SYNC-SOM
Double-layer Oscillatory Network for Cluster Analysis

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Abstract: Despite partial synchronization in the oscillatory networks based on Kuramoto model can be used for cluster analysis, convergence rate of synchronization processes depends on number of oscillators and number of links between oscillators. Moreover result of clustering depends on radius of connectivity that should be chosen in line with input data. We propose double-layer oscillatory network for the two problems. Our network relevant in situation when fast solution is required and when input data should be clustering without expert estimations. In this paper, we presented results of experiments that confirmed better quality then traditional algorithms.

1 INTRODUCTION

Recent researches have suggested that synchronization among neurons in the brain is used to implement the cognitive functions, for example, vision, motion, memory (Haken, 2007). The oscillatory networks provide biologically plausible and parallel methods of modeling cognitive functions. The synchronization processes in oscillatory networks have been applied to various problems such as image segmentation, cluster analysis, sound and image recognition (Basar, 1998); (Cumin et al., 2006); (Benderskaya et al., 2009).

The Kuramoto equation is one of the successful models of synchronization among phases of oscillators (Kuramoto, 1984). However, convergence of synchronization processes depends on number of oscillators and degree of connectivity between oscillators in networks that are based on Kuramoto model. For example, clusters may be elongate and placed close to each other, in this case radius of connectivity (determines oscillators that should be connected) should be chosen exactly before starting algorithm of clustering. Improper radius will cause a false allocation of clusters. Obviously, that a small radius is the cause of small number of connections between the oscillators in the network and as a result it is cause of low level of convergence rate.

In this paper, we proposed a double-layer oscillatory network SYNC-SOM that ensure faster convergence rate without any estimation such as radius of connectivity. The input layer is based on self-organized feature map (SOM) that encodes input features and the output layer based on oscillatory network that uses Kuramoto model (Sync) performs cluster analysis.

2 PRELIMINARIES

2.1 Self-organized Feature Map

Self-organized feature map is special class of artificial neural networks that are based on unsupervised competitive learning (Kohonen, 2001). Each neuron competes for its activation. Self-organization algorithm is divided into three steps: competition, cooperation and adaptation.

Competition process finds the best vector **w** that represents the weight with the smaller distance to the input vector **x** (Haykin, 1999):

$$i(\mathbf{x}) = \arg\min_{j} \left\| \mathbf{x} - \mathbf{w}_j \right\|. \quad (1)$$

The neuron-winner determines the spatial location of the topological neighborhood – cooperation process:

$$h_{j,i(\mathbf{x})}(t) = \exp\left(\frac{\left\| \mathbf{r}_j - \mathbf{r}_i \right\|^2}{2\sigma^2(n)} \right). \quad (2)$$

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Parameter σ is effective width that affects the number of neurons that will be involved in the adaptation process, and $\mathbf{r}_i$ denotes the location of neuron i on the map grid. Synaptic adaptation is the last step that allows excited neurons (that are located in topological neighborhood) to adjust its weight. In other words excited neurons move closer to the input vector:

$$\mathbf{w}_j(t+1) = \mathbf{w}_j(t) + \eta(t)\left(\mathbf{x} - \mathbf{w}_j(t)\right). \tag{3}$$

2.2 Kuramoto Model

Kuramoto model is able to ensure various type of synchronization in networks with various structures (Acebron et al., 2005); (Arenas et al, 2008). Dynamic of the model described by following equation (Kuramoto, 1984):

$$\dot{\theta}_i = \omega_i + \frac{K}{N}\sum_{j=1}^{N}\sin\left(\theta_j - \theta_i\right). \tag{4}$$

Phase of oscillator θ_i is basic state variable that disposes in the range from 0 to 2π. Frequency ω_i can be considered as offset parameter. Coupling strength K affects the rate and the type of synchronization. High value of coupling strength ensures global synchronization and low value of coupling strength ensures local synchronization or desynchronization.

The degree of synchronization between oscillators can be evaluated by estimate r that helps to define state of synchronization (Kuramoto, 1984):

$$r = \left|\frac{1}{Ne^{i\cdot\varphi}}\sum_{j=1}^{N}e^{i\cdot\theta_j}\right|, \quad \varphi = \frac{1}{N}\sum_{j=1}^{N}\theta_j . \tag{5}$$

The state of global synchronization occurs when $r \to 1$, global de-synchronization occurs when $r \to 0$. Partial synchronization occurs in case:

$$r \to \sqrt{1 - K_c/K} . \tag{6}$$

The degree of partial synchronization depends on value of critical coupling strength K_c that is depends on the width of the frequency distribution of oscillator: $K_c = 2\gamma$.

An important feature of the Kuramoto model is possibility to provide synchronization processes in networks with various communication structures. We performed experimental study using numerical simulations and found that states of global and partial synchronization can be successfully sets in the oscillatory networks with communication structures such as grids, stars, bidirectional list and unidirectional circular list.

2.3 Oscillatory Networks based on Kuramoto Model

Oscillatory networks are nonlinear dynamic systems where neuron (unit) is oscillating element that is called an oscillator. The dynamic of the oscillatory network is characterized by the type of synchronization: global, local (partial) and desynchronization. Local synchronization can be interpreted as a case of clustering where each ensemble synchronous oscillators corresponds to one cluster.

The adapted model for oscillatory network that is intended for cluster analysis (Miyano et al., 2007); (Bohm et al., 2010):

$$\dot{\theta}_i = \frac{K}{N(\theta_i)}\sum_{j\in N(\theta_i)}\sin\left(\theta_j - \theta_i\right). \tag{7}$$

Each oscillator corresponds to only one input vector from data set and coordinates of oscillator equals to coordinates of corresponding object. Parameter $N(\theta_i)$ defines number of neighboring oscillators for oscillator i. The set of oscillator neighbors depends on connectivity radius ϵ that should be chosen in line with input data.

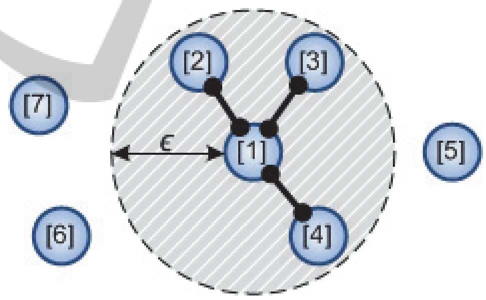

Figure 1: Illustration of how connections are established for oscillator in line with radius ϵ.

The connection is established between oscillators if Euclidian distance (it can be other metric) between less than connectivity radius ϵ.

We have investigated possibilities of the network for cluster analysis using the widespread data set FCPS (Ultsch, 2005). Experiments have shown problems with elongate or with non-uniform density clusters that are located closely next to each other, for example, samples EngyTime, TwoDiamonds and WingNut. It can be hard to choose right radius connectivity or even impossible.

Moreover convergence rate of synchronization processes depends on number of oscillators in network based on Kuramoto model. For example, the oscillatory network with grid structure has quadratic dependence $O(n^2)$ and network with unidirectional list structure has cubic dependence $O(n^3)$. Therefore solution of data clustering may have cubic complexity in worst case.

3 THE SYNC-SOM NETWORK

The proposed network SYNC-SOM consists of the input and the output layers. The architecture of the network presented on the figure 1.

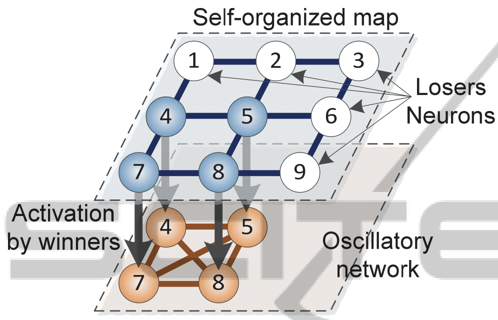

Figure 2: The architecture of the SYNC-SOM oscillatory network. The input layer encodes features from input data set and the output layer performs clustering.

The input layer based on self-organized feature map that reduces high dimensional input space to a lower dimensional map space. Final state of input layer defines number of active oscillators in the output layer that usually equals to number of neurons-winners. The input layer contains several principle differences from conventional self-organized map that will be described further.

Initial values of weights are initialized by random values in the conventional algorithm and it has a high influence on learning process (self-organization process). In this case a neuron-winner is random and spatial location of the topological neighborhood that is defined each step of learning becomes random too. And as a result it is the immediate cause of maps with different topologies at the end of learning process with the same data set. Moreover, random initialization is cause of the formation of areas in which high and low concentration of neurons can occurs. Therefore some clusters cannot be allocated properly by the second layer. It is especially significant shortcoming for the sample TwoDiamonds.

We propose to perform initialization of weights by "uniform grid" in line with input data set. The "uniform grid" represent rectangular grid that covers input data in first two dimensions and distance between the nodes is the same in each of the two dimensions of data. Further the "uniform grid" should be aligned with the center in other dimensions of data. Thus coordinates of nodes of the "uniform grid" define initial weights of neurons.

Our approach for the initialization ensures stable results of learning process and prevents formation of areas that are crowded by neurons, whereas in other areas there is a lack of them. Example of difference of formed featured maps is presented on figure 3. Also we offer to abandon using of permutations of objects of input data set during training on each step as this reduces complexity of learning process, because complexity of the permutation is $O(n!)$.

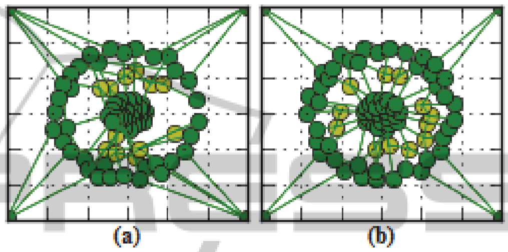

Figure 3: Difference of formed feature maps in cases (a) the random and (b) the "uniform grid" initialization of weights for the sample "Target".

The output layer of SYNC-SOM is based on the oscillatory network whose dynamics described by the following model:

$$\dot{\theta}_i = \frac{K}{N_i} \cdot \sum_{j \in N_i} \sin\left(\theta_j - \theta_i\right). \qquad (8)$$

Number of oscillators N is defined by number of winner-neurons in the input layer. Each neuron-winner corresponds to only one oscillator and coordinates of neurons from the input layer corresponds to coordinates of oscillators in the output layer. The proposed architecture ensures faster solution than basic oscillatory neural networks based on Kuramoto model because the output layer uses significantly fewer oscillators due to using neurons-winners of the input layer instead of objects of an input data.

Connections between oscillators are formed if Euclidean distance between them is less than average distance between approximately no more than ten percent of the total number neurons-winners in input layer. Additionally, U-matrix P-matrix (Ultsch, 2005), information about active and dead neurons (losers) from the first layer can be used for forming more accurate structure of the output layer.

Evaluation of the end of the process clustering r_c is described as follows (Novikov et al., 2013):

$$r_c = \left| \sum_{i=1}^{N} \frac{1}{N_i} \sum_{j \in N_i} e^{\theta_j - \theta_i} \right|. \qquad (9)$$

Ending process synchronization (clustering) is indicated when $r_c \rightarrow 1$. Oscillators whose phases are approximately equal to each other belong to the same cluster with high probability, in other words each ensemble of synchronous oscillators corresponds to one cluster of data.

4 EXPERIMENTAL RESULTS

To illustrate how SYNC-SOM is used for cluster analysis, we have performed study using data set FCPS. Comparison has been performed with algorithms such as K-Means (MacQueen, 1967), ROCK (Guha et al., 2000), Hierarchical (Anil et al., 1988), Sync (Bohm et al., 2010) and DBSCAN (Ester et al., 1996). All experiments have been performed on a workstation with Intel Core i5-2300 CPU 2.8 GHz and 4.0 GB RAM.

We can confirm that the oscillatory network SYNC-SOM is able to ensure accurate results of clustering for all samples from the FCSP data set. We have used 100 neurons in the input layer and coupling strength K in the output was used equal to 1. Several SYNC-SOM results of clustering are presented on figure 4. It's important to note that clusters can be allocated not only by final state of the output layer. Sometimes global synchronization can be reached and only one cluster can be allocated in this case. Analysis of dynamics of the output layer should be performed by dendogram that shows hierarchical organization of clusters where uniting time of clusters is main feature to determine the actual number of clusters (Wang el al., 2009).

K-Means is not able to allocate clusters properly from samples Lsun, Target, WingNut and several others sample where clusters don not have Gaussian or spherical distribution. Hierarchical algorithm has problems with clustering Lsun, Target, Chainlink due to using only minimization of the distance between objects, i.e. has troubles with elongated clusters that are close to each other. Illustration that shows shortcomings of K-means and Hierarchical algorithm is presented on figure 5.

DBSCAN, Sync and ROCK algorithms successfully allocate clusters for all samples. DBSCAN requires finely tuned parameters (number of trusted neighbors and connectivity radius), especially, it's hard to find properly parameters for successful clustering samples TwoDiamonds and WingNut, and small parameter changes can lead to incorrect results. But obvious DBSCAN advantage is high performance. Sync and ROCK are parameterized by connectivity radius too, but they are more robust. The Sync is robust due to possibility to allocate clusters by the mentioned before dendogram in case of global synchronization. And ROCK is robust due to depth analysis of structures. However, they solve the problem slowly compared with other considered algorithms. Table 1 demonstrates comparison of rate solving between the algorithms.

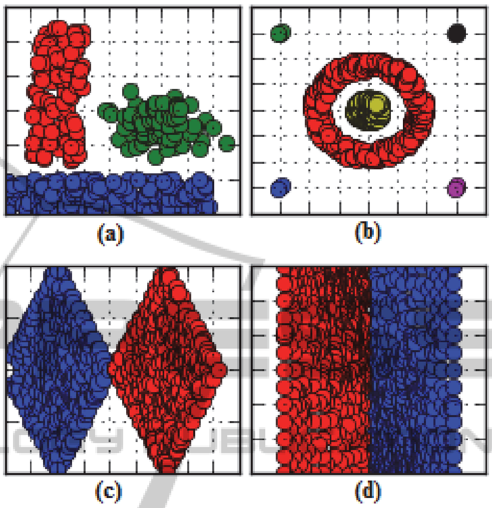

Figure 4: SYNC-SOM results of clustering. (a) Three clusters for Lsun. (b) Six clusters for Target. (c) Two clusters for TwoDiamonds. (d) Two clusters for WingNut.

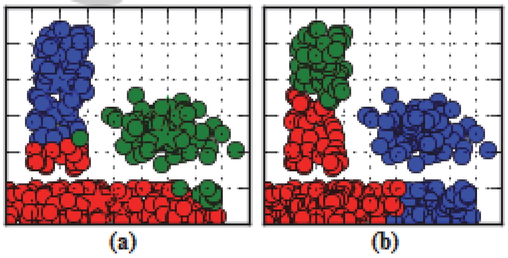

Figure 5: Illustration of shortcomings. (a) K-means. (b) Hierarchical.

Table 1: Execution time of various algorithms.

Sample	Algorithm (Execution time)			
	syncsom	sync	dbscan	rock
Lsun	6.12	26.6	0.37	24.0
Target	12.8	74.3	1.78	174
Two Diamonds	14.8	299	1.43	194
WingNut	21.1	423	2.30	397
Chainlink	21.7	72.0	2.96	383
Hepta	2.58	0.87	0.12	3.65
Tetra	5.70	127	0.39	24.5

The SYNC-SOM is much faster than Sync and ROCK algorithms. Sync can be faster than SYNC-SOM only for very small input data sets, for example, the sample Hepta, because our algorithm spends some time for encoding features.

5 CONCLUSIONS

In this paper we have proposed novel oscillatory network SYNC-SOM for cluster analysis that is based on Kuramoto model and on SOM. We have investigated problems with convergence rate in the conventional oscillatory network based on Kuramoto model and problems with learning processes in SOM. We have performed comparison with various algorithms such as K-Means, DBSCAN, ROCK, Sync and Hierarchical. Our experimental results have confirmed ability of SYNC-SOM to perform fast successful clustering.

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