PERSONALISED E-LEARNING PROCESS
The Case of Geometry in IWT

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Abstract: This paper is concerned with the personalisation of teaching/learning paths in mathematics education. Such personalisation would exploit the features offered by the e-learning platform IWT, which allows to manage both the knowledge domain and the student’s profile. We will present the implementation of a personalised course of “Geometry” in IWT, describing the given representation of the knowledge domain and on the other side the design of various learning objects both meaningful from educational viewpoint and according to different learning style. The genesis of the course is based on the integration of research in mathematics education and e-learning. The course has been experimented at the Faculty of Engineering of the University of Salerno (Italy). The analysis of the outcomes is in progress taking into account both pedagogical and technical issues.

1 BACKGROUND

This paper starts from one of the main hypotheses of e-learning (Nichols, 2003), which considers the facilitation of education processes, providing the learners with many personalised learning opportunities. According to this perspective, we want to present the case of the Geometry learning at University level.

In the next sections we will focus on some issues which are regarded as critical by research in education and in particular in mathematics education and could be dealt with in a more appropriate way with the help of an e-learning platform.

1.1 Personalisation/Individualisation of the Learning Process

The individualisation of teaching is one of the most critical issues in instructional practice. It is well known that some instructional strategies are more or less effective for particular individuals depending upon their specific abilities. According to Cronbach & Snow (1997) the best learning achievements occur when the instruction is exactly matched to the aptitudes of the learner. At first, we can say that individualisation regards how much the instruction fits students’ characteristics, creating learning situations suitable to different students. In particular we refer to the individualisation at the teaching level which, according to Baldacci (1999), means the adjustment of the teaching to the individual students’ characteristics, by means of specific and concrete teaching practices. Another major goal is the personalisation of the teaching, which refers to the set of activities directed to stimulate each specific person in order to achieve the maximum of his/her intellectual capability. It is clear that neither individualisation nor personalisation are possible at undergraduate level, especially with large classes of freshman students, if teaching is still based on standard lectures.

From the viewpoint of individualisation the teaching procedures included in the platform should get the students to attain the basic skills, by means of a choice of different learning paths, whereas from that of personalisation teaching activities should be planned in order to allow each student to get his/her own way to excellence, through specific opportunities to develop his/her own cognitive potential. In order to develop each student’s specific skills of, it is necessary to let him/her free to move, to choose, to plan and to manage some suitable cognitive situations.
1.2 Mathematics e-Learning

In the case of mathematics, e-learning offers new, almost unexplored opportunities, especially as concerns personalisation, cooperative and constructive methods, language and representations (Albano & Ferrari, 2008).

At the International Conference of Mathematicians (2006), a discussion panel has been devoted to the integration of e-learning with the teaching and the learning of mathematics. The object is to understand how to direct the technological potential in order to improve quality and quantity of mathematics learning. Clearly enough needs are different according to the kind and the level of instruction considered, and general answers are not available.

Bass (2006) recognizes five topics in mathematics education which can be helped by technology:

1) Drawing mathematically accurate and pedagogically valid graphs. Graphs can be used for different purposes: exploring, investigating what happens if some elements vary, prove/show ideas, explanations, solutions.

2) Keeping trace of the classroom work and errors. This from the one hand provides indications (mainly to the teacher) to re-direct subsequent work, from the other hand allows (mainly the student) to "see" his/her own improvements increasing his/her sense of self-efficacy (Zan, 2000).

3) Coordinating lectures and textbooks. Technology gives the opportunity to design tasks and additional activities for the student.

4) Easy access for the teacher. Technology gives the opportunity to adopt a flexible timetable for meeting students. Also the exchanges of messages between teacher and student contribute to trace each student's history and his/her advancements.

5) The repetitive nature of individual, out-of-schedule sessions. Most often some understanding problems cyclically recur and the teacher is compelled to replicate his/her explanations each time. FAQ's and for a allow teachers to make accessible to all students topical discussions potentially useful.

2 THE CASE OF THE GEOMETRY COURSE

In this section we will see in details the work underlain to allow IWT to support students with personalised learning paths. IWT (Intelligent Web Teacher), realised at Italian Pole of Excellence on Learning&Knowledge, is a distance learning platform, whose innovative features are openness, flexibility and extensibility, in particular given the presence of three models (Didactic, Student, Knowledge) allowing the student to reach the defined didactical objectives delivering a personalised course with respect to his/her specific needs, previous own knowledge, preferred learning styles, didactical model more suitable to the knowledge at stake and to the mental model (then engagement) of the learner.

The case we will examine regards the scientific domain of the Geometry addressed the first year University level. The work has been consisted on one side in representing the knowledge domain and on the other side in designing and implementing various learning objects both meaningful from educational viewpoint and according to different learning style.

2.1 The Representation of the Knowledge Domain

One of the fundamental steps of the model is the construction of a sufficiently rich structure on the raw set of data, notions and exercises. Indeed, while a traditional textbook’s structure is essentially linear, a more ramified type of backbone appears necessary in this context. In fact, the richer the structure, the greater the information that can be recovered from data and feedback – the simplest example of this being a time series as a part of the real line, where both algebraic and order structures play a meaningful role and convey information.

According to this very general idea, the IWT Knowledge Model allows the experts to define and structure disciplinary domains, by constructing domain dictionaries, composed by a list of terms representing the relevant concepts of the disciplinary domain that we are modelling, and constructing some ontologies on such dictionaries that are modelled using graphs structure. The first step is to choose a suitable level of granularity in splitting the various types of knowledge into atomic parts. An irreducibility criterion appears to be reasonable in the notion joining sense. More precisely, we may define a semigroup structure on notions, where the internal operation is given by joining notions; irreducibility now means that a given notion is a "prime" i.e. it cannot be expressed (in a non-trivial way) as the product of two or more notions. Of course we may not have a unique factorisation so
that, although irreducible elements appear to be well defined, an arbitrary element may be factorised in more than one way into “primes” – thus possibly requiring random choices during the factorisation process, or additional nodes and links to account for. According to this, a suitable decomposition of the Geometry domain has been done. A dictionary consisting of about 150 terms, i.e. elementary concepts, has been created. A graph, whose nodes are the elements of the dictionary, has been designed.

The arcs connecting the nodes are mainly related to two order relations called “Is Required By” (prerequisite) and “Suggested Order”, and a decomposition relation called “Has Part”. The following figure zooms in on the created ontology, to better show the relations:

![Figure 1: A zoom on the Geometry ontology.](image)

As you can see, the concept “Matrici” (i.e. Matrices) has been split into five sub-concepts, which are connected by the relation “Has Part” with “Matrici”. Among these nodes, some order relation is mandatory, e.g. you need to know what is a determinant (node “Determinante”) in order to learn what is the rank of a matrix. So the relation “Is Required By” connects the node “Determinante” to the node “Rango” (i.e. rank). On the other hand, there is no pre-requisite relation between the concepts of rank and echelon matrix (node “MatriceAScalini”). Anyway the author of the ontology (an expert of the knowledge domain) may suggest a preference, according to his/her educational experience or to the addressed educational context. This is why in the figure you can see that the node “Rango” is linked to the node “MatriceAScalini” by the relation “Suggested Order”. If this latter relation is present, the platform will take into account, otherwise a random choice is done.

### 2.2 The Learning Objects

In this section we will describe the various types of learning objects (LO) created for each concept of the knowledge domain, and the educational ratio of their creation. The IWT Knowledge Model allows to annotate each LO with a metadata, which requires to specify a concept (or more than one) inside a domain which the content of the learning object itself is referring to. In this way, it is possible to link the learning object to the concepts of the ontologies, indeed, by associating a learning object with one or more concepts, we can assume that the content of such learning object “explains” the correlated concepts.

#### 2.2.1 Hypermedia

In the school practice it is evident the change in the students’ style of studying/working, which is too often based on patterns of mnemonic learning and on a very focused study, neglecting variation and connections. So the study appears strongly split, pieces of knowledge are memorised being absolutely disjointed from the context where they born and live and often the involved concepts themselves become “words with no sense” repeated as they appear on the textbook.

As stressed by the National Council of Teachers in Mathematics (2000), when the students are able to see connections among various mathematical contents, they arrive to have a global and integrated vision of the mathematics. It is important, as the students learn new concepts, to make evident the connections with the knowledge they already have. The connections they develop give them a grater mathematical power.

Linked to what said above, we also cite the need of putting each content in a suitable framework, which means to give the right level of detail (e.g. some technical steps are fundamental because they allow to understand, some others no or not always). Moreover we consider the reification, that is how to compress pieces of knowledge or procedures, uncompressing them only if necessary).

According to the above framework, some learning objects which are a generalization of the hypermedia have been constructed. They are composed of a main HTML text with keywords. The links bring to other learning objects, which differ as both typology of resources (e.g. plain texts, animated slides, exercises or algorithms, figures, simulations, video, etc.) and educational parameters (didactical approach, semantic density, difficulty,
level of interactivity, etc.).

The links in the main text have been designed in order to allow the students to make connections among different topics of the mathematical knowledge, and in particular of the geometry one; to see the same concept from different viewpoint (e.g. geometrical meaning of an algebraic concept such as the determinant of a matrix); to deepen historical or motivational references; to explicit technical details (e.g. in a proof); to use various semiotic representations and their coordination (Duval, 2006), that is to make a treatment in a fixed semiotic system (e.g. algorithmic procedures) or a conversion from a semiotic representation to another one (e.g. among verbal formulation, symbolic one and figures), the latter being the key of the comprehension in mathematics; to recall definitions or theorems which are pre-requisites of the topic at stake.

2.2.2 Structured Video

According to Rav (1999), the whole mathematical know-how is plunged in the proofs, which contains all the mathematical methodologies, concepts, strategies for problem solving, connections among theories and so on. Based on the Rav thought, some reflections have guided the creation of suitable learning objects regarding proofs. Our starting remark is that in general a proof is not a whole inseparable text, but it is possible to single out a structure composed by several autonomous blocks, which have a proper meaning and a specific role within the proving path (e.g. sub-goals). Each of such blocks can be considered as a module which is possible to refer to in a concise manner or in wide manner depending on the advisability. The composition of more modules leads to the construction of new knowledge, that is it allows to prove the thesis of the theorem at stake. It is worthwhile to note that various theorems may share same modules within their proofs. Moreover some proofs have a non linear path, that is some pieces are non depending one on the other, so the ordering of the corresponding modules is not univocally determined and thus the proving flow also.

Thus it becomes crucial for effective learning that the students are able to identify such modules and to understand their role in the context, because this allows them from one hand to look at the text with more different levels (a whole text, a list of modules, list of expanded modules), and on the other hand it makes evident proving strategies and solving techniques.

In the above framework, some learning objects consisting in structured video, realised with a multimedia blackboard, have been designed and implemented. The videos reproduce something like a face-to-face lecture, focused on the written steps and their audio comments. Various colours have been used to address attention balancing. Pieces of previous knowledge (even in a different digital format) can be stored in other pages of the blackboard and then suitably recalled.

According to what said above, in order to make evident the modules constituting the proof, the videos have been further managed. They have been split into more pieces corresponding to an educational splitting of the proof, that are the modules. Each piece has a title, which is a synthetic phrase describing the characterisation of the module (e.g. the sub-goal the module bring to). The list of these titles constitutes a lateral index, moving along it the student can access directly the related part of the video. It is obvious that, where more decompositions are possible, just one choice has presented and it will be given the students as homework to create other possible lateral indices. Moreover, we note that various granularity can be chosen in splitting the videos. Some realised videos have a very fine granularity, whilst other ones have macro-decomposition. This is because we want to leave up to students as homework to go on by using subsequent refinements.

A similar reasoning has been done with respect to solving techniques. So videos, illustrating step by step how to solve some exercises or how to apply an algorithm, have been created. The videos are supplied with a lateral index corresponding to elementary steps the procedure can be split into.

2.2.3 Static and Dynamic Exercises

In order to cover the knowledge domain with problem solving competences, attention has been paid to offer learning objects on basic solving techniques. Thus two type of exercises have been implemented: a static one and a dynamic one.

The first one consists in a solving model in plain text for various exercises, supplied with many comments and theoretical recalls in order to contrast the mnemonic acquisition of some procedures usually applied automatically from the students, without a previous analysis of the exercise at stake. This means that students often does not think of the correctness of the application of a certain solving procedure, which often leads to incorrect outcomes as the chosen procedure is not applicable. Moreover,
often they do not take into account some specific conditions of the exercise at stake and the automatic application of standard procedure leads to waste time (even if they reach the correct result), which is an important variable to be faced to in a written time-restricted examination.

Dynamic exercises have also been designed and implemented. To this aim, Mathematica and WebMathematica have been used to create suitable algorithms generating on the fly infinity (and always different) exercises. All the algorithms are based on the divide and conquer strategy, splitting each exercise into one or more elementary steps; that is the student is guided to the solution facing easier sub-problems. We define “elementary” step as a sub-problem which is seen as first time (very fine granularity) or a sub-problem which corresponding to an exercise already developed step by step. For instance, the exercise “Echelon Form of a matrix” is split into elementary steps corresponding to the steps described by the Gauss algorithm. At the same time “Echelon Form of a matrix” is an elementary step if it is used to prove the linear independence of vectors. At each elementary step a hint is given and an interaction is required, so that students have to give an answer to the current sub-problem. An automatic evaluation of the correctness of the given answer is done, using Mathematica. The algorithms have been suitably thought in order to recognize and distinguish errors of a (most probably) theoretical character (e.g. logical inconsistencies) and computational errors. Correspondingly, a different warning message is generated, suggesting the most likely nature of the error and suitable means of correcting it. This feature proved particularly useful in saving time during the error correction phase, since students did not have to uselessly repeat the whole theoretical background in case of mere computational errors and, conversely, receiving a timely warning when they needed to get a better understanding of the underlying theory.

Moreover if the student is wrong in his answer, the system at the first time force to re-insert the answer in order to stimulate the students to try again, then if the student made mistake again, the system gives the chance of viewing the correct result if the student wants.

2.2.4 Animated Slides

Animated slides are particular meaningful when some figures comes into play. The construction of a figure is often the first and the key task to correctly solve a problem. To this aim, the conversion between verbal description and figural representation is crucial. Ferrari (2004) note that a large share of students’ failures can be ascribed to linguistic issues. The animation and the synchronisation between the textual description and the corresponding graphical representation allow to guide the student in such conversion. Also in this case, the animation has been designed according to some suitable elementary steps. The main topics treated in this way concern the analytic geometry. Here the conversion among verbal, graphic and symbolic representations has been treated by suitable animations, which allow to see step by step for instance the construction of the equations of the line or the plane in two and three dimension through a continuous migration from the graphical situation to the verbal description and to the algebraic formula. This way the learner experiences the genesis of the known equation of the line and at the same time gains experience in the coordination of different semiotic systems. The latter is a worthwhile learning activity, as such coordination is not spontaneous and it is the key of comprehension in mathematics (Duval, 2006).

2.2.5 Lessons

According to the viewpoint of having various learning objects with different granularity, we have created some modules, called Lesson, which consists of a collection of elementary learning objects among the types seen above.

2.2.6 Junction Elements

A further element to enrich the connection structure available in IWT, besides the arcs of the ontology and the links of the hypermedia, consists in the so called “Junction Elements”. They allow to add a new learning object acting as connectors between adjoining learning objects which are apparently disjointed, giving the learning path a non homogeneous look. For instance, this is the case of a plain text based on historical approach to a given concept followed by a dynamic exercise. Then a junction element allow to bridge the gap between them. IWT offers three types of junction elements according the following goals: to fix the objective, in order to bridge the gap between theoretical notions and their applications; to settle the learning process, fostering curiosity, connections and so on; to stimulate the fruition of further learning objects which make evident interesting aspects related to the concept at stake.

Some few experimental junction elements of the
cited previous types have been designed, respectively in form of: a student report of his/her experience in using his/her acquired knowledge to solve a given problem; a simulated forum, in order to stimulate curiosity; a collection of questions, with or without answers, in order to address the students to interact with other specific learning objects.

2.3 The Personalised Geometry Course

Starting from what said above, we will see how IWT is able to create a personalised Geometry course. At first the teacher will select the Geometry ontology, the target concepts for his/her course and, eventually, some milestones (e.g. intermediate tests). When student accesses to the course the first time, IWT is able to automatically generate for each student the best possible learning path according to the information available in the Student Model, to the course specifications and to the learning objects available in the repository (Albano et al., 2007). At first the ontology is used to create the list of the concepts needed to reach the target concept of the course. Then the information of the Student Model is used to update this list according to the cognitive state and to choose the more suitable learning objects according to the learner preferences. The choice is made possible taking the learning objects whose metadata better matches with the learner preferences data. Moreover the platform is able to dynamically update the learning path according to the outcomes of the intermediate tests.

The student has also chance to personalize himself the course. In fact, for each didactical resource of the course he/she has the possibility to access alternative resources, so to explore and choose what he/she considers the more suitable to better understand the topic at stake. Moreover, he/she can create his/her own resources, adding annotations (textual or multimedia), and also decide to let them public or not. In such a way students interact with the learning material in a tri-dimensional relationship: they do not restrict themselves to receive and elaborate some objects (such as in the case of the book), but produce new learning objects starting from the ones placed at their disposal by the platform (Maragliano, 2000).

3 FUTURE TRENDS

In this paper we have presented a personalised Geometry course based on the integration of research in mathematics education and e-learning. It has been experimented at the University of Salerno. The data already available on IWT show a highly level of interactions of the students with the course material. Some first feedbacks report their enthusiasm for the wide range of different resources available, and their preference for videos, interactive exercises and hypermedia. Specific tasks have been designed in order to test the pedagogical efficacy of the different kind of the created resources, which have been assigning to the students along this term. The analysis of the outcomes is in progress taking into account: average trend of the tasks and done mistakes; academic achievements comparison with standard course; answers to a submitted questionnaire to explore their feeling regarding the personalised course; IWT reports about interaction with the learning objects.

REFERENCES