$\mathcal{FTE}$: A Fuzzy Timed Action Language

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Abstract: This paper proposes a fuzzy approach for reasoning about action and change in timed domains. In our method, actions and world states are modeled as fuzzy sets over time axis. Thus, their temporal relations and time constraints can be model as fuzzy rules. So, our method handles well the issue that action happens at an approximate time and then the states change also at an approximate time, which has not been solved well in existing work. Finally, our method is used to solve the classic problem of rail road crossing control in a fuzzy environment. The theoretic and simulation analysis shows that the controller using our method works well.

1 INTRODUCTION

The study of Reasoning about Action and Change (RAC) (Thielscher, 2011; Varzinczak, 2010; Mueller, 2009; Reiter, 2001; Shanahan, 1997; Sandewall, 1994) has been an active topic since the early days of Artificial Intelligence (van Harmelen et al., 2008). The main concern of RAC is to describe the evolution of a possible world by formalising actions and their effects in timed domains (Shen et al., 2010), which means that actions are required to be performed within a certain amount of time or after a certain amount of time that has elapsed.

However, few systems of RAC are developed for the use in fuzzy environments. Nevertheless, the real world is often fuzzy (Zadeh, 1965; Luo et al., 2002; Luo et al., 2007; Huang et al., 2012). For example, an action can be taken only at an approximate time or the state is not very clear at an accurate time point. So, it is necessary to extend these RAC systems to ones that can be used in fuzzy environments. On the other hand, Zadeh proposed fuzzy set theory (Zadeh, 1965), which composes a form of many-valued logic (Zadeh, 1975; Luo et al., 2002). In fuzzy theory, reasoning is approximate rather than crisp and fuzzy logic variables have a truth degree that ranges in-between 0 and 1 rather than 0 or 1 only. So, fuzzy theory is a powerful tool for us to develop a method for fuzzy RAC in timed domains. Thus, based on the work of (Shen et al., 2010; Wan, et al. 2012), this paper employs fuzzy set theory to develop a reasoning language about fuzzy actions and fuzzy states over time.

The rest of the paper is organized as follows. Section 2 reviews some necessary points in fuzzy theory. Section 3 presents our fuzzy timed action language. Section 4 discusses how to use fuzzy logic to implement fuzzy temporal relation in our timed action language. Section 5 shows how to use our method to solve the problem of rail road crossing control in fuzzy environments. Section 6 discusses the related work to show how our work advance the state-of-art in the field. Section 7 concludes our work and points out the future work.

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2 PRELIMINARIES

This section will review some basics of fuzzy set theory (Nanda and Das, 2010; Zadeh, 1965), which are necessary for our system.

Definition 1. *A fuzzy set, denoted as A, on domain U is characterized by a* **membership function** $\mu_A : U \mapsto [0,1]$, *and* $\forall u \in U$, $\mu_A(u)$ *is called the* **membership degree** *of u in fuzzy set A.*

The following definition is about the implication of the Mamdani method (Nanda and Das, 2010):

Definition 2. *Let A_i be a Boolean combination of fuzzy sets $A_{i1}, \cdots, A_{im}$, where $A_{ij}$is a fuzzy set defined on $U_{ij}, (i=1,\cdots,n; j=1,\cdots,m)$, and B_i be fuzzy set on $U', (i=1,\cdots,n)$. Then when the input is $\mu_{A_{i1}}(u_{i1}),\cdots,\mu_{A_{im}}(u_{im})$, the output of such fuzzy rule $A_i \to B_i$ is fuzzy set B'_i, which is defined as: $\forall u' \in U'$,*

$$\mu_i(u') = min\{f(\mu_{A_{i1}}(u_{i1}),\cdots,\mu_{A_{im}}(u_{im})), \mu_{B_i}(u')\}, \quad (1)$$

where f is obtained through replacing A_{ij} in A_i by $\mu_{ij}(u_{ij})$ and replacing "and", "or", "not" in A_i by "min", "max", "$1-\mu$", respectively. And the output of all rules $A_1 \to B_1, \cdots, A_n \to B_n$, is fuzzy set M,

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which is defined as: $\forall u' \in U'$,

$$\mu_M(u') = max\{\mu_1(u'), \cdots, \mu_n(u')\}. \quad (2)$$

By Definition 2, the result what we get is still a fuzzy set. To defuzzify the fuzzy set, we have following centroid method (Nanda and Das, 2010):

Definition 3. *The centroid point u_{cen} of fuzzy set M given by formula (2) is:*

$$u_{cen} = \frac{\sum_{j=1}^{n} u_j \mu_M(u_j)}{\sum_{j=1}^{n} \mu_M(u_j)}. \quad (3)$$

Actually, u_{cen} in above is the centroid of the area, that is formed by the curve of membership function μ_M and its horizontal ordinate.

3 FUZZY TIMED ACTION LANGUAGE

This section will propose our fuzzy timed action language.

3.1 The Syntax

Definition 4. *A* **fuzzy timed action language**, *denoted as $\mathcal{FTE}$, is a tuple $\langle \mathbb{N}, \Delta, \Phi, \Theta, \mathcal{B}(\Theta) \rangle$, where $\mathbb{N}$ is the set of natural numbers called* time points, *Δ is a non-empty set of* action symbols *representing fuzzy actions at some approximate time points, Φ is a non-empty set of* state symbols *representing fuzzy states, Θ is a non-empty set of clock variables over $\mathbb{N}$, $\mathcal{B}(\Theta)$ is the set of all* clock constraints *over Θ, each clock constraint $\psi \in \mathcal{B}(\Theta)$ is an expression of the form $x \bowtie n$ or $x - y \bowtie n$, where $x, y \in \Theta$, $\bowtie \in \{\le, <, =, >, \ge\}$ and $n \in \mathbb{N}$.*

The various forms of propositions in $\mathcal{FTE}$ are defined as follows:

Definition 5. *Giving a fuzzy timed action language $\langle \mathbb{N}, \Delta, \Phi, \Theta, \mathcal{B}(\Theta) \rangle$, let $A \in \Delta$ represent a fuzzy action over time axis, $F \in \Phi$ represent a fuzzy state over time axis, $\lambda \subseteq \Theta$ be a set of clocks, C be a set of fuzzy states, $\Psi \subseteq \mathcal{B}(\Theta)$ be a set of clock constraints, and $T \in \mathbb{N}$. Then:*

1. *A* **fuzzy C-proposition** *is in the form of*

$$A \textbf{ initiates } F \textbf{ resets } \lambda \textbf{ if } C \textbf{ when } \Psi. \quad (4)$$

2. *A* **fuzzy H-proposition** *is in the form of*

$$A \textbf{ happens-at about } T. \quad (5)$$

3. *A* **fuzzy O-proposition** *is in the form of*

$$F \textbf{ holds-at about } T. \quad (6)$$

A domain description or theory in $\mathcal{FTE}$ is a finite set of fuzzy C-propositions, fuzzy H-propositions, and fuzzy O-propositions.

The fuzzy C-proposition means that if clock constraint Ψ and state precondition C are met, then fuzzy action A happens, and causes fuzzy state F holds and the set of clocks λ are reset. The fuzzy H-proposition describes that fuzzy action A happens at about time point T. Similarly, the fuzzy O-proposition describes that fuzzy state F holds at about time point T.

3.2 The Semantics

Definition 6. *For a fuzzy timed action language of $\langle \mathbb{N}, \Delta, \Phi, \Theta, \mathcal{B}(\Theta) \rangle$:*

- *A* **clock interpretation** *is a mapping of $\nu : \Theta \mapsto \mathbb{N}$. We say that ν satisfies a clock constraint of ψ, written as $\nu \models_c \psi$, if ψ holds under ν according to the standard arithmetic semantics. If there exists such a ν, then ψ is satisfiable and ν is a solution to ψ. For the set of clock constraints Ψ, $\nu \models_c \Psi$ if $\forall \psi \in \Psi$, $\nu \models_c \psi$.*
- *A* **clock operation** *is that given $\delta \in \mathbb{N}$, $\nu + \delta$ is the clock interpretation that maps every $x \in \Theta$ to $\nu(x) + \delta$; and for $\lambda \subseteq \Theta$, $\nu[\lambda := 0]$ is the clock interpretation that maps each $x \in \lambda$ to 0 and every clock $y \in \Theta \backslash \lambda$ remains unchanged. In particular, ν_0 is a clock interpretation that maps every clock to 0.*

For example, let $\Theta = \{\lambda'\}$, then $\nu(\lambda') = 2$ satisfies clock constraint $1 < \lambda' < 3$. And $\nu + 3 = \nu(\lambda') + 3 = 5$, meaning that the time of the clock λ' representing is 5 now.

Definition 7. *The* **interpretation of fuzzy O-proposition** *(6) is a membership function given as follows:*

$$\mu(t) = \begin{cases} 0 & \text{if } t > (T + r_2), t < (T - r_1), \\ \frac{1}{r_1}(t - T) + 1 & \text{if } (T - r_1) \le t \le T, \\ \frac{1}{r_2}(T - t) + 1 & \text{if } T < t \le (T + r_2), \end{cases} \quad (7)$$

where $\mu(t) \in [0, 1]$ represents that fuzzy state F holds to the degree of $\mu(t)$ at the time point of t, and r_1 and r_2 represent the fuzzy ranges. **Fuzzy H-proposition** *(5) can be similarly interpreted as formula (7).*

Definition 8. *The* **interpretation of fuzzy C-proposition** *(4) is as the following fuzzy rule:*

$$((C \wedge \Psi) \wedge A) \rightarrow (F \wedge \nu[\lambda := 0]) \quad (8)$$

By the Mamdani method (see Definition 2), from (8) we can have:

$$((C \wedge \Psi) \wedge A) \wedge (F \wedge \nu[\lambda := 0]) \qquad (9)$$

Then, we can define the fuzzy action and fuzzy state interpretation as follows:

Definition 9. The interpretation of fuzzy action A **and fuzzy state** F *is a mapping* $\mu_{AF} : \mathbb{N} \mapsto [0,1]$*, where* $\mu_{AF}(t)$ *means that fuzzy action A happens or fuzzy state F holds at t to the degree of* $\mu_{AF}(t)$ *where* $(T - r_1) \leq t \leq (T + r_2)$*, T is the time point in the H-proposition or O-proposition, and* r_1 *and* r_2 *represent the fuzzy ranges.*

Definition 10. *A* **fuzzy interpretation of an** $\mathcal{FTE}$ **theory** *is defined as a pair of* $\langle \mu_{AF}, \nu \rangle$*, where* μ_{AF} *is a fuzzy action and fuzzy state interpretation and* ν *is a clock interpretation.*

Definition 11. *Let D be an* $\mathcal{FTE}$ *theory,* $\langle \mu_{AF}, \nu \rangle$ *be an interpretation of* $\mathcal{FTE}$ *and* $F \in \Phi$*, time point* $T \in \mathbb{N}$ *and* $(T - r_1) \leq t \leq (T + r_2)$*, where* r_1 *and* r_2 *represent the fuzzy ranges. Let* λ' *be the clock with respect to t, and* $\nu(\lambda') = t'$*. For the interpretation of H-proposition and O-proposition (7) and the interpretation of fuzzy action A and fuzzy state F:*

1. *If there exists a subinterval* $[t_1, t_2] \subseteq [T - r_1, T]$ *such that if* $t' > t$ *and* $t \in [t_1, t_2]$ *then* $\mu_{AF}(t') > \mu_{AF}(t)), \forall t' \in [t_1, t_2]$*, then the points in* $[t_1, t_2]$ *are called* **initiation points**.
2. *If there exists a subinterval* $[t_1, t_2] \subseteq [T, T + r_2]$ *such that if* $t' > t$ *and* $t \in [t_1, t_2]$ *then* $\mu_{AF}(t') < \mu_{AF}(t), \forall t' \in [t_1, t_2]$*, then the points in* $[t_1, t_2]$ *are called* **termination points**.

Actually, formula (7) in the interpretation of H-proposition and O-proposition usually has the increasing part and the decreasing part. The initiation points and the termination points represent the increasing part and the decreasing part, respectively.

Furthermore, if the set of λ *in C-proposition is non-empty, then t is called a* **resetting point** *for* λ *in* $\langle \mu_{AF}, \nu \rangle$ *with respect to D.*

Definition 12. *Let D be an* $\mathcal{FTE}$ *theory. Let* λ' *be the clock with respect to t, and* $\nu(\lambda') = t'$*. Then the interpretation of* $\langle \mu_{AF}, \nu \rangle$ *is called* **a model for** *D if* $\forall F \in \Phi, A \in \Delta, T \in \mathbb{N}$*, and* $t \in [t_1, t_2] \subseteq [T - r_1, T + r_2]$ *(where* r_1 *and* r_2 *represent the fuzzy ranges), the following conditions hold:*

1. *For each fuzzy O-proposition in D in the form of "F* **holds-at about** *T",* $\mu_{AF} : \mathbb{N} \mapsto [0,1]$*, where* $\mu_{AF}(t)$ *means that fuzzy state F at t holds to the degree of* $\mu_{AF}(t)$*.*
2. *For each fuzzy H-proposition in D in the form of "A* **happens-at about** *T",* $\mu_{AF} : \mathbb{N} \mapsto [0,1]$*, where* $\mu_{AF}(t)$ *means that fuzzy action A at t happens to the degree of* $\mu_{AF}(t)$*.*
3. *If there are no initiation points or termination points in* $[t_1, t_2]$*,* $\forall t' \in [t_1, t_2]$*, if* $t' > t$*, then* $\mu_{AF}(t') = \mu_f(t)$*.*
4. *If the time points in interval* $[t_1, t_2]$ *are initiation points,* $\forall t' \in [t_1, t_2]$*, if* $t' > t$*, then* $\mu_{AF}(t') = max\{\mu_{AF}(t), \mu_{AF}(t')\}$*.*
5. *If the time points in interval* $[t_1, t_2]$ *are termination points,* $\forall t' \in [t_1, t_2]$*, if* $t' > t$*, then* $\mu_{AF}(t') = min\{\mu_{AF}(t), \mu_{AF}(t')\}$*.*
6. *If t is not a resetting point in* $[t_1, t_2]$ *for clock* λ'*,* $\forall t' \in [t_1, t_2]$*, if* $t' > t$*, then* $\nu(t') = \nu(t) + (t' - t)$*.*
7. *If t is resetting point in* $[t_1, t_2]$ *for clock* λ'*, and there exists no other resetting points in* $[t, t']$*, then* $\nu(t') = \nu[\lambda' := 0] + (t' - t)$*.*
8. *if the fuzzy C-proposition holds at t, then there exists some corresponding* $\mu_{AF} > 0$ *and* $\nu(t) \models_c \Psi$*.*

4 FUZZY MODELING

This section will explain our fuzzy modeling of action and change in timed domains.

Proposition 1. *In the* $\mathcal{FTE}$ *theory, the fuzzy action can change the fuzzy state over time and in the time constraints through* **fuzzy C-proposition**.

In Proposition 1, the relation of the fuzzy action and the fuzzy state is represented by the fuzzy C-proposition. Now the further relation on the time axis can be shown as following proposition:

Proposition 2. *In the* $\mathcal{FTE}$ *theory, for the fuzzy action and the same agent's corresponding fuzzy state that it causes, the fuzzy action can be represented by the corresponding fuzzy state on the time axis.*

Although a fuzzy action can be represented by the corresponding fuzzy state on the time axis, if different C-propositions are with respect to different agents, the preconditions about states and time constraints still need to be considered.

Proposition 3. *In the* $\mathcal{FTE}$ *theory, the resetting time points and the time constraints can be represented by the time axis.*

Thus, when all of variables are put on the time axis, the resetting time points and the time constraints do not need to be considered by Proposition 3. Now we transform the fuzzy C-proposition to a simplified fuzzy rule on the time axis.

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Proposition 4. *In the $\mathcal{FTE}$ theory, the fuzzy C-proposition on the time axis can be modeled as:*

$$F \; If \; C \tag{10}$$

By Proposition 4, we can change fuzzy C-proposition (8) as the following fuzzy rule:

$$C \rightarrow F \tag{11}$$

Further, by the Mamdani method (see Definition 2), we have:

$$C \wedge F \tag{12}$$

Then the fuzzy C-proposition are transformed to the relation of the initiate states and the final state on the time axis, which is easier to be handled. Then we can use Mamdani method (2) and centroid method (3) to defuzzify fuzzy sets C and F. The input is fuzzy set C, and the output is fuzzy state set F. Thus, we can see the clear change of F with C.

5 SCENARIO OF RAIL ROAD CROSSING CONTROL

This section shows how to use our fuzzy approach to solve the classical scenario of rail road crossing control (Alur, 1999) in a fuzzy environment with the help of Matlab's Fuzzy Inference System.

5.1 Fuzzy Modeling of Rail Road Crossing Control

In the problem of rail road crossing control, there are three agents: *Train*, *Controller* and *Gate*. At a state, the train can send a signal to the controller, then the controller must send the corresponding signal to the gate within 1 minute, and finally the gate must close or open within 1 minute. In particular, the train should send the corresponding signal to the controller at least 2 minutes before it enters the crossing and it must exit after it enters the crossing within 3 minutes.

The train's possible actions are *Approach*, *Enter* and *Exit*, and its states are *Far-approaching*, *Approaching*, *Close-approaching*, *Close-exit*, *Exit* and *Far-exit*.

- *Far-approaching* means that the train is approaching but it is a little far from the crossing.
- *Approaching* means that it is between *Far-approaching* and *Close-approaching*.
- *Close-approaching* means that the train is approaching (i.e., very close to the crossing).

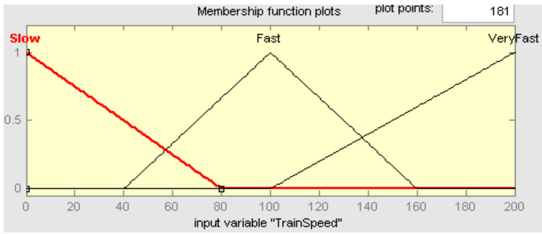

Figure 1: The train's speed membership functions.

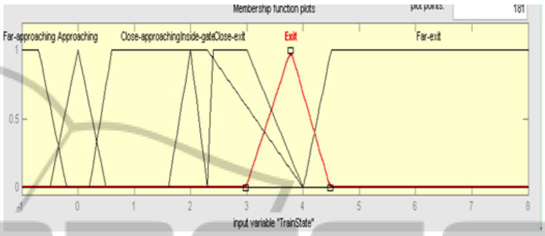

Figure 2: The train's states membership functions.

The train's states *Close-exit*, *Exit* and *Far-exit* have the similar meaning. And *Inside-gate* means that the train is in the gate.

The controller has three states, which describe how long to wait for sending the signal to the gate after it receives a train signal. These states are *Immediately*, *Soon*, and *Delay*. Similarly, the gate also has three states, which describe how fast to close or open the gate. These states are *Fast*, *Normal*, and *Slow*. We assume that the signals of *Far-approaching*, *Approaching*, *Close-approaching*, and *Inside-gate* make the gate closed and the signals of *Close-exit*, *Exit,* and *Far-exit* make the gate open. The controller can take one action (i.e., *Send*), and the gate can take two actions (i.e., *Close* and *Open*).

In the real world, we should consider the speed of the train. Actually, when the train is approaching, the higher the train's speed, the faster the train gets closer to the crossing, and so the faster the controller and the gate needs to take actions for safety and saving time for passerbies. Therefore, different train's speeds need to be put into account. Thus, we add the speed of the train as an input variable, which values are *Slow*, *Fast*, and *Veryfast* and their membership functions are as shown in Figure 1.

Let λ_1 be the clock for the train, λ_2 be the clock for the controller, and λ_3 be the clock for the gate. Suppose we can have two fuzzy H-propositions in the form of formula (5) as follows:

$$\textit{Send} \ \textbf{happens-at about} \ T \tag{13}$$
$$\textit{Close} \ \textbf{happens-at about} \ T' \tag{14}$$

where $T \in \mathbb{N}$ is the time point at about which the controller sends a signal, and $T' \in \mathbb{N}$ is the time point

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at about which the gate is closed. And we can have two fuzzy C-propositions in the form of formula (4) as follows:

$$\textbf{if } \mathit{Slow} \textbf{ and } \mathit{Far\text{-}approaching} \textbf{ when } 0.55 \leq t$$
$$\textbf{then } \mathit{Send} \textbf{ initiates } \mathit{Delay} \textbf{ resets } \lambda_3, \quad (15)$$
$$\textbf{if } \mathit{Slow} \textbf{ and } \mathit{Far\text{-}approaching} \textbf{ when } 0.65 \leq t'$$
$$\mathit{Close} \textbf{ initiates } \mathit{Slow}. \quad (16)$$

where $(T - r_1) \leq t \leq (T + r_2)$ and $(T' - r_1') \leq t' \leq (T' + r_2')$ (r_1, r_2, r_1', and r_2' represent the fuzzy ranges).

By Proposition 1, we can have two fuzzy O-propositions in the form of formula (6) as follows:

$$\mathit{Delay} \textbf{ holds-at about } T \quad (17)$$
$$\mathit{Slow} \textbf{ holds-at about } T' \quad (18)$$

where $T \in \mathbb{N}$ is the time point at about which the controller's state *delay* holds, and $T' \in \mathbb{N}$ is the time point at about which the gate's state *Slow* holds. Similarly, we describe other states in this way.

By Propositions 2 and 3, we can represent fuzzy actions by fuzzy states, and put the clock and clock constraints on the time axis. Then, we can operate them in the Matlab's Fuzzy Inference System. If we set that the time of 2 minutes before the train enters the crossing is the time of zero, then the train must exit at the time of 5 as shown in Figure 2. The controller and the gate must do something within 1 minute and so we represent this by two output variables as shown in Figures 3 and 4. We have one input variable as describing the states of the train. The train states' membership functions as shown in Figure 2.

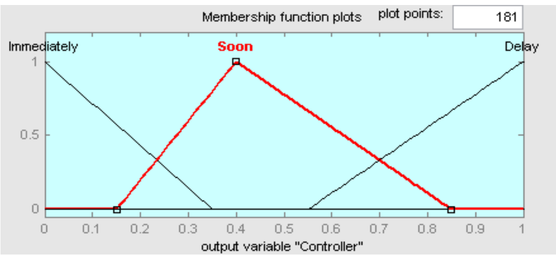

Figure 3: The controller's membership functions.

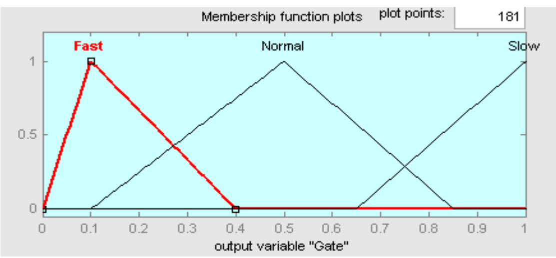

Figure 4: The gate's membership functions.

Now we use the membership functions (see Figures 3 and 4) to describe our two output variables. By Proposition 4, we can model C-propositions (15) and (16) as follows:

$$\mathit{Delay} \textbf{ if } \mathit{Slow} \textbf{ and } \mathit{Far\text{-}approaching}, \quad (19)$$
$$\mathit{Slow} \textbf{ if } \mathit{Slow} \textbf{ and } \mathit{Far\text{-}approaching}. \quad (20)$$

Now we consider the rules between the inputs and outputs. The main rules are that when the train is approaching, the higher speed and the closer to the crossing the train, the faster the controller and the gate take actions for safety and saving time for passerbies. From rules (19) and (20), we can get our first rule:

$$\mathit{Delay} \textbf{ and } \mathit{Slow} \textbf{ if } \mathit{Slow} \textbf{ and } \mathit{Far\text{-}approaching}. \quad (21)$$

That is:

1. **If** (TrainSpeed is *Slow*) **and** (TrainState is *Far-approaching*) **Then** (Controller is *Delay*) **and** (Gate is *Slow*).

Similarly, we can get other rules in the problem and put them into Matlab's Fuzzy Inference System.

5.2 Analysis of the Fuzzy Model in Safety and Effectiveness

By using formula (3) to the fuzzy set of formula (12), we can get all the rules' result of defuzzification. Now we can see the output of the fuzzy reasoning by Surface Viewer of Matlab as shown in Figures 5 and 6. For example, when the train speed is 100 km per hour and the time is 0, the controller should send the signal to the gate in 0.354 minute and the gate should be closed in 0.388 minute. Actually, Figure 5 shows how the controller changes with the train speeds and the train states; and Figure 6 shows how the gate changes with the train speeds and the train states.

For a rail road crossing control system, it should satisfy the criteria of safety and effectiveness. That is, to be safe, the gate should be closed as soon as possible when the train is approaching, and the gate should be open gently when the train is leaving; to be effective, (i.e., in order to save time for passerbies), the gate should not be closed too early when the train is approaching, and the gate should be opened for passerbies as soon as possible when the train left far away. In our system, the gate is controlled by the controller according to the gate's states. Figures 5 and 6 show that our system satisfies the criteria of safety and effectiveness. We have assumed that the train enters the gate at about the time point of 2. So, when the time is close to the left of the time point of 2 (i.e., the train approaches very closely to the gate), it is dangerous for passerbies and so the controller and the gate

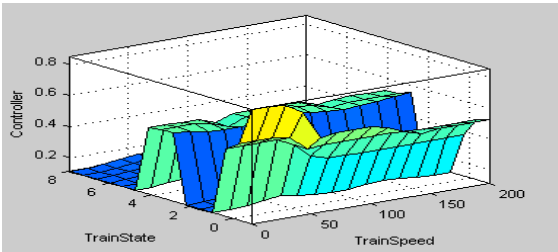

Figure 5: Controller's states change with inputs.

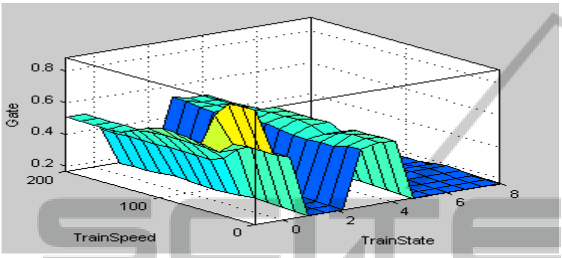

Figure 6: Gate's states change with inputs.

should take some measures as quickly as possible. When the time is far from the left of the time of 2, the controller should delay some time for passerbies, and the gate need close gently for being safer. However, when the time point is close to the right of the time point of 2, which means that the train is leaving but it is still close to the gate, or the tail of the train may be still on the crossing. Thus, the controller should delay some time to send the signal, and the gate can close gently for safety. When the train is leaving, and far from the gate (i.e., far from the right of time point of 2), it is safe for passerbies, and thus the controller and the gate should take some measures as quickly as possible to save the time for passerbies.

Figs. 7(*a*) and (*b*) illustrate that how the controller and the gate synchronize with the train's states at a specific train's speed. The curves decrease first, then do not change when it reaches the curve bottom, which means that the train is approaching to the gate and the controller and the gate all should take actions as quickly as possible. Then the curve increase suddenly, which means that the train is leaving, but it is still closed to the gate, and so the controller and the gate need not to do the things quickly. Then the curve is going down just as the discussion above.

Figs. 7(*c*) and (*d*) illustrate how the controller and the gate synchronize with the train's speeds when we have a specific train's state at the time point of 0. From both two figures, we know that when the train's speed is higher, then the controller and the gate both should take the measures as soon as possible. Actually, we know that if anything wrong, the train needs to stop. The faster the train, the more time the train

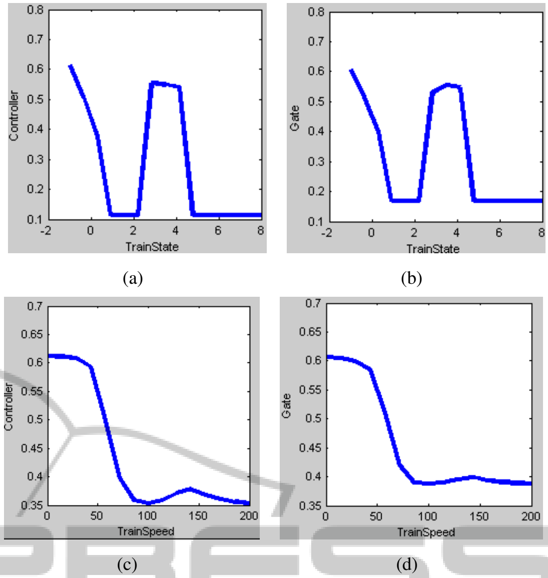

Figure 7: (*a*) and (*b*): The controller's state and the gate's state change with the train's states when the train's speed is 100 km/h; (*c*) and (*d*): The controller's state and the gate's state change with the train's speeds at the time point of 0.

needs to stop. So, our system can ensure this situation safe.

From above illustration, we can have some properties for our system as follows:

Theorem 1. *In the rail road crossing control system, for any different train speeds, if the train sends the corresponding signal at least 2 minutes before the train enters the crossing, meaning that the time point is at the left of zero point, the gate can be closed when the train enters the crossing.*

Theorem 2. *In the rail road crossing control system, for any different train speeds, the gate can be open within 2 minutes after the train sends the corresponding signal to the controller when the train is leaving.*

Theorem 1 shows that our system is safe, and the Theorem 2 shows that our system is effective. From the above analysis, we can conclude that our system satisfies the criteria of safety and effectiveness.

6 RELATED WORK

Fuzzy reasoning about action and change in timed domains is related to fuzzy automata theory, fuzzy finite state machine, fuzzy timed transition system, fuzzy representation and control. In the following, we will check them one by one.

Firstly, we examine the research about fuzzy automata theory. In (Tiwari et al., 2012), fuzzy automata

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theory based on a lattice-ordered monoid is proposed and the associated topology is studied. They introduce the separatedness and connectedness properties of these fuzzy automata. In (Li, 2011), proposed are the lattice-valued finite automata, which generalize the fuzzy automata with membership values in a distributive lattice. In (Stamenković and Ćirić, 2012), an effective method for constructing an equivalent fuzzy finite automaton from a given fuzzy regular expression is provided. However, all of these fuzzy automata only consider the states only (do not consider fuzzy action over time), while our work considers states and actions both.

Secondly, we discuss the research about fuzzy finite state machine. In (Gómez et al., 2011), Gómez et al. propose a fuzzy finite state machine and a fuzzy transformation semigroup with the interval truth structure of the transition function. In (Alvarez-Alvarez et al., 2011), fuzzy finite state machines are used for body posture recognition. In (Alvarez-Alvarez et al., 2012), an automatic method based on fuzzy finite state machines and genetic algorithms are used to model the human gait. In these fuzzy finite state machines, states and time have been considered, but they do not consider the fuzzy time constraints and how fuzzy actions change the states. Rather, we do.

Thirdly, we check the research about timed transition. Our model of fuzzy action and state reasoning can be actually viewed as a fuzzy timed transition system because the transitions are based on approximate time. In (Andrés et al., 2011), a novel approach is presented to self-adaptive systems by a fuzzy-time formal model, whose main concepts are clocks and clock constrains. The system can reflect the behavior of fuzzy timed systems, but unlike us, they do not model how the actions change the states. In (Cao et al., 2011), a fuzzy transition system, which deals with how the actions change states, is used to measure the behavioral similarity of states. However, they did not concern fuzzy time constraints, while ours does not have the problem. In (Acampora et al., 2010), a timed fuzzy controller is developed to manage the temporal component by pairing the initial location of a timed automaton with a fuzzy controller, representing the system's initial control configuration, and associating each automaton transition with a collection of operators. This system can maximize performances and robustness. Although this system has applied to many real situations and it has considered time constraints and states' change, it does not consider actions and how actions change the states, while we do in this paper.

Fourthly, we have a look at the research on fuzzy representation and control. In (Schiffe, 2011), the method of fuzzy action representations and control for robot is proposed by extending the action language, which is developed for the high-level control of agents and robots. However, this fuzzy representation does not consider fuzzy time and time constraints, and so it cannot handle the problem that our method can. In (Schiffer, 2012), the fuzzy action representation is improved by a thorough integration of qualitative representations and reasoning for positional information, but they still do not handle fuzzy time and time constraints. In (Barbosa et al., 2010), a new way of fuzzy reasoning is proposed by combining the feature of fuzzy controllers with the feature of fractional controller of PID-type. It makes the controllers better at superior robustness and wider domain of application. However, in real world, we cannot neglect that actions change states under time constraints. However this mixed controller fails to incorporate them, while we do. In (Ribaric and Hrkac, 2012), a model of fuzzy spatio-temporal knowledge representation is proposed. Nevertheless, unlike our method in this paper, it cannot deal with that the fuzzy actions change states based on constraints of state and time.

7 SUMMARY

In this paper, we propose a fuzzy approach for reasoning about action and change in timed domains. In our method, actions, states and time constraints are represented by fuzzy sets over time axis and the temporal rules are modeled by fuzzy rules. This is significant because when the actions change the states flowing the time flash and in the time constraints, all of these often happen in the fuzzy environment of real life. So, fuzzy reasoning about action and change in timed domains is more realistic in the real world, and thus we can use it in many real situations. To illustrate this, we use our approach to solve the problem of classic rail road crossing control in a fuzzy environment and analyse the problem with the help of Matlab. Moreover, our simulation and theoretical analysis show our treatment on the problem is safe, effective and efficient. In the future, we will consider more complex situations to improve our system and put it into practice.

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