Using Forward-backward Contractors to Identify Parasitic Parameters of Electrical Circuits Working in High Frequency

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Abstract: Parasitic parameters in electrical networks are usually sources of intolerant electromagnetic interference in their near environment. In order to understand better the undesirable phenomenon, the values of these unknown parameters must be estimated with a good accuracy. This work shows how interval analysis can help designing set-membership algorithm that is able to solve with numerical guarantee the kind of issue. A simple example, namely second order filter, is studied and our method shows promising performances for dealing with complex circuits.

1 INTRODUCTION

Electromagnetic Interference (EMI) filters are widely used in power electronics. In order to efficiently predict their performance, EMI filters should be characterized and specially their parasitic parameters, which assume an important role in high frequency. The effects on filter performance of the parasitic capacitance in an inductor and the parasitic inductance in a capacitor are well-known (Chen et al., 2008). These effects can be measured with an impedance analyzer (Labarre and Costa, 2011) or a network analyzer (Wang et al., 2005) and correctly modeled. Eventually, these parasitic effects can be modeled using 3D simulators. The parasitic effects related to the layout and the packaging are more difficult to grasp and then to model. In the literature, few works attempted to solve this issue. For example, in (Kotny et al., 2009) the authors present a fitting approach using impedance measurements. However, this approach is based on approximations and does not take into account the noises that constantly affect the measured values. Moreover, the user of this fitting method must manually analyze the experimental data in order to be able to apply the proposed approximations. Motivated by these problems, we present in this work a set-membership estimation algorithm that can deal with corrupted data (Jaulin et al., 2001). In fact, our set-membership estimation method assumes that the corrupted data are unknown but belong to a set with known bounds. Then, it discards in a rigorous way portions of the search space where the parasitic parameters are inconsistent with the impedance models, the experimental data and the priori known error bounds (Jaulin et al., 2001). That means, the set-membership method computes an outer approximation of the feasible parameters set. Additionally, this method makes the uncertainty in the estimated parameters directly available from the shape and the size of the outer approximation of the solution set. Indeed, the projections of the outer approximation onto the parameters axes constitute uncertainty intervals for the identified parameters. Moreover, compared to the classical identification methods (Buiatti et al., 2007) based on stochastic assumptions, the set-membership approach requires much less of experimental data and takes into account the deterministic modeling error. In fact, the classical identification methods are relevant only when an explicit characterization of the measurement noise is available. It is important to note that, for our parameters estimation problem, deterministic modeling and measurement error cannot adequately be described by random variables. Add to this, the use of the set-membership approach to identify electrical parameters is stimulated by the fact that the effect of each parasitic parameter appears only in one segment of the frequency range. That means, for each frequency segment, the input impedance model of the studied circuit is significantly simplified and so the conservativeness related to the set computation is greatly reduced.

This paper is structured as follows. Section 2
is devoted to state the studied problem and a set-membership formulation for the parasitic parameters identification problem is established. In section 3 some definitions and notions about interval analysis are briefly introduced and thus the principle of contracting a box under set of constraints using interval analysis is discussed. Our set-membership identification algorithm is presented in section 4 and then simulated test is included to illustrate the merits of this approach.

2 PROBLEM STATEMENT

The aim of the paper consists in estimating the parasitic parameters associated to passive electrical circuits working in high frequency. Before presenting a set-membership formulation for our parameters identification problem, let us briefly recall the definition of the input impedance concept of electrical circuit. In fact, the input impedance of an electrical circuit is the equivalent impedance seen by a power source connected to that circuit. If the source provides known the equivalent impedance seen by a power source constitutes the input impedance concept of electrical circuit. As well, the real behavior of a simple coil is modeled in a given circuit and let \( Z(p, w) \) be its input impedance model. Then, our purpose consists in characterizing in guaranteed way the following set of parameters

\[
P = \{ p \in \mathbb{D} \mid \forall i = 1, \ldots, N, \ Z(p, w_i) \in \mathbb{Z}_i \} \tag{1}
\]

where \( Z_i \) is the feasible domain of the measured input impedance at the frequency \( F_i \) and \( \mathbb{D} \) is the initial research space of parameters. Note that, here the guarantee term means that no consistent parameters can be lost during the pruning phase. Now, assume that for all \( i \in \{1, \ldots, N\} \) the feasible domain of measurement \( Z_i \) is built as follows

\[
Z_i = Z_i + \mathbb{E}_i \tag{2}
\]

where \( Z_i \) represents the real measurement and \( \mathbb{E}_i \) represents the feasible domain of error, which includes both modeling and measurement error. To reach our goal, in the unknown but bounded error framework, we will use a set-membership approach based on interval analysis (Jaulin et al., 2001). Note that, this approach tries to compute a tighter outer approximation \( [p] \) of the parameters set (1). That is \( [p] \) contains all the elements of the set \( p \supseteq [p] \). In the next, before presenting our identification algorithm, we introduce some definitions and tools of interval analysis (Moore, 1966).

3 INTERVAL ANALYSIS

Interval analysis was initially developed to account for the quantification errors introduced by the floating point representation of real numbers with computers and was extended to reliable computations (Moore, 1966). Denote by \( [x] = [\underline{x}, \overline{x}] \) a real interval which is a connected and closed subset of \( \mathbb{R} \) where the real numbers \( x \) and \( \overline{x} \) are respectively the lower and the upper bound of \( [x] \). So, the set of all real intervals of \( \mathbb{R} \) is denoted by \( \mathbb{I} \). Over \( \mathbb{R} \) an interval arithmetic was built by an extension of the real arithmetic operations. That means, for each operator \( \circ \in \{+, -, \times, \div\} \) and for each couple of intervals \([x] \) and \([y] \) one defines

\[
[x] \circ [y] = \{a \circ b \mid a \in [x], \ b \in [y] \} \tag{3}
\]

The width of an interval \([x] \) is defined by \( w([x]) = \overline{x} - \underline{x} \). As well, an interval vector or box denoted by \([x] \) is a subset of \( \mathbb{R}^n \) defined as the Cartesian product of \( n \) closed intervals. The set of all interval vectors of order \( n \) will be denoted by \( \mathbb{I}^n \). The width of an interval vector of dimension \( n \) is defined by \( w([x]) = \max_{1 \leq i \leq n} w_i([x]) \). Likewise, we define the vector width of an interval vector by \( w_v([x]) = (w([x_1]), w([x_2]), \ldots, w([x_n])) \). That means the components of the real vector \( w \) are the widths of each component of the interval vector \([x] \).

3.1 Inclusion Functions

Consider the real function \( f : \mathbb{R}^n \to \mathbb{R} \). The range of

\[
[\mathbb{Z}_i] = [Z_i] + \mathbb{E}_i
\]
this function over an interval vector \([x]\) is given by:
\[
f([x]) = \{ f(a) \mid a \in [x] \}
\] (4)

Then, one calls an inclusion function denoted by \([f]\) for the real function \(f\) an interval application that satisfies the following inclusion
\[
\forall [x] \in IR^n, \ f([x]) \subset [f([x])]
\] (5)

In practice, the simplest manner to obtain an inclusion function \([f]\) for real function \(f\) consists in replacing each occurrence of a real variable by the corresponding interval and each standard function by its interval counterpart. The resulting function is called the natural inclusion function and the tightness of the enclosure provided by \([f]\) depends on the formal expression of \(f\). In fact, it is well known if the same variable \(x_i\) has many occurrences in the mathematical expression of \(f\), the dependence effect (Moore, 1966) (Jaulin et al., 2001) will induce pessimism while computing an enclosure of the range of the real function. Hence, formal pre-processing of the function expression is advisable in order to minimize the number of variable occurrences. In the next paragraph, we will show how thanks to interval computation one can solve easily some constraint satisfaction problems (CSP) for continuous domains (Jaulin et al., 2001).

### 3.2 Constraint Satisfaction Problem

Historically the constraint satisfaction problems were firstly formulated and studied in the context of discrete domains, viz the variables belonged to finite sets (Mackworth, 1977). Then, they have been extended to continuous domains where the variables are part of subsets of \(\mathbb{R}\) or intervals (Sam-Haroud and Faltings, 1996). In continuous setting one defines a CSP as follows
\[
H : (g(x) = y, \ \mathbf{x} \in [\mathbf{x}], \mathbf{y} \in [\mathbf{y}])
\] (6)

where the vectors \(\mathbf{x}\) and \(\mathbf{y}\) belong respectively to \(\mathbb{R}^n\) and \(\mathbb{R}^m\) with \(n\) not necessarily equal to \(m\). Furthermore, the set of constraints can be assimilated to a subset \(S\) of \(\mathbb{R}^n\) defined by
\[
S = \{ \mathbf{x} \in [\mathbf{x}] \mid g(\mathbf{x}) \in [\mathbf{y}] \}
\] (7)

In practice, thanks to special algorithms (contractors) (Jaulin et al., 2001) based on interval analysis and consistency techniques, one can get with a polynomial complexity in time and space, an outer approximation \(\tilde{S}\) of the solution set \(S \subseteq \tilde{S}\). Moreover, the outer approximation set \(\tilde{S}\) is said global and guaranteed in the sense that it encloses all the possible solutions of the CSP \(H\). Thus, contracting an interval vector (box) \([\mathbf{x}]\) under the set of constraints \(H\) consists in computing a smaller box \([\mathbf{x}]\), which contains in guaranteed way the set resulting from the following intersection \(\tilde{S} \cap [\mathbf{x}] \subset [\mathbf{x}]\). That means, one attempts to reduce the width of the initial box \([\mathbf{x}]\) according to the set of constraints \(H\). In the literature several types of contractors are developed in order to deal with specific CSP. For instance, we can cite those inspired from point algorithms such as Gauss elimination, Gauss-Seidel algorithm, Krawczyk method, linear programming and Newton algorithm (Jaulin et al., 2001). Hereafter, let \(C\) be a contractor and note that all contractors have the following properties:
\[
\forall [\mathbf{x}], \ C([\mathbf{x}]) \subset [\mathbf{y}] \text{ (contractance),}
\]
\[
\forall [\mathbf{x}], \ [\mathbf{x}] \cap [\mathbf{y}] \subset C([\mathbf{x}]) \text{ (correctness),}
\]
\[
[\mathbf{x}] \subset [\mathbf{y}] \Rightarrow C([\mathbf{x}]) \subset C([\mathbf{y}]) \text{ (monotonicity)}
\] (8)

and a fixed point of a contractor \(C\) is a box \([\mathbf{x}]\) such that \(C([\mathbf{x}]) = [\mathbf{x}]\). To close this section, we present in the following a contractor based on the forward-backward constraint propagation.

### 3.3 Forward-backward Contractor

In this paragraph, we illustrate the core idea of contracting a box \([\mathbf{x}]\) according to a set defined by \(H\) using the constraints propagation on intervals (Benhamou et al., 1999). Note that, for the kind of contractor the dimension of the vector function \(g\) is no longer necessarily equal to the dimension of the variable vector \(\mathbf{x}\). Assume that the mathematical expression \(g(\mathbf{x}) = \mathbf{y}\) is decomposable into a sequence of primitive constraints. Roughly speaking, a primitive constraint is a constraint involving only a single operator \(\{+, -, \times, \div\}\) or single function \(\{\sin, \cos, \exp, \ldots\}\). The forward-backward contractor, here denoted by \(C_{fb}\), incorporates two procedures called forward propagation and backward propagation. Using interval computation, a direct evaluation of all the primitive constraints are achieved in the forward propagation procedure in order to obtain in a sequential way an inclusion function for \(g\). Then, using consistency techniques (Sam-Haroud and Faltings, 1996) the width of the computed inclusion function is reduced in accordance with the given box \([\mathbf{y}]\).

After this correction a backward propagation is performed from the corrected inclusion function to its elementary arguments gathered in the interval vector \([\mathbf{x}]\). Now, let us explain how works this kind of contractor through a simple example. Consider the non-linear constraint
\[
g(x_1, x_2) = x_1 \exp(x_2) = y, \ (x_1, x_2) \in [x_1] \times [x_2], \ y \in [Y]
\] (9)

which can be decomposed into sequence of two primitive constraints
\[
\begin{align*}
a_1 & = \exp(x_2) \\
y & = x_1 a_1
\end{align*}
\] (10)
where $a_1$ is an intermediate variable. Then the forward propagation is obtained by an intervalization of the point sequence (10) as follows

\[
\begin{align*}
[a_1] = & \exp([x_2]) \\
[y] = & [x_1] \ast [a_1]
\end{align*}
\] (11)

The correction stage consists in reducing the size of $[y]$ by eliminating its inconsistent part with $[Y]$. That means, one computes the following intersection

\[
[y] = [Y] \cap [y]
\] (12)

Thus, from the corrected box $[y]$, the backward propagation computes the interval variables $[x_1]$ and $[x_2]$ in the following way

\[
\begin{align*}
[a_1] = & \dfrac{[y]}{[x_1]} \cap [a_1] \\
[x_1] = & \dfrac{[y]}{[a_1]} \cap [x_2] \\
[x_2] = & \log([a_1]) \cap [x_2]
\end{align*}
\] (13)

To resume, the algorithm below gives the finite list of instructions that must be performed by the forward-backward contractor $C_{fb}$ to get an outer approximation of the solution set of (9).

**Algorithm.** $C_{fb}([x_1],[x_2],g,[Y])$

1. Forward Propagation
   - $[a_1] := \exp([x_2])$
   - $[y] := [x_1] \ast [a_1]$
2. Correction
   - $[y] := [Y] \cap [y]$
3. Backward Propagation
   - $[a_1] := \dfrac{[y]}{[x_1]} \cap [a_1]$
   - $[x_1] := \dfrac{[y]}{[a_1]} \cap [x_2]$
   - $[x_2] := \log([a_1]) \cap [x_2]$
4. Return $([x_1],[x_2])$

Thus, in the next section, using the forward-backward contractor we design a new set-membership algorithm to estimate the parasitic parameters associated to electrical components working in high frequency.

## 4 ESTIMATION ALGORITHM

Recall that our purpose consists in developing a specific algorithm that is able to identify all the parasitic parameters that appear in electrical circuits. In other terms, we want to build more realistic models for electrical circuits in order to well understand the undesirable EMI effects. To do so, we propose to measure for several frequencies the input impedance of the studied circuit working in different configurations. For instance, we consider the open circuit $(o)$, the short circuit $(s)$ and the closed circuit $(c)$ configurations. Then, for each measured impedance we apply the forward-backward contractor. To be more clear, let us assume that for each configuration we measure the real and the imaginary parts of the input impedance. That is, at each frequency measurement $F_i, i \in \{1, \ldots, N\}$, we have

\[
Z_i^m = R_i^m + jI_i^m
\] (14)

where $m \in \{o, s, c\}$ and $R_i^m, I_i^m$ represent respectively the real and the imaginary parts of the measured input impedance. The feasible domains of measurements are given by

\[
\begin{align*}
\mathbb{R}_i^m &= R_i^m + E_i^R, \quad \mathbb{I}_i^m &= I_i^m + E_i^I
\end{align*}
\] (15)

where $E_i^R$ and $E_i^I$ represent the priori known feasible domains of error. Then, we want to determine a tight outer approximation of the following set of parameters

\[
\mathcal{P} = \left\{ \left. \begin{array}{c} p \in \mathbb{D} \\
\forall m \in \{o, s, c\}, \forall i \in \{1, \ldots, N\} \end{array} \right\| \begin{array}{c} R_i^m(p, w_i) \in \mathbb{R}_i^m \\
I_i^m(p, w_i) \in \mathbb{I}_i^m \end{array} \right\}
\] (16)

where $R_i^m(p, w)$ and $I_i^m(p, w)$ represent respectively the real and the imaginary part of the input impedance models

\[
Z_i^m(p, w) = R_i^m(p, w_i) + jI_i^m(p, w)
\] (17)

Moreover, the above parameters set (16) is also defined by the following CSP

\[
\mathcal{H} : \left\{ \begin{array}{c}
\forall m \in \{o, s, c\}, \forall i \in \{1, \ldots, N\} \\
R_i^m(p, w_i) = R_i^m, \quad p \in \mathbb{D}, R_i^m \in \mathbb{R}_i^m \\
I_i^m(p, w_i) = I_i^m, \quad p \in \mathbb{D}, I_i^m \in \mathbb{I}_i^m
\end{array} \right\}
\] (18)

Then, based on the forward-backward contractor, the algorithm below gives the procedure to be followed for computing an outer approximation of the set of parameters (16).

**Algorithm.** $SM-Ident$(Input: $\mathbb{D}$, Output: $[p]$)

\[
\begin{align*}
[p] & := \mathbb{D} \\
\varepsilon_1 & := w_i([p]) \\
\varepsilon_2 & := 0.9 \ast \varepsilon_1 \\
\text{while} \ (\varepsilon_2 < \varepsilon_1) \ & \\
\varepsilon_1 & := w_i([p]) \\
\text{for} \ m := o, s, c \\
\varepsilon_1 & := w_i([p]) \\
\text{for} \ i := 1, \ldots, N \\
R_i^m & := R_i^m + E_i^R \\
\mathbb{D}_i^m & := C_{fb}(p_i, R_i^m(p, w_i), R_i^m) \\
\mathbb{I}_i^m & := I_i^m + E_i^I \\
\mathbb{D}_i^m & := C_{fb}(p_i, I_i^m(p, w_i), I_i^m) \\
\text{end} \\
\varepsilon_2 & := w_i([p]) \\
\text{end}
\end{align*}
\]
end
\[ \varepsilon_2 := w_v(p) \]
end

Remark. For this algorithm the comparison operator < applied between real vectors should be understood as a collection of inequalities applied component by component.

For a given initial research space of parameters \( \mathbb{D} \), the SM-Ident algorithm uses the forward-backward contractors to reduce its size throughout several stages. Indeed,

- for each experience: the open circuit, the short circuit and the closed circuit configuration (ordered by the first for loop),
- for each value of the measured input impedance (14) with its feasible domain (15) (ordered by the second for loop),
  
  the forward-backward contractor is applied successively to reduce \( p \) under the constraints defined by the real and imaginary parts of the input impedance model (17). Thus, the box contracting procedure is repeated until no size reduction is observed. That means, the real width vector \( w_r \) becomes constant, and so one leave the two while loops.

4.1 Discussion about Pessimism

In fact, in the field of system identification, it is well known that the applied interval methods suffer from pessimism generated by the dependence phenomenon of interval computation (Moore, 1966) (Jaulin et al., 2001). Therefore, the use of interval approach to solve practical problems is limited. In this paragraph, we try to explain why we claim that in the case of electrical circuits the interval identification approaches give good results.

In practice, when we analyze the behavior of the input impedance of an electrical circuit, we find that the effect of each passive element of the studied circuit is significant only on a well defined frequency range. This fact is justified by the following properties: (i) each coil of the circuit behaves as a low pass filter with a specific cutoff frequency; (ii) each capacitor of the circuit behaves as a high pass filter with a specific cutoff frequency; (iii) the impedance of an RLC circuit in resonance is equal to its resistance. According to these properties, we can safely conclude that on each frequency segment, the impedance model of the studied circuit is significantly simplified and so the conservativeness related to the pessimism of set computation is greatly reduced even if the impedance model contains unknown parameters with many occurrences.

To illustrate the merits of our estimation method, the next subsection is devoted to its application to identify the parasitic parameters associated to a simple second order electrical filter.

4.2 Illustrative Example

Consider the electrical second order filter depicted in Figure 2.

![Figure 2: Real second order filter constructed by a capacitor with capacitance \( C = 1\mu F \) and a coil with inductance \( L = 300\mu H \).](image)

Taking into account the physical existence of the parasitic parameters, the real filter is equivalent to the circuit depicted in Figure 3. For our experience, we assume that the values of the main components \( L \) and \( C \) are well known \( L = 300\mu H \) and \( C = 1\mu F \). Then, we attempt to estimate the vector \( p = [R_c, L_c, R_l, C] \). To do that, we will use the SM-Ident algorithm. The experimental data are collected from three experiences: the open circuit, the short circuit and the closed circuit experiences. Note that, for the last case we close the circuit by a resistive load \( R = 50\Omega \) and for each experience the real and imaginary parts of the input impedance are measured for several frequencies.

As shown previously, the basic components of the SM-Ident algorithm are the forward-backward contractors \( C_{fb} \). To design these contractors, we must compute the analytical expressions of the input impedances. For instance, in the case of short circuit experience, by direct computation we obtain the following expression of the real part of the input impedance

\[
R((p, w) = \frac{(AC + BD)}{(C^2 + D^2)} \tag{19}
\]

where \( A = \frac{1}{\varepsilon} [LR_c + (L - \frac{1}{w})R_l], B = \frac{1}{\varepsilon} [L(R_c w - R_l) - \frac{1}{\varepsilon} R_l R_c], C = \frac{1}{\varepsilon} (L + L_c - \frac{1}{w} R_l) + R_l R_c + U R_c (L_c - L) w^2 \) and \( D = R_c L_c + R_l (L_c - L) w^2 \).
Thus, we have to resolve at each measurement the following CSP
\[
(R^i(p, w_i) = r^i, \quad p \in [p], R^i \in \mathbb{R}^n_+)\]
To do that, we use (19) to build a contractor as illustrated in subsection 3.3. Similarly, for each experience we build contractors for the real and imaginary parts of the input impedance. So, all ingredients needed to use SM-Ident algorithm are available. In following a simulation test is achieved to show the performances of the SM-Ident algorithm.

4.3 Simulation Test
For each circuit configuration, \(m \in \{a, s, c\}\), the simulated data are obtained by using the analytical expression of the input impedance \(Z^m(p, w)\) with \(p = [85 m \Omega, 43 \mu H, 150 m \Omega, 1 p F]\). Thus, for a discrete set of frequencies \(\{F_1, F_2, F_3, \ldots, F_N\}\) where \(v_i \in [1, \ldots, N]\), \(F_i \in [10, 10^2] Hz\), we build the simulated measurement signals \((r^m_i, l^m_i)\) with their feasible domains,
\[
[r^m_i, l^m_i] = \begin{bmatrix} r^m_i \pm |\varepsilon| r^m_i \mid, \\ l^m_i \pm |\varepsilon| l^m_i \mid \end{bmatrix}
\]
where \(\varepsilon\) is the maximum error that can be disturbed the measurements. In table 1 some results of the parameter identification procedure are presented. In fact, for different values of the maximal error \(\varepsilon\) the SM-Ident algorithm contracts the initial research space of parameters
\[
\mathbb{D} = ([0, 200] \Omega \times [0.6] \mu H \times [0, 700] \Omega \times [0, 1] F)
\]
until its size will be constant. As shown in table 1,

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2%</th>
<th>5%</th>
<th>10%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_a(\Omega))</td>
<td>[83.3, 86.8]</td>
<td>[80.7, 89.3]</td>
<td>[76.5, 93.6]</td>
</tr>
<tr>
<td>(L_c(\mu H))</td>
<td>[42.93, 43.07]</td>
<td>[42.82, 43.18]</td>
<td>[42.65, 43.35]</td>
</tr>
<tr>
<td>(R_s(\Omega))</td>
<td>[145.2, 154.9]</td>
<td>[138.3, 162.3]</td>
<td>[127.2, 175.1]</td>
</tr>
<tr>
<td>(L_a(\mu F))</td>
<td>[0.997, 1.003]</td>
<td>[0.992, 1.008]</td>
<td>[0.985, 1.016]</td>
</tr>
</tbody>
</table>

for each value of \(\varepsilon\) the sought parameter vector \(p^*\) is enclosed in the identified parameter interval vector; and its accuracy is given by its width. Moreover, the identified parameter interval vector contains an outer approximation of the uncertainty associated to each identified parasitic parameter. Finally, note that for initial parameters boxes that do not contain the sought parameter vector \(p^*\), the SM-Ident algorithm returns as expected an empty set of possible solutions.

5 CONCLUSIONS
In this paper, we developed and tested an identification algorithm for electrical circuits modeled by their input impedance. Our algorithm works in the unknown-but-bounded-error context and it used interval analysis to solve, in rigorous way, the issue of parasitic parameters estimation. For future works, we plan to apply the algorithm to more complex electrical networks.

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