

Iterative Robust Registration Approach based on Feature Descriptors Correspondence

Application to 3D Faces Description

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Abstract: In this paper, we intend to introduce a fast surface registration process which is independent from the original parameterization of the surface and invariant under 3D rigid transformations. It is based on a feature descriptors correspondence. Such feature descriptors are extracted from the superposition of two surfacic curves: geodesic levels and radial ones from local neighborhoods defined around reference points already picked on the surface. A study of the optimal number of those curves thanks to a generalized version of Shannon theorem is developed. Thus, the obtained discretized parametrisation (ordered descriptors) is the basis of the matching phase that becomes obvious and more robust comparing to the classic ICP algorithm. Experimentations are conducted on facial surfaces from the Bosphorus database to test the registration of both rigid and non-rigid shapes (neutral faces vs. faces with expressions). The Hausdorff distance in shape space is used as an evaluation metric to test the robustness to tessellation. The discriminative power in face description is also estimated.

1 INTRODUCTION

Global registration is a fundamental issue in shape acquisition and shape modeling. The principle is as follows: given two shapes, the model and the data, the goal is to find the optimal rigid transformation that readjusts or registers the data to the model. Many research works focused on this topic while it is applied in several fields, essentially two key ones: the combination of partial models of a same object in order to obtain a numerical complete model and 3D shapes recognition.

Several approaches have been proposed to solve the problem of registration of two tridimensional shapes (a cloud of points, triangulated surfaces, implicit or parametric surfaces)(Fallavollita, 2009).

The most popular algorithm is the Iterative Closest Point (ICP). Its principle consists on iteratively alternating two steps: a matching step and a transformation estimation one.

Indeed, the matching phase necessitates finding for each point p on a first surface, the "best" matching point q on the second one. The second step is an optimization of the rigid transformation that aligns temporarily the data to the model. This process is it-

erated until a convergence criterion is reached.

The main limitation of ICP is that a local convergence could not necessarily guaranty an optimal global convergence. To overcome this limitation, the registration process can be improved by the use of geometric descriptors. In fact, a local geometric descriptor is a quantity defined for each point and based on the shape of a local neighborhood around that point.

The idea is to suggest that the points whose descriptors correspond, could potentially correspond. Thus, the choice of those points is based on the computation of a given geometric descriptor.

In fact, the description of 3D shapes remains an open issue and many works have been proposed. The three-dimensional surface description methods can be classified into four major categories: the transform based approaches, the 2D views, the graph ones and those based on statistical features.

For the transform based approaches, after the conversion of the surface onto 3D voxels or a spherical grid, specific transformations are applied. The most known ones are 3D Fourier, the 3D Radon, the rotation-invariant spherical harmonics (Dutagaci et al., 2005).

In the two dimensional view based methods, a

collection of 2D projections from canonical viewpoints is realized. Then, planar image descriptors are computed as Zernike moments and Fourier descriptors (Daoudi and Ghorbel, 1998).

The graph based approaches have the potential to code geometrical and topological shape properties in an intuitive manner. The usually used descriptors are Reeb graphs and the skeletal ones (Tung and Schmitt, 2005).

We especially focus on the fourth class of methods that are based on the extraction of numerical attributes (local or global) of the 3D objects. We cite in particular, the works for the determination of high curvature, the 3D moments, and the canonical 3D Hough transform descriptor (Hallinan et al., 1999).

Authors in (Scovanner et al., 2007) have proposed to generalize the SIFT algorithm, well known for 2D images, to the 3D field.

Many works have introduced geometric descriptors extracted from curves defined on 3D surfaces. Several representations of curves were proposed in the literature:

(Maalej et al., 2011) define levels of curves with reference to a function distance taken to be the Euclidean distance or the geodesic one.

(Samir et al., 2006) and (Mpiperis et al., 2007) defined local coordinates by the exponential map around a point belonging to the surface. This coordinate system is obtained by wrapping a neighborhood of that point by the polar coordinates of the tangent plane at it.

Indeed, the registration algorithm accuracy doesn't only depend on the choice of points used but remains also tributary to the number of those points. The key idea is that only an optimal number of selected points would ensure the effectiveness of the registration process.

Contributions

In this paper, we present an iterative registration approach based on features correspondence using geometric descriptors. The main contributions of the proposed approach are as follows:

- The shape descriptor introduced is constructed to ensure invariance to small variations in shape and is relatively isotropic towards the 3D motion group.
- The proposed process ensures more robustness in the correspondence phase thanks to the identification of particular feature points obviously correspondent.
- The number of points involved in the registration process is fixed as result of an optimization sampling technique.
- The proposed approach shows effective discrimination in face description.

Thus, this paper will be structured as follows: We

present in the second section the descriptor construction steps. In particular, its mathematic formulation and its optimal resolution are explained. We show in the next section the registration algorithm phases. Then, the used similarity metric to compare shapes (the Hausdorff distance in the shape space) is clarified in section 4. In section 5, the experimentation results are showed, particularly the robustness to sampling and the applicability to face description are tested. In the last section, a conclusion and some perspectives of the work are envisaged.

2 CONSTRUCTION OF THE DESCRIPTOR

This section is devoted to explain the extraction procedure of the local surface representation in order to obtain the geometric descriptors.

2.1 Determination of Descriptor Points Relatively to a given Reference Point

Let S be a differential manifold of dimension two and r a reference point taken on the surface.

We denote a function $U_r : S \rightarrow R$ to be a surface distance function from r to any point p on the surface. In fact, $U_r(p)$ is the length of the shortest geodesic path joining r to p (Yin et al., 2006).

We define the level set of the geodesic curves around r as the n level sets of the function $U_r(p)$ as following:

$$\xi_r^{\lambda_j} = \{p \in S, U_r(p) = \lambda_j\} \subset S \quad (1)$$

such that $\lambda_j \in [0, \infty)$ and $1 \leq j \leq n$

Now, we note the geodesic path $C(t)$ as the solution of the system (Γ) where ∇ is the gradient. This curve is called radial line respectively to the angular direction θ (Yin et al., 2006).

$$(\Gamma) \begin{cases} \frac{dC(t)}{dt} = -\nabla U_r(C) \\ C(0) = r \\ \frac{dC(t)}{dt} \Big|_{t=0} = \theta \end{cases} \quad (2)$$

Consequently, the set of these curves emanating from the reference point r , respectively to an angular direction $\theta_{1 \leq k \leq m}$ are denoted by $\xi_r^{\theta_k}$. It is important to notice that the reference angular direction ($\theta = 0$) could be chosen arbitrary.

The superposition of both level sets of curves $\xi_r^{\theta_k}$ and $\xi_r^{\lambda_j}$ constitutes a curvilinear local coordinate system.

Then, the intersections of the sets of radial lines $\xi_r^{\theta_k}$ and the geodesic level curves $\xi_r^{\lambda_j}$ define a set of points relatively to a given reference point r that are given by:

$$N_r = \{p_{j,k} \in \xi_r^{\lambda_j} \cap \xi_r^{\theta_k} \mid 1 \leq j \leq n, 1 \leq k \leq m\} \quad (3)$$

2.2 Determination of Descriptor Points Relatively to Several Reference Points

Let's consider now several reference points $\{r_{i,1 \leq i \leq n_r}\} \in S$ and let denote by $\xi_{r_i}^{\theta}$ and $\xi_{r_i}^{\lambda}$ their respective set of geodesic level curves and radial lines with reference to each reference point r_i .

The resulting set of local coordinates respectively to each reference point r_i is defined as follows:

$$N = \{N_{r_i}; 1 \leq i \leq n_r\} \quad (4)$$

– To be useful in registration process, a descriptor should be invariant towards rigid transformations and reflects the local geometry of the surface. Thus, the obtained set of points is relatively isotropic towards the 3D motion group. They are also parametric (and thus ordered) because each point is indexed by the level of geodesic curve and radial line it belongs to; relatively to each reference point.

Moreover, the volume of points participating in the registration process is of a paramount importance. In fact: More the number of points involved in the registration process is bigger; more the consuming time is greater. Thus, the issue is to guaranty that a minimum number of points picked from the surface is sufficient to ensure an optimal registration result. The determination of that minimal number of points is defined as follows.

2.3 Optimal Descriptor Points Number

The variation of both the number of geodesic curves and the number of radial ones makes several resolutions of the discretized representation.

The key idea now is to find the optimal number of those curves that ensures a good description of the given surface sufficiently precise.

So, we intend to propose a criterion that allows fixing the optimal number of curves composing the parametric system. That criterion consists in applying a generalized version of the Shannon theorem to the surface representation already developed (Daoudi and Ghorbel, 1998). The principle is as follows: Given a surface S that is a differential manifold of dimension two and r_i a reference point taken on S .

There exists necessarily a map (V_{r_i}, ϕ) where V_{r_i} is a neighborhood of r_i and ϕ is a α -diffeomorphism between a disk D of R^2 and V_{r_i} .

$$\phi(u, v) = (x(u, v), y(u, v), z(u, v))^t \quad (5)$$

With (x, y, z) the three-dimensional components of the extracted parametric points.

(u, v) the two-dimensional parametrization $0 \leq u \leq 1$ and $0 \leq v \leq 1$.

t means the transpose.

We denote by F_ϕ the two-dimensional Fourier Transform of each coordinate given by:

$$F_\phi(\vartheta_u, \vartheta_v) = (F_x, F_y, F_z)(\vartheta_u, \vartheta_v) \quad (6)$$

Where

$$F_x(\vartheta_u, \vartheta_v) = \int \int_D x(u, v) e^{-2i\pi(u\vartheta_u + v\vartheta_v)} dudv \quad (7)$$

The spectrum of the map is obtained by calculating the standard Fourier transform of ϕ :

$$\|F_\phi(\vartheta_u, \vartheta_v)\| = \sqrt{(F_x(\vartheta_u, \vartheta_v))^2 + (F_y(\vartheta_u, \vartheta_v))^2 + (F_z(\vartheta_u, \vartheta_v))^2} \quad (8)$$

We propose to apply the Shannon theorem to the local parametrisation ϕ in order to fix the frequencies ϑ_u^0 and ϑ_v^0 so that:

$$\text{supp}\{F_\phi(\vartheta_u, \vartheta_v)\} \subseteq [-\vartheta_u^0, \vartheta_u^0] \times [-\vartheta_v^0, \vartheta_v^0] \quad (9)$$

Where $\text{supp}(g)$ is the support of the function g . So, with this way and thanks to Shannon theorem, we fix the optimal number of geodesic level curves and radial lines and therefore the optimal number of points extracted from the intersection of those curves.

–So, with this way and thanks to an optimization sampling process based on the Shannon theorem, we fix the optimal number of geodesic level curves and radial lines and therefore the optimal number of points extracted from the intersection of curves. Only this optimal number of points will participate in the registration process. The feature points extraction procedure is summarized in figure 1.

3 REGISTRATION ALGORITHM

Here, we will illustrate the matching algorithm procedure.

3.1 Generation of Homologous Points Pairs

Only the parametric points extracted respectively from the model shape and the data shape, are involved

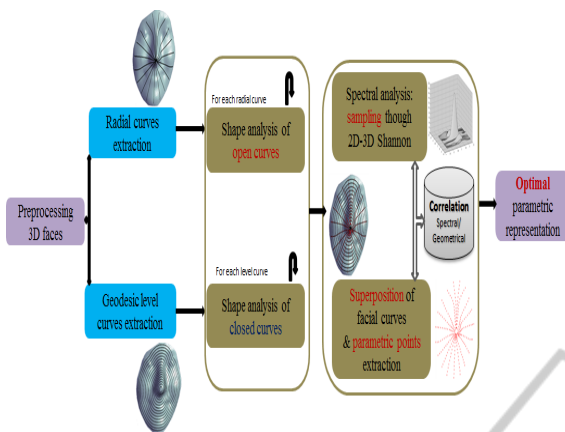


Figure 1: Optimal parametric points extraction.

in the registration procedure and thus in the matching phase. Those selected feature points are chosen such a way to make the correspondence phase easier and robust. In fact, each point is indexed by the geodesic curve and the radial line owned to. Thus, a pair of corresponding points is those belonging to the same level of radial line and geodesic curve respectively to the test and the reference surface.

3.2 Iterative Transformation Estimation

We notice that an initialization step is realized by the computation of the inertia matrix in order to globally readjust the two surfaces (this makes the surfaces in frontal position).

Then, the correspondence pairs of points are used to estimate temporarily the rigid transformation (rotation and translation) existing between the two shapes. The rotation is estimated thanks to the SVD technique and the translation is estimated between gravity points positions.

Such process is iterated until a convergence criterion is reached.

–That fine registration of two 3D shapes is based on several iterative local registrations around the given reference points.

This technique allows the registration of two objects with non-rigid transformations thanks to several rigid local registration processes around small neighborhoods.

4 SIMILARITY METRIC

It is important to define the used similarity metric to compare between different shapes. The well known Hausdorff shape distance introduced by Ghorbel

in (Ghorbel, 1998) is chosen. Following the same process, we denote by G the group representing all possible normalized parametrisations of surfaces which can be the real plane R^2 or the unit sphere S^2 . We consider the space of all surface pieces as the set of all 3D objects assumed diffeomorphic to G which can be assimilated to a subspace of $L^2_{R^3}(G)$ formed by all square integrated maps from G to R^3 . The direct product of the Euler rotations group $SO(3)$ by the group G , acts on such space in the following sense:

$$SO(3) \times G \times L^2_{R^3}(G) \rightarrow L^2_{R^3}(G) \quad (10)$$

$$\{A, (u_0, v_0), S(u, v)\} \rightarrow AS(u + u_0, v + v_0)$$

The 3D Hausdorff distance Δ can be written for every S_1 and S_2 belonging to $L^2_{R^3}(G)$ and g_1 and g_2 to $SO(3)$ as follows:

$$\Delta(S_1, S_2) = \max(\rho(S_1, S_2), \rho(S_2, S_1)) \quad (11)$$

Where:

$$\rho(S_1, S_2) = \sup_{g_1 \in SO(3)} \inf_{g_2 \in SO(3)} \|g_1 S_1 - g_2 S_2\|_{L^2} \quad (12)$$

$\|S\|_{L^2}$ denotes the norm of the functional banach space $L^2_{R^3}(G)$.

Due to the fact that the euclidean rotations preserve this norm, it is easy to show that this distance is reduced to the following quantity:

$$\Delta(S_1, S_2) = \inf_{h \in SO(3)} \|S_1 - h S_2\|_{L^2} \quad (13)$$

After that, we consider a normalized version of Δ so that the variations of this normalized distance are confined to the interval $[0,1]$.

5 EXPERIMENTAL RESULTS

This section presents experiments used to investigate the following questions:

- How robust is the descriptor with respect to point sampling?
- How well does the algorithm discriminate the human faces?

5.1 Robustness to Sampling

In this section, we aim to test the robustness of the descriptor by evaluating how efficient those features points extracted from discrete points sampling converges to those picked from the surface.

Because the number of points (as proved in section 2.3) is so important in the registration algorithm

and also to evaluate our proposal to fix the optimal number of descriptor points thanks to the Shannon theorem. The experimental protocol is as follows:

For a given model S , we generate a set of points S^N consisting of N randomly placed samples on the surface S (the same process as described in (Osada et al., 2001)). We compute then the Hausdorff distance in the Shape space (as described in section 4) between the features points of S and those of S^N . Figure 2 visualizes the distance values for Laurana model for different resolutions of sampling points.

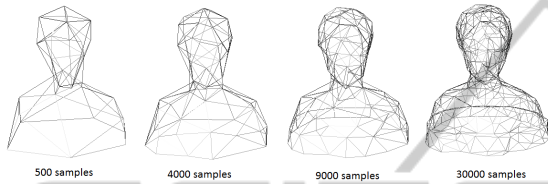


Figure 2: Several samples of Laurana model.

Figure 3 shows the results of the experiments for the Hausdorff distance where the values of N vary from 100 to 10^4 . We notice that even when using the set of samples that is not bigger, the descriptor obtained from the points set approximates well the one computed on the surface. The robustness of the descriptor to point sampling indicates that it can be used even when the initial input of the model is a sampling set of points from the surface (for example for data obtained by scanning).

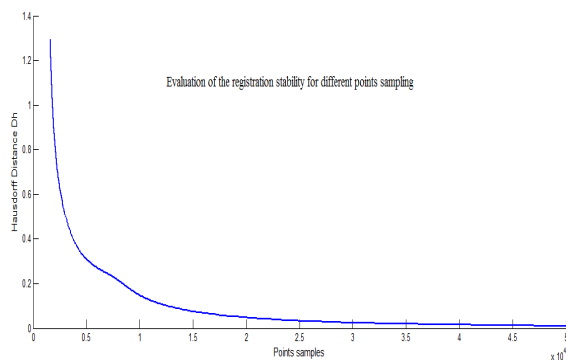


Figure 3: Robustness to sampling.

5.2 Application to 3D Face Description

The experimentations were conducted on facial surfaces from the Bosphorus database (Savran et al., 2008). We have used a collection of faces belonging to two classes. A first class contains faces of the same person with different expressions (fear, sadness, happiness, surprise and disgust) in addition to the neutral expression. And, the second class represents faces of different persons.

Indeed, the construction of the parametric representation supposes that the surface is a two dimensional differential manifold. Although, in numerical three-dimensional imaging field, the most common surface representation is the 3D triangular mesh. Moreover, the reference points are chosen as landmarks taken on the facial surface. An MPEG4 specification of 83 facial feature points that reflect key positions on human faces has been presented in (Yin et al., 2006).

So, we have calculated the parametric representation respectively to reference points taken to be the two inner and outer corners of the eyes, the tip nose and the two centers of the cheek (calculated as the middle of the geodesic path between the tip nose and the outer corner of the eye). Then, the whole registration process was computed using a subset of models respectively from the gallery and the test dataset; the same process as in (Gadacha and Ghorbel, 2013).

-Complexity and Computation time

The complexity of the matching phase is of $O(n)$ order where n is the number of points involved in the process. The complexity of the same step in the classic well-known Iterative Closest Point algorithm (ICP) is of $O(N^2)$ order where N is the number of points on the surface ($N \gg n$). Thanks to the calculation accuracy, the algorithm is less costly in time consuming.

In order to evaluate the accuracy and effectiveness of the proposed approach, the Hausdorff distance in shape space is used as an evaluation metric. The pairwise normalized distances between ten faces is computed (figure 4): only are involved the parametric features points. The first class belongs to the same person while the others correspond to different individuals. In this display, the distances have been scaled to values between 0 and 1.

As expected, the matrix shows that the distances between the faces of the same person are smaller than the ones respectively to different individuals. This proves that our parametric representation has a good discriminative power allowing applications of facial recognition.

6 CONCLUSIONS

In this work, we have focused on 3D rigid registration issue. We have presented a global registration approach that aligns two three-dimensional shapes. The proposed registration approach is based on many iterative registration processes operated on local neighborhood around given interest points already taken on the surface. The approach has proved its efficiency in comparison to the most popular algorithm (the ICP

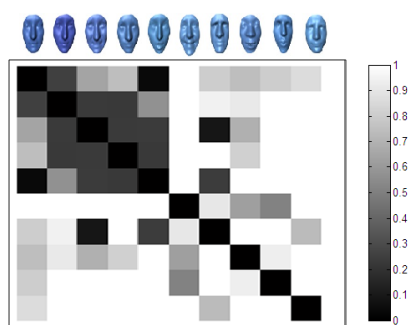


Figure 4: Matrix of pairwise normalized shape distance between facial surfaces. The first class of faces belongs to the same person while the others correspond to different individuals.

one) thanks to the robustness of the matching phase. Indeed, the features points extracted present several desirable properties such as parametrizations over a canonical domain, stability and invariance to scale and 3D motion group. They are also ordered. This makes the correspondence step useless for registration application. Moreover, the neighborhood resolution (number of geodesic level curves and radial lines around a given interest point) affects the accuracy and quality of the matching results. Therefore, a study on the optimal resolution of the curves has been fixed thanks to a generalized version of the Shannon theorem. Thus, the relationship between the size of the features and the performance registration has been studied. Because the features points properties are robust towards tessellation, an application in 3D imaging field especially 3d face description has been chosen. A good discriminative power in face description has been noticed over experimentation on 3D facial database. This works suggests a number of questions to be addressed in future research such that adopting database with several class of objects to be applied in other fields (medical imaging, indexing, etc).

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