Wiener-Hopf Analysis of the Diffraction by a Finite Sinusoidal Grating: The Case of H Polarization

Toru Eizawa and Kazuya Kobayashi
Department of Electrical, Electronic, and Communication Engineering, Chuo University, 1-13-27 Kasuga, Bunkyo-ku, Tokyo 112-8551, Japan
eizawa@mth.biglobe.ne.jp, kazuya@tamacc.chuo-u.ac.jp

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Abstract: The diffraction by a finite sinusoidal grating is analyzed for the H-polarized plane wave incidence using the Wiener-Hopf technique combined with the perturbation method. The scattered far field is evaluated with the aid of the saddle point method, and scattering characteristics of the grating are discussed via numerical examples of the far field intensity.

1 INTRODUCTION

The analysis of the diffraction by gratings is important in electromagnetic theory and optics. Various analytical and numerical methods have been developed and the diffraction phenomena have been investigated for many kinds of gratings (Ikuno and Yasuura, 1973; Shestopalov et al., 1973; Hinata and Hosono, 1976; Petit, 1980; Okuno, 1993).

The Wiener-Hopf technique is known as a rigorous approach for analyzing wave scattering problems related to canonical geometries, and can be applied efficiently to the analysis of the diffraction by specific periodic structures.

Most of the analyses in the relevant works done in the past are restricted to periodic structures of infinite extent and plane boundaries and hence, it is important to investigate the problems without these restrictions. In our previous paper, we have considered a finite sinusoidal grating as an example of gratings of finite extent and non-plane boundaries, and analyzed the E-polarized plane wave diffraction based on the Wiener-Hopf technique combined with the perturbation method (Kobayashi and Eizawa, 1991). This problem is important in investigating the end effect of finite gratings as well as the applicability of the Wiener-Hopf technique to obstacles with non-plane boundaries.

In this paper, we shall consider the same grating geometry as in Kobayashi and Eizawa (1991), and analyze the diffraction problem for the H-polarized plane wave incidence. Assuming that the corrugation amplitude of the grating is small compared with the wavelength, the original problem is approximately replaced by the problem of the H-polarized plane wave diffraction by a flat strip with a certain mixed boundary condition. We also expand the unknown scattered field using a perturbation series and separate the diffraction problem under consideration into the zero-order and the first-order boundary value problems.

Introducing the Fourier transform for the unknown scattered field and applying boundary conditions in the transform domain, the problem is formulated in terms of the zero- and first-order Wiener-Hopf equations, which are solved exactly via the factorization and decomposition procedure. However, the solution is formal in the sense that branch-cut integrals with unknown integrands are involved. These branch-cut integrals are then evaluated asymptotically for the width of the grating large compared with the wavelength, leading to an explicit high-frequency solution. Taking the Fourier inverse of the solution in the transform domain and applying the saddle point method, the scattered far field in the real space is derived. Based on these results, we have carried out numerical computation of the far field intensity for various physical parameters. Scattering characteristics of the grating are discussed in detail via numerical examples.

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

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2 STATEMENT OF THE PROBLEM

We consider the diffraction of an H-polarized plane wave by a finite sinusoidal grating as shown in Fig. 1, where the grating surface is assumed to be infinitely thin, perfectly conducting, and uniform in the y- direction, being defined by

$$x = h \sin mz \ (m > 0,\ h > 0) \tag{1}$$

for $|z| \le a.$ In view of the grating geometry and the characteristics of the incident field, this is a two-dimensional problem.

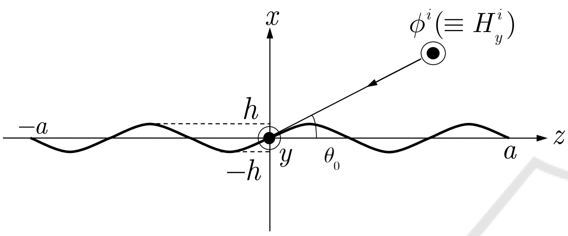


Figure 1: Geometry of the problem.

Let us define the total magnetic field $\phi^t(x,z)$ $[\equiv H_y^t(x,z)]$ by

$$\phi^t(x,z) = \phi^i(x,z) + \phi(x,z), \tag{2}$$

where $\phi^i(x,z)$ is the incident field of H polarization given by

$$\phi^i(x,z) = e^{-ik(x\sin\theta_0 + z\cos\theta_0)} \tag{3}$$

for $0 < \theta_0 < \pi / 2$ with $k[\equiv \omega(\varepsilon_0\mu_0)^{1/2}]$ being the free-space wavenumber. The scattered field $\phi(x,z)$ satisfies the two-dimensional Helmholtz equation

$$(\partial^2/\partial x^2 + \partial^2/\partial z^2 + k^2)\phi(x,z) = 0. \tag{4}$$

Once the solution of (4) has been found, nonzero components of the scattered electromagnetic fields are derived from the following relation:

$$(H_y, E_x, E_z) = \left(\phi, \frac{1}{i\omega\varepsilon_0}\frac{\partial\phi}{\partial z}, \frac{i}{\omega\varepsilon_0}\frac{\partial\phi}{\partial x}\right). \tag{5}$$

According to the boundary condition, tangential components of the total electric field $E_{\tan}^t$ satisfies

$$E_{\tan}^t = \frac{\partial\phi^t(h\sin mz, z)}{\partial n} = 0, |z| < a, \tag{6}$$

where $\partial/\partial n$ denotes the normal derivative on the grating surface. We assume that the corrugation depth $2h$ is small compared with the wavelength, and expand (6) in terms of a Taylor series around $x = 0.$ Then by ignoring the $O(h^2)$ terms from the Taylor expansion, we obtain that

$$\frac{\partial\phi^t(0,z)}{\partial x} + h\left[\sin mz \frac{\partial^2\phi^t(0,z)}{\partial x^2} - m\cos mz \frac{\partial\phi^t(0,z)}{\partial z}\right] + O(h^2) = 0 \tag{7}$$

for $|z| < a.$ Equation (7) is the approximate boundary condition used throughout the rest of this paper.

We express the unknown scattered field $\phi(x,z)$ in terms of a perturbation series expansion in h as

$$\phi(x,z) = \phi^{(0)}(x,z) + h\phi^{(1)}(x,z) + O(h^2), \tag{8}$$

where $\phi^{(0)}(x,z)$ and $\phi^{(1)}(x,z)$ are the zero-order and the first-order terms contained in the scattered field, respectively. Substituting (8) into (4) and using (2), (3), and (7), the original problem can be separated into the two perturbation problems.

The zero-order and first-order scattered fields $\phi^{(n)}(x,z)$ for $n = 0,1$ satisfy

$$(\partial^2 / \partial x^2 + \partial^2 / \partial z^2 + k^2)\phi^{(n)}(x,z) = 0, \tag{9}$$

where the boundary conditions are given by

$$\phi^{(0)}(+0,z) = \phi^{(0)}(-0,z)[\equiv \phi^{(0)}(0,z)], \tag{10}$$

$$\frac{\partial\phi^{(0)}(+0,z)}{\partial x} = \frac{\partial\phi^{(0)}(-0,z)}{\partial x}\left[\equiv \frac{\partial\phi^{(0)}(0,z)}{\partial x}\right], \tag{11}$$

$$\phi^{(1)}(+0,z) = \phi^{(1)}(-0,z)\left[\equiv \phi^{(1)}(0,z)\right], \tag{12}$$

$$\frac{\partial\phi^{(1)}(+0,z)}{\partial x} = \frac{\partial\phi^{(1)}(-0,z)}{\partial x}\left[\equiv \frac{\partial\phi^{(1)}(0,z)}{\partial x}\right] \tag{13}$$

for $|z| > a,$ and

$$\phi^{(0)}(+0,z) - \phi^{(0)}(-0,z) = j^{(0)}(0,z), \tag{14}$$

$$\frac{\partial\phi^{(0)}(0,z)}{\partial x} = ik\sin\theta_0 e^{-ikz\cos\theta_0}, \tag{15}$$

$$\phi^{(1)}(+0,z) - \phi^{(1)}(-0,z) = j^{(1)}(0,z), \tag{16}$$

$$\frac{\partial\phi^{(1)}(0,z)}{\partial x} + \sin mz \frac{\partial^2\phi^{(0)}(0,z)}{\partial x^2} - m\cos mz \frac{\partial\phi^{(0)}(0,z)}{\partial z} = \frac{ik}{2}\left[k\sin^2\theta_0 \sum_{n=1}^{2}(-1)^n e^{-ikz\cos\theta_n} - m\cos\theta_0 \sum_{n=1}^{2} e^{-ikz\cos\theta_n}\right] \tag{17}$$

for $|z| < a$ with

$$\cos\theta_{1,2} = \cos\theta_0 \mp m / k. \tag{18}$$

In (14) and (16), $j^{(0)}(0,z)$ and $j^{(1)}(0,z)$ are the zero-order and first-order terms of the unknown surface currents induced on the grating surface, respectively. As seen from the above discussion, the zero-order problem corresponds to the diffraction by a perfectly conducting flat strip. On the other hand, the first-order problem is important since it contains the effect due to the sinusoidal corrugation.

3 WIENER-HOPF EQUATIONS

For convenience of analysis, we assume that the medium is slightly lossy as in $k = k_1 + ik_2$ with $0 < k_2 \ll k_1$. Using the radiation condition, it follows from (8) that the zero- and first-order scattered fields $\phi^{(n)}(x,z)$ for $n = 0,1$ show the asymptotic behavior

$$\phi^{(n)}(x,z) = O(e^{-k_2|z|\cos\theta_0}),\ |\,z\,| \to \infty. \tag{19}$$

Let us introduce the Fourier transform of the scattered field $\phi^{(n)}(x,z)$ with respect to z as in

$$\Phi^{(n)}(x,\alpha) = (2\pi)^{-1/2}\int_{-\infty}^{\infty}\phi^{(n)}(x,z)e^{i\alpha z}dz, \tag{20}$$

where $\alpha = \mathrm{Re}\,\alpha + i\,\mathrm{Im}\,\alpha\,(\equiv \sigma + i\tau)$. It follows from (19) and (20) that $\Phi^{(n)}(x,\alpha)$ for $n = 0,1$ are regular in the strip $|\,\tau\,| < k_2\cos\theta_0$ of the complex α- plane. We also introduce the Fourier integrals as

$$\Phi_{\pm}^{(n)}(x,\alpha) = \pm(2\pi)^{-1/2}\cdot\int_{\pm a}^{\pm\infty}\phi^{(n)}(x,z)e^{i\alpha(z\mp a)}dz, \tag{21}$$

$$\Phi_1^{(n)}(x,\alpha) = (2\pi)^{-1/2}\int_{-a}^{a}\phi^{(n)}(x,z)e^{i\alpha z}dz, \tag{22}$$

$$J_1^{(n)}(0,\alpha) = (2\pi)^{-1/2}\int_{-a}^{a}j^{(n)}(0,z)e^{i\alpha z}dz. \tag{23}$$

Then it is seen that $\Phi_{\pm}^{(n)}(x,\alpha)$ are regular in $\tau \gtrless \mp k_2\cos\theta_0$ whereas $\Phi_1^{(n)}(x,\alpha)$ and $J_1^{(n)}(0,\alpha)$ are entire functions. It follows from (20)-(22) that

$$\Phi^{(n)}(x,\alpha) = e^{-i\alpha a}\Phi_-^{(n)}(x,\alpha) + \Phi_1^{(n)}(x,\alpha) + e^{i\alpha a}\Phi_+^{(n)}(x,\alpha). \tag{24}$$

Taking the Fourier transform of (9) and making use of (19), we derive that

$$[d^2/dx^2 - \gamma^2(\alpha)]\Phi^{(n)}(x,\alpha) = 0, \tag{25}$$

where $\gamma(\alpha) = (\alpha^2 - k^2)^{1/2}$ with $\mathrm{Re}\,\gamma(\alpha) > 0$. The solution of (25) is expressed as

$$\begin{aligned}\Phi^{(n)}(x,\alpha) &= A^{(n)}(\alpha)e^{-\gamma(\alpha)x},\ x > 0,\\ &= B^{(n)}(\alpha)e^{\gamma(\alpha)x},\ x < 0\end{aligned} \tag{26}$$

for $n = 0,1$, where $A^{(n)}(\alpha)$ and $B^{(n)}(\alpha)$ are unknown functions. Setting $x = \pm 0$ in (26) and arranging the results, we obtain that

$$\begin{aligned}&\Phi^{(n)}(+0,\alpha) - \Phi^{(n)}(-0,\alpha)\\ &\quad = A^{(n)}(\alpha) - B^{(n)}(\alpha),\end{aligned} \tag{27}$$

$$\begin{aligned}&\Phi^{(n)\prime}(+0,\alpha) - \Phi^{(n)\prime}(-0,\alpha)\\ &\quad = -\gamma(\alpha)[A^{(n)}(\alpha) - B^{(n)}(\alpha)],\end{aligned} \tag{28}$$

where the prime denotes differentiation with respect to x. Using the boundary conditions as given by (10)-(17), (26) is now expressed as

$$\Phi^{(0)}(x,\alpha) = \pm(1/2)J_1^{(0)}(\alpha)e^{\mp\gamma(\alpha)x}, \tag{29}$$

$$\begin{aligned}\Phi^{(1)}(x,\alpha) &= \pm(1/2)J_1^{(1)}(\alpha)e^{\mp\gamma(\alpha)x}\\ &+ [4i\gamma(\alpha)]^{-1}\{[\gamma^2(\alpha+m) - m(\alpha+m)]\\ &\cdot J_1^{(0)}(\alpha+m)\\ &- [\gamma^2(\alpha-m) + m(\alpha-m)]\\ &\cdot J_1^{(0)}(\alpha-m)]\}e^{\mp\gamma(\alpha)x}\end{aligned} \tag{30}$$

for $x \gtrless 0$. Equations (29) and (30) are the zero- and first-order scattered fields in the Fourier transform domain, respectively.

Setting $x = \pm 0$ in (29) and (30) and carrying out some manipulations with the aid of the boundary conditions, we are led to

$$\begin{aligned}&e^{-i\alpha a}U_-(\alpha) + K(\alpha)J_1^{(0)}(0,\alpha)\\ &\quad + e^{i\alpha a}U_{(+)}(\alpha) = 0,\end{aligned} \tag{31}$$

$$\begin{aligned}&e^{-i\alpha a}V_-(\alpha) + K(\alpha)J_1^{(1)}(0,\alpha)\\ &\quad + e^{i\alpha a}V_{(+)}(\alpha) = 0\end{aligned} \tag{32}$$

for $|\,\tau\,| < k_2\cos\theta_0$, where

$$U_-(\alpha) = \Phi_-^{(0)\prime}(0,\alpha) + \frac{A_0}{\alpha - k\cos\theta_0}, \tag{33}$$

$$U_{(+)}(\alpha) = \Phi_+^{(0)\prime}(0,\alpha) - \frac{B_0}{\alpha - k\cos\theta_0}, \tag{34}$$

$$V_-(\alpha) = \Psi_-(\alpha) - \sum_{n=1}^{2}(-1)^n\frac{A_nC_n}{\alpha - k\cos\theta_n}, \tag{35}$$

$$V_{(+)}(\alpha) = \Psi_+(\alpha) + \sum_{n=1}^{2}(-1)^n\frac{B_nC_n}{\alpha - k\cos\theta_n}, \tag{36}$$

$$K(\alpha) = \gamma(\alpha)/2 \tag{37}$$

with

$$\Psi_{\pm}(\alpha) = \Phi_{\pm}^{(1)\prime}(0,\alpha) + (1/2i) \cdot \{[\gamma^2(\alpha+m) - m(\alpha+m)] \cdot e^{\pm ima}\Phi_{\pm}^{(0)}(0,\alpha+m) - [\gamma^2(\alpha-m) - m(\alpha-m)] \cdot e^{\mp ima}\Phi_{\pm}^{(0)}(0,\alpha+m) + (2\pi)^{-1/2} m \cos ma \phi^{(0)}(0,a)\}, \quad (38)$$

$$\left.\begin{matrix} A_0 \\ B_0 \end{matrix}\right\} = -\frac{k \sin\theta_0 e^{\pm ika\cos\theta_0}}{(2\pi)^{1/2}}, \quad (39)$$

$$\left.\begin{matrix} A_n \\ B_n \end{matrix}\right\} = \frac{e^{\pm ika\cos\theta_n}}{(2\pi)^{1/2}}, \; n = 1,2, \quad (40)$$

$$C_n = (k/2)[k\sin^2\theta_0 - (-1)^n m\cos\theta_0], \quad n = 1,2. \quad (41)$$

Equation (31) and (32) are the zero- and first-order Wiener-Hopf equations, respectively.

4 EXACT AND ASYMPTOTIC SOLUTIONS

In this section, we shall solve the zero- and first-order Wiener-Hopf equations to obtain exact and asymptotic solutions. First we note that the kernel function $K(\alpha)$ is factorized as

$$K(\alpha) = K_+(\alpha)K_-(\alpha), \quad (42)$$

where $K_{\pm}(\alpha)$ are the split functions defined by

$$K_{\pm}(\alpha) = 2^{-1/2}e^{-i\pi/4}(k \pm \alpha)^{1/2}. \quad (43)$$

Multiplying both sides of (31) by $e^{\mp i\alpha a}/K_{\pm}(\alpha)$ and applying the decomposition procedure with the aid of the edge condition, we arrive at the exact solution with the result that

$$U_{(+)}(\alpha) = K_+(\alpha) \cdot \left\{ -\frac{B_0}{K_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{1}{2}[u_s(\alpha) - u_d(\alpha)] \right\}, \quad (44)$$

$$U_-(\alpha) = K_-(\alpha) \cdot \left\{ \frac{A_0}{K_-(k\cos\theta_0)(\alpha - k\cos\theta_0)} + \frac{1}{2}[u_s(-\alpha) + u_d(-\alpha)] \right\}, \quad (45)$$

where

$$u_{s.d}(\alpha) = \frac{1}{\pi i}\int_k^{k+i\infty} \frac{e^{2i\beta a}U_{(+)}^{s,d}(\beta)}{K_-(\beta)(\beta+\alpha)} d\beta, \quad (46)$$

$$U_{(+)}^{s,d}(\alpha) = U_{(+)}(\alpha) \pm U_-(\alpha). \quad (47)$$

Equations (44) and (45) are formal since branch-cut integrals with the unknown integrands $U_{(+)}^{s,d}(\beta)$ are involved. Applying a rigorous asymptotic method developed by Kobayashi (2013), we obtain a high-frequency solution explicitly as in

$$U_-(\alpha) \sim K_-(\alpha) \cdot \frac{A_0}{K_-(k\cos\theta_0)(\alpha - k\cos\theta_0)} + K_-(\alpha)[C_1^u\xi(-\alpha) + B_0\eta_0^b(-\alpha)], \quad (48)$$

$$U_{(+)}(\alpha) \sim -K_+(\alpha) \cdot \frac{B_0}{K_+(k\cos\theta_0)(\alpha - k\cos\theta_0)} + K_+(\alpha)[C_2^u\xi(\alpha) + A_0\eta_0^a(\alpha)] \quad (49)$$

for $ka \to \infty$, where

$$C_{1,2}^u = \frac{K_+(k)}{1 - K_+^2(k)\xi^2(k)} \cdot [\chi_0^{a,b}(k) + K_+(k)\xi(k)\chi_0^{b,a}(k)], \quad (50)$$

$$\xi(\alpha) = -\frac{2a^{1/2}e^{2ika}}{\pi}\Gamma_1(1/2, -2i(\alpha+k)a), \quad (51)$$

$$\eta_0^{a,b}(\alpha) = \frac{\xi(\alpha) - \xi(\pm k\cos\theta_0)}{\alpha \mp k\cos\theta_0} \quad (52)$$

with

$$\chi_0^a(\alpha) = A_0\eta_0^a(\alpha) + B_0P_0^b(\alpha), \quad (53)$$

$$\chi_0^b(\alpha) = B_0\eta_0^b(\alpha) + A_0P_0^a(\alpha), \quad (54)$$

$$P_0^{a,b}(\alpha) = \frac{1}{\alpha \pm k\cos\theta_0} \cdot \left[\frac{1}{K_+(\alpha)} - \frac{1}{K_{\mp}(k\cos\theta_0)}\right]. \quad (55)$$

In (51), $\Gamma_1(\cdot,\cdot)$ is the generalized gamma function (Kobayashi, 1991) defined by

$$\Gamma_m(u,v) = \int_0^\infty \frac{t^{u-1}e^{-t}}{(t+v)^m} dt \quad (56)$$

for $\operatorname{Re} u > 0$, $|v| > 0$, $|\arg v| < \pi$, and positive integer m. This completes the solution of the zero-order Wiener-Hopf equation (31).

A similar procedure may also be applied to the first-order Wiener-Hopf equation (32). Omitting the whole details, we arrive at a high-frequency solution with the result that

$$V_-(\alpha) \sim -\sum_{n=1}^{2}(-1)^n \cdot \frac{C_n e^{ika\cos\theta_n}(\alpha-k)^{1/2}}{(2\pi)^{1/2}(k\cos\theta_n-k)^{1/2}(\alpha-k\cos\theta_n)} + K_-(\alpha)[D_1^v\xi(-\alpha) + B_1\eta_1^b(-\alpha)+B_2\eta_2^b(-\alpha)], \quad (57)$$

$$V_{(+)}(\alpha) \sim \sum_{n=1}^{2}(-1)^n \cdot \frac{C_n e^{-ika\cos\theta_n}(\alpha+k)^{1/2}}{(2\pi)^{1/2}(k\cos\theta_n+k)^{1/2}(\alpha-k\cos\theta_n)} + K_+(\alpha)[D_2^v\xi(\alpha) + A_1\eta_1^a(\alpha)+A_2\eta_2^a(\alpha)] \quad (58)$$

as $ka \to \infty$, where

$$D_{1,2}^v = \frac{K_+(k)}{1-K_+^2(k)\xi^2(k)} \cdot \sum_{n=1}^{2}\left[\chi_n^{a,b}(k)+K_+(k)\xi(k)\chi_n^{b,a}(k)\right] \quad (59)$$

and

$$\eta_n^{a,b}(\alpha) = -(-1)^n C_n \frac{\xi(\alpha)-\xi(\pm k\cos\theta_n)}{\alpha \mp k\cos\theta_n}, \quad (60)$$

$$\chi_n^a(\alpha) = A_n\eta_n^a(\alpha) - (-1)^n B_n C_n P_n^b(\alpha), \quad (61)$$

$$\chi_n^b(\alpha) = B_n\eta_n^b(\alpha) - (-1)^n A_n C_n P_n^a(\alpha), \quad (62)$$

$$P_n^{a,b}(\alpha) = \frac{1}{\alpha \pm k\cos\theta_n} \cdot \left[\frac{1}{K_+(\alpha)} - \frac{1}{K_\mp(k\cos\theta_n)}\right] \quad (63)$$

for $n=1,2$. Equations (48), (49) and (57), (58) yield high-frequency asymptotic solutions of the zero- and first-order Wiener-Hopf equations (31) and (32), respectively.

5 SCATTERED FAR FIELD

We shall now derive analytical expressions of the scattered field by using the results obtained in Section 4. The scattered field $\phi^{(n)}(x,z)$ with $n=0,1$ in the real space can be derived by taking the inverse Fourier transform of (20) according to the formula

$$\phi^{(n)}(x,z) = (2\pi)^{-1/2} \cdot \int_{-\infty+ic}^{\infty+ic}\Phi^{(n)}(x,\alpha)e^{-i\alpha z}d\alpha, \quad (64)$$

where c is a constant satisfying $|c| < k_2\cos\theta_0$. Introducing the cylindrical coordinate

$$x=\rho\sin\theta,\ z=\rho\cos\theta,\ -\pi<\theta<\pi \quad (65)$$

and applying the saddle point method with the aid of (29)-(32), we derive, after some manipulations, that

$$\phi^{(0)}(\rho,\theta) \sim \mp[e^{ika\cos\theta}U_-(-k\cos\theta) + e^{-ika\cos\theta}U_{(+)}(-k\cos\theta)] \cdot \frac{k\sin|\theta|}{2K(-k\cos\theta)}\frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}}, \quad (66)$$

$$\begin{aligned}\phi^{(1)}(\rho,\theta) \sim{} & \mp[e^{ika\cos\theta}V_-(-k\cos\theta) + e^{-ika\cos\theta}V_{(+)}(-k\cos\theta)] \cdot \frac{k\sin|\theta|}{2K(k\cos\theta)}\frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}} \\ & + \sum_{n=1}^{2}\frac{(-1)^n}{K(k\cos\theta^{(n)})}[4K^2(k\cos\theta^{(n)}) - (-1)^n mk\cos\theta^{(n)}] \\ & \cdot[e^{ika\cos\theta^{(n)}}U_-(-k\cos\theta^{(n)}) + e^{-ika\cos\theta^{(n)}}U_{(+)}(-k\cos\theta^{(n)})] \cdot \frac{k\sin|\theta|}{8iK(k\cos\theta)}\frac{e^{i(k\rho-\pi/4)}}{(k\rho)^{1/2}} \end{aligned} \quad (67)$$

with $x \gtrless 0$ as $k\rho \to \infty$, where

$$\theta^{(1),(2)} = \cos^{-1}(\cos\theta \mp m/k). \quad (68)$$

It is to be noted that (66) and (67) are uniformly valid for arbitrary incidence and observation angles.

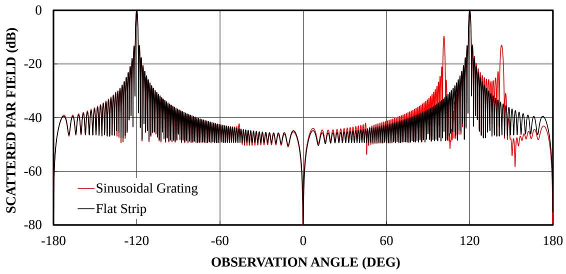
6 NUMERICAL RESULTS AND DISCUSSION

In this section, we shall present numerical examples on the far field intensity and discuss scattering characteristics of the grating. For convenience, let us introduce the normalized far field intensity as in

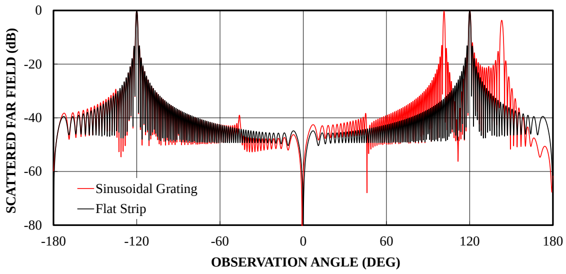
$$|\phi(\rho,\theta)|[\text{dB}] = 20\log_{10}\left[\frac{\lim_{\rho\to\infty}|(k\rho)^{1/2}\phi(\rho,\theta)|}{\max_{|\theta|<\pi}\lim_{\rho\to\infty}|(k\rho)^{1/2}\phi(\rho,\theta)|}\right], \quad (69)$$

where

$$\phi(\rho,\theta) = \phi^{(0)}(\rho,\theta) + h\phi^{(1)}(\rho,\theta). \quad (70)$$



(a) $2h = 0.1\lambda$.



(b) $2h = 0.3\lambda$.

Figure 2: Normalized far field intensity $|\phi(\rho,\theta)|$ [dB] for $\theta_0 = 60^\circ$, $2a = 50\lambda, m/k = 0.3$.

Figure 2 shows numerical examples of the scattered far field intensity as a function of observation angle for $\theta_0 = 60^\circ$ and $2a = 50\lambda$, where red and black curves denote the sinusoidal grating with $2h = 0.1\lambda, 0.3\lambda$, $m/k = 0.3$, and a perfectly conducting flat strip. We see that the effect of sinusoidal corrugation is noticeable in the reflection region $90^\circ < \theta < 180^\circ$, and the far field intensity has sharp peaks at two particular observation angles around the specular reflection direction at $\theta = 120^\circ$. These angles are $101.5^\circ, 143.1^\circ$ and are found to be coincident with the directions of $\pi - \theta_1$ and $\pi - \theta_2$ deduced from (18), which correspond to the propagation directions of the (-1) and $(+1)$ order waves involved in the Floquet's space harmonic modes, respectively.

7 CONCLUSIONS

In this paper, we have analyzed the H-polarized plane wave diffraction by a finite sinusoidal grating using the Wiener-Hopf technique combined with the perturbation method. Our final solution is valid under the condition that the width and the depth of the grating is large and small compared with the wavelength. Based on the results, we have carried out numerical computation of the scattered far field with the choice of typical physical parameters, and investigated the effect of the sinusoidal corrugation of the grating. The results presented in this paper may be useful in design of corrugated reflector antennas.

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