t(\cdot)$, and $CP$.

In summary, the current task requires the generative decomposition of conditional probabilities into a contextually coherent, 5-label combination on the basis of a query label that is included in the set. The context is accurately reproduced if at least one of the original images contains the same 5-label combination produced by the agent. If none of the original images contain the label combination, we assume the context is not coherent.

We hypothesize that our software agent, called Coherence Net, will outperform Coherencer (a competing software agent, described below) by capturing the best of both serial and parallel functionality described in Vertolli and Davies (2013; 2014) and Thagard (2000). Coherence Net effectively explores a larger portion of the search space with a decreased chance of getting stuck on local optima by capitalizing on a global, parallel architecture with local serial transitions.

## 4 IMPLEMENTATION

We proceed by giving a formal outline of Coherencer following Vertolli and Davies (2013; 2014) and a description of Coherence Net.

Coherencer is the visual coherence subsystem of the SOILIE imagination architecture (Breault, Ouellet, Somers, & Davies, 2013; Vertolli, Breault, Ouellet, Somers, Gagné, & Davies, 2014). In this modality, it is tasked with generating contextually coherent label sets corresponding to a single word input. Coherencer is a serial algorithm that implements a heuristic local hill search.

$$Y = \text{Coherencer}(CP, q)$$

1. $\text{Initialize } Q, R, C, Y_1$
2. For $n = 1$ to $|Q| - |Y_1|$
3. Set $K_n, K_n^y$
4. $Y_{n+1} = \{}$
5. If $\sum_{v \in K_n} P_v > \lambda$ do
6. $t(Y_n) = 0$
7. Else
8. $t(Y_n) = \min(f(Y_n, y), y \forall y \in Y_n)$
9. For $y \in Y_n$ do
10. If $f(Y_n, y) > t(Y_n)$ do
11. $Y_{n+1} = Y_{n+1} \cup \{y\}$
12. If $|Y_{n+1}| = 5$ do
13. Return $Y_{n+1} \cup \{q\}$
14. Else
15. $R \cup (Y_n - Y_{n+1})$
16. $C = Q - R$
17. If $|C| > 0$ do
18. $Y_{n+1} = Y_{n+1} \cup \text{rand}(C)$
19. $n = n + 1$
20. Else
21. Return $g_1(q) \cup \{q\}$

Figure 1: Pseudocode for Coherencer.

Coherencer’s algorithm proceeds as follows (see Figure 1 for pseudocode). First, the label set $Y_1$ of the initial hypothetical instance $x_1$ in $f(\cdot, \cdot)$ is defined as the top-4 labels with the highest conditional probability with the query $q$ or

$$Y_1 = \{z_k | y \in Y . P(y|q) \leq P(z_k|q) \leq P(z_j|q), 1 \leq j < k \leq 4\}.$$  

The function $f(\cdot, \cdot)$ acts on the subset of $CP$, called $K_n$, that contains the elements in the current $Y_n$ and their corresponding conditional probabilities (i.e.,

$$K_n = \{(a, b, P(a|b)) \in CP | a, b \in Y_n \cup \{q\}\}).$$

Specifically, the function $f(\cdot, \cdot)$ sums over the conditional probabilities of the subset of triples in $K_n$ that contain $y$,

$$f(Y_n, y) = \sum_{u \in K_n} P_u$$

(5)

The function $t(\cdot)$ evaluates the current total context $K_n$ and, if it passes a threshold ($\lambda$), outputs 0.
allowing the algorithm to return \( Y_n \cup \{ q \} \) as a valid set of labels for the generated instance \( X_n \). Otherwise, it outputs the minimum \( f(\cdot) \) value in \( Y_n \), effectively discarding the associated label. We can express this formally as

\[
t(Y_n) = 0, \text{if } \sum_{u \in X_n} P_u > \lambda \]

\[
t(Y_n) = \text{argmin}(f(Y_n, y)), \forall y \in Y_n, \text{if } \sum_{u \in X_n} P_u < \lambda
\]

where argmin returns the lowest \( f(\cdot) \) value.

Since \(|g_1(q)| = 5\) is a condition for the termination of the search, a new label \( y' \) is randomly selected from \( C = Q - R \), where \( Q = \{y' \mid P(y' \mid q) > 0\} \),

\[
R = \{y'' \mid f(Y_n, y'') < t(x_n), 1 \leq n \leq (|Q| - |Y_n|)\}
\]

If at any point \(|C| = 0\), the result will be \( g_1(q) = \text{argmax} \left( \sum_{u \in X_n} P_u \right) \),

\[
g_1(q) = \text{argmax} \left( \sum_{u \in X_n} P_u \right) \text{, where } 1 \leq n \leq (|Q| - |Y_n|)
\]

where argmax \( \cdot \) selects the index with the highest corresponding value.

Coherence Net is a hybrid of a number of features from evolutionary strategies, genetic programming, and neural networks. It was inspired by an attempt to give an artificial neural network representation to the standard evolutionary algorithm approach, which was originally inspired by DNA replication in a population of chromosomes (Holland, 1975).

Coherence Net represents the standard chromosomal abstraction of evolutionary algorithms as a five-tiered tree of nodes similar to the derivation trees used in grammar guided genetic programming (GGGP; for review, see McKay, Hoai, Whigham, Shan, & O’Neill, 2010). Each tier \( T_y \) for \( 1 \leq y \leq 5 \) contains an ordered list of nodes. Each node contains a set of integers of cardinality 1, 2, or 3, for each respective tier. The integers are all in base-10 except for the first tier, which is in base-2, and they index nodes of the next lower tier \( T_{y-1} \). The parameter \( \omega \) is the minimum number of bits needed to create an ordered list with the elements of \( Q \) randomized, or \( N_2 \times Q \). Extra indices for a given \( \omega \) are re-indexed modulus \( |Q| \). For example, \( \omega = 12 \) encodes 4096 options but only 2974 are needed; if a number, \( z \), is greater than 2973, we take \( z \mod 2974 \) instead.

Functionally, Coherence Net can be thought of as a parallelized version of Coherencer (see Figure 2 for pseudocode). In place of \( Y_n \), we have a collection of instances of \( Y_n, Y_n = (Y_{n1}, Y_{n2}, \ldots, Y_{nm}) \) for \( 1 \leq m \leq |T_4| \) that is represented by the nodes of \( T_4 \). All the elements of \( Y_n \) are randomly initialized from \( Q \) with the possibility of repetition. \( K_n^* \) and \( x_n^* \) are defined similarly with reference to \( m \). The functions \( f(\cdot, \cdot) \) and \( t(\cdot) \) differ by acting on \( Y_n \subseteq Y^* \): a random, 5 member subset of \( Y_n^* \) that indicates a generalized hypothetical instance. Each \( Y_n^* \) is represented by a node in \( T_5 \). The resulting function is

\[
f(\bar{Y}_n^*, Y_n^*) = \left( \sum_{u \in X_n} P_u \right) + [u \in K_n^*] (9)
\]

Note that the sum added to the sum in \( f(\cdot, \cdot) \), by acting on \( \mathbb{N} \) while the sum acts on \( [0,1) \), places greater emphasis on all pairs of labels co-occurring at least once (i.e., \( P(a \mid b) > 0 \)) than the conditional probability sums. This is the first major difference from Coherencer. The function \( t(\cdot) \) indicates a constant threshold \( (t) \) that determines acceptance of the entire set and termination of the search (like Coherencer’s \( t \)). If the threshold is not passed, then

\[
t(\bar{Y}_n^*) = \text{argmax} \left( f(\bar{Y}_n^*, Y_n^*) \right) - \epsilon.
\]

where \( \epsilon \) is an infinitesimal. This function effectively filters the subset \( Y_n^* \) to its maximum member in a variation of five member tournament selection (for the genetic algorithm equivalent, see Miller & Goldberg, 1995). If at any point the filtration results in the the number of unique tier-4 nodes being less than forty percent of \( m \), or \(|\{N \mid \forall N \in T_4\}| < 0.4m\), a destabilization step repopulates \( T_4 \) such that \(|\{N \mid \forall N \in T_4\}| = m\).

\[
Y = \text{CoherenceNet}(E^*, q)
\]

1. **Initialize** \( Q \), \( C^* \), \( Y_1^* \)
2. **For** \( n = 1 \) to \( C^* \) **do**
3. **Set** \( K_n^* \)
4. **For** \( m = 1 \) to \( |T_4| \) **do**
5. **If** \( f(Y_n^*, Y_n^*) > t(Y_n^*) \) **do**
6. **Return** \( Y_n^* \)
7. \( Y_{n+1} = \emptyset \)
8. **While** \( |Y_{n+1}| < |Y_n^*| \) **do**
9. **Set** \( Y_n^* \)
10. **If** \( f(Y_n^*, Y_n^*) > t(Y_n^*) \) **do**
11. \( Y_{n+1} \leftarrow Y_n^* \)
12. **While** \( |\{N \mid \forall N \in T_4\}| < 0.4m \) **do**
13. \( T_4^* \leftarrow \text{rand}(N_{\leq 2974} \times N_{\leq 2974}) \)
14. **For** \( b \in T_4^* \) **do**
15. **If** \( t(Y_{nm}^*) < 1/|T_4^*| \) **do**
16. \( b = (b + 1) \mod (2) \)
17. **n = n + 1 \)
18. **Return** \( g_1(q) \)

**Figure 2**: Pseudocode for Coherence Net.

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In order to search in parallel across the entire collection of \( Y_n \)'s, we abandoned the sequential search in favor of a purely stochastic ‘noise’ step. The noise step causes stochastic fluctuations in the elements of \( Q \) that are being instantiated in \( Y_n \) over the course of \( n \). Since \( Q \) can never be exhausted in this format, we define a new iteration cap \( C^* = 50 \) for \( n \). As with Coherencer, we define a variation of \( g_t(\cdot) \)

\[
g_t^*(q) = \{y \in Y_n \mid \arg \max_{1 \leq n \leq C^*} f^*(Y_{nm}, y) \} \cup \{q\} \quad (11)
\]

We then define two new functions, \( f^*(\cdot, \cdot) \) and \( t^*(\cdot) \), in order to account for the noise step.

The function \( f^*(\cdot, \cdot) \) is defined on the local context \( (Y_{nm}) \) and computes the current probability that the label \( y \in Y_{nm} \) will change from fluctuations at \( T_1 \) using the inclusion-exclusion principle. Formally this can be described by

\[
f^*(Y_{nm}, y) = P(U_{nm}^1 A_1) = \sum_{i=1}^{}\omega P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - \ldots - (-1)^{n-1} P(\bigcap_{1 \leq i \leq n} A_i) \quad (12)
\]

where \( \omega \) is the cardinality of sets in \( T_2 \) and \( A_i = 1/|T_2|, \forall i \) is defined as the probability of a bit changing in the noise step. Since the probability of each bit changing is independent and, thus, the probability of multiple bits changing is the product of the constant \( A_i \), equation 10 can be simplified to

\[
f^*(Y_{nm}, y) = \sum_{k=1}^{\omega} (-1)^{k-1}(n\choose k)1/|T_2|^k \quad (13)
\]

(see Brualdi, 2010). The function \( t^*(Y_{nm}) = \text{rand}(0,1) \) acts as the pseudo-random number generator for the stochastic function \( f^*(\cdot, \cdot) \). Since the noise step can result in \( Y_{nm} \notin CP \), mainly \( \{\{a, b, P(a \mid b)\} | a, b \in Y, a = b\} \), we expand \( CP \) to \( CP^* = CP \cup \{\{a, b, -100\} | a, b \in Y, a = b\} \) in order to avoid these invalid sets.

5 METHOD

There are two models that were compared: Coherencer and Coherence Net. The entire Peekaboom database was filtered to remove all images with fewer than five labels and any labels that only occurred in those images. A total of 8,372 labels and 23,115 images remained after filtration. The images were compressed to \( CP \) and \( CP^* \).

Each of the 8,372 labels was processed by both models 5 times per threshold. Each query plus four returned labels are the elements of a new, hypothetical image instance. The results for each of the algorithms were assessed with regard to the original images. If at least one of these images contained the five labels that were selected by a particular algorithm, including the query, the algorithm scored one point. If there were no images containing the five labels, they did not score a point. The results were averaged for each threshold.

6 RESULTS

The results are reported in Figure 3 for each of the models across half of the parameter space, which ranges from 0 to 1. Both models level off after a threshold of 0.5. The adjusted Wald confidence interval for binomial (success or fail) proportions was used (see Reiczigel, 2003). The max average percent correct for Coherence Net is 90.0 \((n = 7533.6)\) and for Coherencer is 79.3 \((n = 6639.8)\). Pearson chi-square test demonstrates that the difference in the max number correct was statistically significant, \( \chi^2(1, N = 16744) = 363.63, p <.000, \varphi = 0.15 \).

![Figure 3: Percent correct for each model across the threshold parameter space.](image)

7 DISCUSSION

The results support the notion that Coherence Net outperforms Coherencer at generating hypothetical image label sets. It provides one of the first machine learning techniques designed for generative cognition. This, in turn, lends greater support to both the related theories described by Vertolli and Davies (2013; 2014) and Thagard (2000).

Thagard (2000) proposed both serial optimization techniques and parallel or connectionist techniques as valid approaches for dealing with contextual coherence. Thagard argues that the parallel structure better approaches the global
optimum by avoiding local optima. However, before concluding his discussion, Thagard states explicitly that serial algorithms are important for understanding bounded rationality in humans.

Vertolli and Davies (2014) have shown that functionally serial processing techniques can be better than parallel algorithms when the feature set is low level (e.g., conditional probabilities), low dimensionality (e.g., only one feature), and high combinatoric load. However, they leave open the possibility that some combination of parallel and serial techniques could explain how bounded rationality approaches optimal functionality.

The current work supports and extends these authors by implementing a parallelized serial processing system with similarities with connectionist approaches in its artificial neural network representation: CoherenceNet. As Thagard predicted, greater parallelization increased the optimality of the system as a whole. We have also extended their work by providing a formal description of the task and algorithms.

It is worth noting that, outside of the quantitative testing metric, it is challenging to interpret the literal output of each of the models. At times, it is clear why CoherenceNet outperformed Coherencer. For example, given the query ‘robber,’ CoherenceNet returned ‘steal,’ ‘thief,’ ‘mask,’ and ‘jail’ while Coherencer returned ‘steal,’ ‘thief,’ ‘money,’ and ‘square.’ CoherenceNet’s result occurs in an image and Coherencer’s does not. However, for the query ‘bank,’ CoherenceNet returned ‘fruit,’ ‘away,’ ‘keeps,’ and ‘an’ while Coherencer returned ‘hand,’ ‘atm,’ ‘credit,’ and ‘keeps.’ Though CoherenceNet’s output does occur in an image and Coherencer’s does not, it is not obvious which result is actually more desirable as a model of imagination.

Another caveat is that many of the parameters used, especially the number the nodes at each tier, are arbitrary. Generally, more nodes improved the search space but increased the search time. We used 1000 nodes for tiers 1 through 3 and 2500 nodes for tiers 4 and 5 as we found these numbers worked well in a reasonable amount of time. Future work will evaluate many of these properties in greater detail.

REFERENCES