ANALYZING DECENTRALIZED GOVERNABILITY OF BUSINESS PROCESSES BY EXTENDED PETRI NETS AND MODAL LOGICS

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Abstract: We introduce a novel notion of decentralized governance structure of event-driven processes together with the notions of their behavioral and structural correctness. The ways for attaining correct process behavior, as well as the notion of decentralized governability, are examined based on temporal logical analyses of process behavior via Petri net representations of process structures. Also, the deontic and temporal logical prescriptions of normative constraints (tasks) on the processes are introduced that are then translated into extended hierarchical Petri net structures. The conflicts among these tasks are examined on this hierarchical structure.

1 INTRODUCTION

As e-businesses grow, users have gained power in obtaining information and combining various service applications. For instance, if a user decides to go on a trip, s/he may use an access map, train information, a hotel search, and a defrayment service. Thus, users are nowadays active, unlike those of the past that only passively followed the line prepared by service providers. In other words, a single service provider is not as powerful now as before, and processes are not as centralized as before.

This paper proposes a theoretical framework for such decentralized multi-agent (in this case, users and service providers) systems that can represent agents’ behavior and their policies or control rules. The behaviors of agents are represented by a Petri net (Petterson, 1981), which offers rich mathematical analysis, and introducing modal logics (Hughes and Cresswell, 1968) enables us to represent policies and control rules. It is known that a Petri net is conventional and now its successors, e.g., a Unified Modeling Language (UML) (Saldhana and Shatz, 2000), are popular tools for system modeling. Applying a Petri net is still a hot topic for those researchers that place emphasis on checking the behavior of system design (Hu and Shatz, 2004). Modal logics, e.g., temporal and deontic logics, are also known as conventional theories. They have been applied to several systems (Blackburn et al., 2006) and their theoretical studies still progress (Nute, 2004).

For further development of e-business, decentralized governance is inevitable. Growing numbers of services and applications may lead to overlap of services, which may sometimes cause interference. Our framework enables us to check the existence of interference among services’ control and user’s policies. Furthermore, it shows us how to eliminate the interference.

The rest of this paper consists of the following sections. Section 2 introduces the outline of the framework. In this framework, components of a target system are classified into two portions from the viewpoint of whether it is a prefixed structural element or an element that may change over time like users’ policies and service providers’ control rules. The former portion is encoded into a conventional Petri net. The latter half is first represented by modal logic formulae, which are then translated into extended Petri nets that we call “task unit graphs.” The two portions are then integrated into a single extended Petri net. In section 3, the framework introduced in section 2 is applied to the modeling of decentralized governance problems. Based on the model, we discuss the correctness of process behavior and centralized/decentralized governability in sections 4 and 5.
2 MODELING SYSTEMS BY PETRI NET AND MODAL LOGIC

This section proposes a method for modeling decentralized systems based on a kind of Petri net and modal logic. Hereafter, we present the procedure of system modeling by using an example of a travelers’ decision on itineraries with the assistance of e-applications.

Example System. Assume that in the near future, many e-applications will work as sophisticated agents, and help users by cooperating with each other. Now a traveler has arrived at Porto station. s/he enters an internet cafe, and launches the following agents: e-landmark map agent (A1), e-hotel search (A2), e-train search information (A3), and an e-defrayment system (A4).

s/he has two itineraries to decide on. One is finding a hotel to stay at and a way to get there by train. The other is finding sightseeing spots and how to access them. In the former mission, s/he first searches for a hotel by using A2, which submits the hotel’s location to A3, as well as her/his current location. A3 determines the route between Porto and the nearest station to the hotel. After the route is decided, defrayment is executed by A4. In the latter mission, s/he decides on two scenic sites s/he wants to visit with the help of A1, which hands over the location of the sites to A3 and defrayment is again executed by A4. Anyway, A3 requires two locations and searches for the optimal route between them.

2.1 Petri Net Representation of System’s Event-Driven Aspect

A Petri net is known as a conventional representation scheme for modeling a physical structure and event-driven behavior of discrete event systems (Karateevich, 2007). It is also known that a k-bounded standard Petri net can be translated into an equivalent 1-bounded Petri net. We employ a 1-bounded one called a token locations” is two (2-bounded). Thus it can be modeled as a C/E system as shown in Fig. 1, where a token in place P1 means the following:

\( P_1 \): Both A1 and A2 are idling,

\( P_2 \): A1 is working,

\( P_3 \): A2 is working,

\( P_4 \): cache memory of A3 stores a location,

\( P_5 \): second memory of A3 stores a location,

\( P_6 \): A4 is idling,

\( P_7 \): A4 is working.

Each of the transitions \( \tau_2 \) and \( \tau_3 \) means a submission of the location of a hotel or a landmark, \( \tau_3 \) means the “data transfer from the cache to the second memory” and “flushing cache,” and the firing of \( \tau_6 \) makes A3 search for a train connection between two locations and give train fees to A4.

![Figure 1: Petri net representation of a decentralized discrete event system.](image)

When we correspond a place of a C/E system to a proposition, we can represent the true/false value of the proposition by putting/removing a token in/from the place. In this case, each transition leads the value alteration of the proposition. For instance, in Fig. 1, the firing \( \tau_6 \) leads \( P_4, P_5, P_6 \) to turning from true to false, and \( P_7 \) to turning from false to true.

2.2 Modal Logic Representation of Tasks

Next, we represent the tasks that state “when the focused state should be true” as propositions by introducing temporal and deontic logic.

2.2.1 Temporal Modalities

A temporal logic is given by the propositional logic, modal operators, and an axiom system. This paper employs the following modal operators:
\(\mathcal{T}A\): A will be true at the next state \(S_1\),
\(\mathcal{G}A\): A will be true from now on \(S_0, S_1, S_2, \ldots\),
\(\mathcal{T}A\): A is true at \(S_0\), or will be true at some time in the future \(S_i (i > 0)\),
\(\mathcal{A}t\mathcal{B}\): B is true at \(S_0\) or A will be true from now on until the first moment when B will be the case,

where \(A, B\) denote logic formulae, and \(S_0/S_i (i > 0)\) mean current/future states (worlds) respectively.

Axiom systems of temporal logic vary depending on the viewpoint of time. This paper employs one of the discrete and linear axiom systems \(K_{SU}\) (Katai and Iwai, 1983), which is an extension of the minimal axiom system \(K_1\) (Rescher and Urquhart, 1971). Introducing \(\mathcal{T}\) (yesterday) as the mirror image of \(\mathcal{T}\) (tomorrow), the axiom system claims that \(\vdash \mathcal{T} \neg A \equiv \neg \mathcal{T} A\), \(\vdash \mathcal{T} \mathcal{T} A \equiv \neg \mathcal{T} \neg A\), and \(\vdash \mathcal{T} \mathcal{T} A \equiv \mathcal{G} \mathcal{T} A \equiv A\). Introducing \(\mathcal{T}\) (since) as the mirror image of \(\mathcal{T}\) (until), \(\mathcal{G} A \equiv A \mathcal{T} \perp\), where \(\perp\) denotes the contradiction, and \(\mathcal{T} A \equiv \neg \mathcal{G} \neg A\). \(K_1\) is rewritten as

\[
\begin{align*}
\vdash & A \mathcal{T} B \equiv B \lor (A \land \mathcal{T} (A \mathcal{T} B)), \\
\vdash & A \mathcal{B} \equiv B \lor (A \land \mathcal{B} (A \mathcal{B} B)), \\
\vdash & \{A \supset \mathcal{T} (A \mathcal{B} B)\} \supset \{A \supset A \mathcal{T} (B \lor C)\}, \\
\vdash & \{A \supset \mathcal{T} (A \mathcal{B} B)\} \supset \{A \supset A \mathcal{T} (B \lor C)\},
\end{align*}
\]

where \(A, B\) and \(C\) denote logic formulae.

Regarding state transitions in the future, there are two aspects, i.e., \(b\) (branching) and \(l\) (linear), thus the modes \(\mathcal{G}\) and \(\mathcal{T}\) are more precisely defined as (Katai, 1981):

- \(\mathcal{G}bA\): A will necessarily be persistent,
- \(\mathcal{T}bA\): A will possibly be persistent,
- \(\mathcal{T}A\): A will necessarily be the case,
- \(\mathcal{G}A\): A will necessarily be the case.

Figure 2 illustrates those modes where each circle denotes a state, each arc denotes a state transition, and the black circles mean that the state holds A.

Furthermore, \(\mathcal{T}\) also can be branching (\(\mathcal{T}b\)) or linear (\(\mathcal{T}l\)) as shown in Fig. 3. In the figure, each black circle means that B is true in that state, and the letter A means that A is true in that state. Among them, the following relations are established: \(\mathcal{G}bA \equiv A \mathcal{T} b (B \land \neg B) \equiv \neg \mathcal{T} b \neg A\), \(\mathcal{T}bA \equiv A \mathcal{T} l (B \land \neg B) \equiv \neg \mathcal{T} l \neg A\).

### 2.2.2 Deontic Modalities

Understanding the system’s behavior by temporal logic is of an “objective” view of the focused proposition. To represent our “subjective” intention or purpose, such as how the propositions should behave, i.e., the control rule (or task), we introduce deontic modalities:

- \(\mathcal{O}\mathcal{T} A\): A is obligatory, \(\mathcal{T}A\): A is permitted.

The axiom system we adopt here for \(\mathcal{O}\) and \(\mathcal{T}\) is that of SDL (standard deontic logic), which defines \(\mathcal{O} A \equiv \neg \mathcal{T} \neg A\), and claims \(\vdash \mathcal{O} A \supset \mathcal{T} A\), and \(\vdash \mathcal{O} (A \supset B) \supset (\mathcal{O} A \supset \mathcal{O} B)\).

Some control rules and specifications of systems can be translated into the combinations of temporal and deontic modes by using “translation templates” such as

\(\mathcal{O} \mathcal{G} A\): A has to be true in the future,
\(\mathcal{O} \mathcal{T} A\): A can always be true.

They correspond to alethic modes \(\mathcal{T} A\) and \(\mathcal{G} A\) respectively.

#### 2.3 Network Representation of Tasks

We translate the task represented by modal logic into an extended Petri net, which we call a “task unit graph,” by introducing four types of special arcs shown in Fig. 4.

![Diagram](image-url)
These arcs are placed from a place to a transition. They function whenever the place holds a token and the transition satisfies the firing condition, but they differ from regular arcs of the conventional Petri net on the following points. First, they do not transfer tokens from places to arcs. Next, if there are multiple special arcs from the same place, all of them are activated simultaneously. As a result, simultaneous firing of multiple transitions is permitted at the same state.

Figure 5: Examples of task unit graph.

Figure 5 shows examples of task unit graphs, and these net representations are derived by a systematic analysis of logical representations of tasks. For instance, consider a task $O(A \cup B)$. A has to be true from now on until $B$ will be the case. If $B$ is the case at $S_0$, this task is accomplished, else if $\neg A \land \neg B$ at $S_0$, this task cannot be accepted due to Eq. (1). If $A \land \neg B$ at $S_0$, $A$ should be maintained and $O(A \cup B)$ also has to be the case at $S_1$. In each case, $O(A \cup B)$ at $S_0$ prohibits the alteration from $A$ to $\neg A$, so an arc of "prohibition of firing" is placed from the place of $O(A \cup B)$ to the "transition of the alteration from $A$ to $\neg A.""

3 DECENTRALIZED SYSTEM STRUCTURE

This paper defines the characteristics of decentralized systems as
- each agent has its own territory,
- each control task is given to one of these agents,
- a task can control a transition within the territory of the agent to whom the task is given.

Figure 1 is an example of decentralized system. It consists of four agents who are in charge of managing the sub tasks denoted by thick broken circles in Fig. 1.

Assume that the traveler has her/his own policy for using e-agents and adopts them as control rules in order to fulfill her/his needs such as:

- PP1: defrayment must not be done simultaneously with submission of a landmark/hotel location;
- PP2: hotel search is always followed by a landmark search;
- PP3: submission of either a hotel or a landmark is accepted only if the first cache of the train search is empty;
- PP4: the system should request a hotel submission some time in the future before the end of a set of executions.

Each task associated with each agent is activated by the firing of the corresponding transition in its territory, e.g., in this case we have the following tasks represented in temporal deontic logical forms,

$$
 PP0: \text{agent } A_2 \text{ should not submit a hotel location until the cache memory flushes its contents to the memory, is resident and translated into:} \\
 O_{2} G ( ( \neg P_{4} ) \cup ( \neg P_{4} \land P_{5} ) ) \cup ( \neg P_{1} ).
$$

Defining $Q \equiv \neg P_{4} \land P_{5}$, $H \equiv \neg P_{5} \cup Q$, $PP0$ can be represented as $O_{2} G ( HS ( \neg P_{1} ) )$, which derives an extended Petri net representation as shown in the left part of Fig. 6 (Katai, 1981).

It consists of module nets reflecting its subtasks, and they are joined with linkage relations prescribing concurrent (simulations) firing of linked relations shown in Fig. 4 (d). It should be noted that the transitive closure of these linkage relations links the task in analysis with the target system, which is shown in the middle part of Fig. 6. In the figure, task unit graphs showing $\tau$ and $\land$ (conjunction) are employed. Their general types are defined as shown in Fig. 7.

4 CORRECTNESS OF SYSTEM AND ITS BEHAVIOR

4.1 Behavioral Corrections

A system can be regarded to behave correctly iff it satisfies the following conditions:
Definition 1 (Correctness of State Transition Sequence). A sequence of state transition is correct if the following cases do not occur along with this infinite sequence:

1. Any proposition $A$ does not occur just after the state where $O_i \mathcal{G} (O_i A, O_i (A U B))$ is given to an agent $i$.

2. $A$ never happens to the case after the state (until $B$ is the case) where $O_i \mathcal{F} A (O_i (A U B))$ is given.

4.2 System Correctness and its Characterization

We have two kinds of system corrections as follows:

Definition 2 (Strong Correctness of System). A system is called strongly correct if we can extend an arbitrary generated state transition sequence so that it is correct by appropriately executing the firing of permitted (legal) transitions. This can be characterized as:

$$
\mathcal{G}b (\neg C), \quad \mathcal{G}b (O_i \mathcal{G} A \supset \mathcal{F}_i A), \quad \mathcal{G}b \{O_i (\neg (A U B)) \supset \neg (A U B)\},
$$

where $C$ is a state of contradiction.

We have practically more important and weaker notions of system correctness as follows:

Definition 3 (Weak Correctness - Centralized Governability). A system is called weakly correct if we can extend an arbitrary generated state transition sequence so that it is correct by appropriately executing the firing of permitted (legal) transitions. This can be characterized as:

$$
\mathcal{G}b (\neg C), \quad \mathcal{G}b (O_i \mathcal{G} A \supset \mathcal{F}_i A), \quad \mathcal{G}b \{O_i (\neg (A U B)) \supset \neg (A U B)\}.
$$

In terms of the state transition diagram, the current notion of strong correctness can be characterized as:

Theorem 1 (Strong Correctness of System). A system is strongly correct if the following hold:

(a) for an arbitrary terminal state of its state transition diagram, there is no task associated with it of the form of $O_i \mathcal{G} A$, and if $O_i \mathcal{F} A$ is present there, then $A$ is also present on that state, and if $O_i (\neg (A U B))$ is present there, then both $\neg A$ and $\neg B$ are also there;

(b) for an arbitrary cycle (circuit) of its state transition diagram, the following hold:
Theorem 2 (Weak Correctness of System). A system is weakly correct iff the following hold:

(c) the same as condition (a) in Theorem 1;
(d) for every terminal strong component of its state transition diagram the following hold:

(d.1) if $o_j \neg A$ is present in a state at the component, there is a state (and possibly another) on which $A$ holds;
(d.2) if $o_j (\neg (A \cup B))$ is present in a state $s$, then there is a state $s'$ on which $\neg A$ holds and there is a path joining $s$ and $s'$ along which $B$ never holds.

5 DECENTRALIZED GOVERNABILITY OF PROCESSES

5.1 Method of System Correction

From the above results, we will have two ways of making an arbitrary system to behave correctly:

(i) to make the system strongly correct,
(ii) first to make the system weakly correct and then to control it so that its behavior (generated state transition sequence) becomes correct.

In the first approach, there is no more need to control it, i.e., any state transition sequence yielded from it is surely correct. In the latter approach, weak correctness itself is merely a precondition on governability and there is still need for supplementary control on permitted transitions. In other words, weak correctness guarantees the possibility of this supplementary control. In this paper we will pursue the latter approach, which seems to be of more practical importance than the former.

5.2 System Correction in Terms of State Transition Diagram

It can be readily seen that the following modifications on state transition diagrams are necessary for making systems be weakly correct:

(ii.1) remove the terminal states from the diagram at which either a task

- $o_j T A$ is present, or
- $o_j F A$ and $\neg A$ are present, or
- $o_j (\neg (A \cup B))$ is present and at least one of $\neg A$ or $\neg B$ is absent;

(ii.2) remove the terminal strong components, which include

- $o_j F A$ but $\neg A$, or
- $o_j (\neg (A \cup B))$ at a state $s$ and $\neg A$ at a state $s'$ such that any path from $s$ to $s'$ includes a state at which $B$ does not hold.

The above operations on state transition diagrams need to be applied repeatedly because removal of terminal states or strong components will yield different terminal states and strong components. The operations proceed until there is no need for them. If we still have remaining states in the diagram, the system is modified to be weakly correct.

An Example of Conflict Detection. The typical conflicts are observed among tasks. Figure 8 shows the diagram of a portion of the sequence of state transitions of the target system with the initial state S0, which holds $P_1$ and $P_6$, and is in charge of tasks PP0, 1, 2, 3, and 4. Table 1 shows the markings of each state where “•” means a normal token and “•” means an active token, which constraints other tokens.

![State transition diagram of the system in table 1](image-url)

The terminal state S5 is removed from the diagram by the operation (ii.1) since $o_j F P_3$ is required but $\neg P_3$ at the state. As a result, there is no “terminal strong component” in the diagram and the system becomes weakly correct.

The conflict in the state S5 can be detected by tracing synchronized firing linkages (broken lines in Fig. 9) as mutual interferences among tasks. In state
S5, place $O_1 P_1 \cup (\neg P_4)$ has a token, which prohibits firing of $\tau_1$ and $\tau_3$. On the other hand, the token in place $O_2 \exists P_3$ requests firing of $\tau_3$. Therefore, there is a conflict of firing $\tau_3$ in state S5. The only way to resolve this conflict is turning $P_4$ to $\neg P_4$, which leads the token in $P_1 \cup (\neg P_4)$ to free. But the establishment of $\neg H$ in state S5 prohibits turning $P_4$ to $\neg P_4$ by tracing synchronized firing linkages from $\neg H S (\neg P_7))$. As a result, this conflict cannot be resolved unless $\neg H$ turns to $H$.

5.3 Derivation of Control Rules

The above modifications on transition diagrams can be translated into control actions on the extended Petri net systems. The removal of terminal states becomes (ii.1') prohibit the firing of transitions just before (leading to) the removed components.

(ii.2') prohibit the firing of transitions just before (leading to) the removed components.

It should be noted that we need supplementary control actions over weakly correct systems for making their behavior correct. More precisely, by referring to conditions (d.1) and (d.2) in Theorem 2, we need the following operations:

(d.1') if we arrive at a state where a task of the form $O_i \exists A$ is present, then we must eventually (surely in the future) arrive at a state where $A$ is realized.

(d.2') if we come to a state where $O_i (\neg (A \cup B))$ is present, then we must eventually arrive at a state where $\neg A$ holds by going through states at which $B$ is not the case.

5.4 Decentralized Governability

In the above control operations, we have to consider the decentralized nature of systems, i.e., each agent
having its own territory over which it has control. Namely, we should set the following:

**Condition (System Decentralization).** Each prohibition of firing of a transition (ii.1’) or (ii.2’) must be caused by a task that is given to an agent whose territory includes this transition.

Thus, all the prohibition operations along with the course of deriving a weakly correct system should be subject to this condition. It should be noted that there may be various ways of deriving weakly correct systems, and only a portion of them may satisfy the above condition. Hence, it is not easy to verify the following property of an arbitrarily given decentralized system.

**Definition 4 (Decentralized Governability).** A system is called “decentralized controllable” if there exists a sequence of operations (ii.1’) and (ii.2’) in which all the prohibitions of transition firing are in accordance with the above condition on system decentralization.

### 6 CONCLUSIONS

We have introduced the notions of decentralized governance of event-driven processes, their behavioral and structural correctness, and centralized and decentralized control for attaining correct behavior, as well as that of decentralized governability. Also the method of deriving correct behavioral processes is shown. The basic framework we adopted was temporal and deontic logical analyses of process behavior and Petri net representations of process structures.

The hierarchical decomposition of tasks elucidates the governance (control) structure of tasks over the event-driven process described by Petri net systems. The control flows descend the hierarchy, while the flows of information reporting the changes of object systems ascend the hierarchy. These flows go along the fire synchronization arcs. Also the conflicts between the control flows are elucidated. These analyses and methods are expected as a basis for treating complex business processes that are subject to high reliability and credibility under complicated decentralized governance structures.

### REFERENCES


