# Reconstructing Archeological Vessels from Fragments using Anchor Points Residing on Shard Fragment Borders 

Zexi Liu ${ }^{1}$, Fernand Cohen ${ }^{1}$ and Ezgi Taslidere ${ }^{1}$<br>${ }^{l}$ Department of Electrical and Computer Engineering, Drexel University, Philadelphia, U.S.A.

Keywords: Intrinsic Geometric Features, Curvature, Inflection, Mending, Archeological Shards, Global Constraint.


#### Abstract

This paper presents a method to assist in the tedious process of reconstructing ceramic vessels from excavated fragments. The method models the fragment borders as 3D curves and uses intrinsic differential anchor points on the curves. Corresponding anchors on different fragments are identified using absolute invariants and a longest string search technique. A rigid transformation is computed from the corresponding anchors, allowing the fragments to be virtually mended. A global constraint induced by the surface of revolution (basis shape) to decide on how all pairs of mended fragments are coming together as one global mended vessel is used. The accuracy of mending is measured using a distance error map metric. The method is tested on a set of 3D scanned fragments ( 313 pieces) coming from 19 broken vessels. $80 \%$ of the pieces were properly mended and resulted into alignment error at the scanner-resolution-level. The method took 59 seconds for mending pieces plus 60 minutes for 3D scans as compared to 12 hours for stitching manually.


## 1 INTRODUCTION

The mending of unearthed archeological ceramic shards to reconstruct vessels that the fragments once formed is currently a tedious and time-consuming process. Nevertheless, it is a vital step in interpreting the archeological record and an important component in understanding and preserving cultural heritage.

There are a variety of existing techniques for characterizing and reconstructing fragments. They can be classified into four categories, which are point-to-point, curve, surface, and shape descriptor matching. The unifying idea rests on finding corresponding parts, matching, aligning, and gluing all matching parts together.

Ucoluk and Toroslu (Ucoluk and Toroslu, 1999) proposed a 3D curve matching approach for the mending of thin-shell fragments, based on string matching of the curvature and torsion values on the discrete 3D curve points. The string matching algorithm is further elaborated in (Rodriguez, Last et al., 2004).

For thick-shell fragments, i.e. broken pieces with large contact surfaces, Papaioannou et al. (Papaioannou and Karabassi, 2003) presented a method based on polygonal surfaces. They introduce a matching error between complementary surfaces
that exploits the z-buffer algorithm. This method is then extended to incorporate curve matching ideas.

In this paper, we select a feature-based approach that identifies corresponding anchor points on different shards to help in the mending process. The main advantages of our method are: (i) the extraction of novel anchor points residing on the fragments' borders. These are differential intrinsic points sometimes not easily seen by the naked eyes, hence the method looks at features that can go beyond human visual mending; (ii) introducing a simple and easy method for the extraction of the fragment border through the use of a Delaunay triangulation on the 3D data, followed by the detection of those triangles edges that are not shared by two abutting triangles. It results into a set of selfinduced ordering of points of the border edges by simply following the end points of the abutting border edges in a clockwise or an anticlockwise direction. These ordered endpoints are the fragment border curve points; (iii) establishing correspondences between anchor points on fragments through the use of absolute invariants and a novel string matching method; (iv) introducing a global constraint induced by the surface of revolution on how all pairs of mended fragments are coming together as one global mended vessel; and finally (v) we run a comparative analysis between our automatic mending method with that of an
expert manually mending the pieces both in terms of time and performance. We found that compared to expert mending, the automatic mending process on the 313 fragments took about 59 seconds for the mending plus approximately 60 minutes for the fragments to be 3D scanned as compared to 12 hours for stitching by the experts.

## 2 ALIGNMENT WITH ANCHOR POINTS

There are several geometrical landmarks, for instance, inflection points, corner points, and zerotorsion points, that are intrinsic and preserved under a rigid transformation. These are the anchor points that we use in this paper for mending the fragments. These anchor points are theoretically pinned in the differential geometry of curves via the Frenet Frames (Kühnel, 2006).

### 2.1 Constructing Absolute Invariants

Since length is preserved under 3D rigid maps and are absolute invariants, we can construct a sequence of absolute invariants by considering the sequence of length of lines between the two consecutive anchor points (e.g., lines between points \#1-2, points \#2-3, etc. as shown in Figure 1). Note that there is an anchor point missing between points \#4 and \#5 on curve $C_{1}$. This will not affect the matching given that there is enough number of anchor points. For a curve with n anchor points, the sequence of length is denoted by $I=\left(L_{1}, L_{2}, \ldots, L_{n-1}\right)$. For the curve $C_{1}$ in Figure 1, there are 7 anchor points on the curve.


Figure 1: Absolute invariants with anchor points with some possibly missing. The algorithm in section 2.3 will always find the longest matching string.

Thus there are 6 lines. When curve $C_{1}$ undergoes a rigid transformation ( $C_{1}^{\prime}=T\left\{C_{1}\right\}$ ), the transformed lines sequence has also the same number of elements $I^{\prime}=\left(L_{1}^{\prime}, L_{2}^{\prime}, \ldots, L_{n-1}^{\prime}\right)$ as the original curve $C_{1}$. And we need to find the sequences of elements in that invariant vectors in $I$ and $I^{\prime}$ that correspond. Towards that end we introduce a "longest string search" technique, which is similar to the "listmatching algorithm" (Bratko, 1990) for establishing the correspondence and declaring the match. In order to recover the rigid transformation and mend the fragments, at least three pairs of matched points is required, i.e., the minimum edge string length should be 2 .


Figure 2: Fragment mending based on anchor points (red dots).

### 2.2 Longest String Match

As the anchor points and the joining edges between them are ordered, we can find the longest string of consecutive edges that lie on two fragments whose respective length absolute invariants match. This is necessary as invariant values on two different fragments coming from different vessels may be equal to within our allowable error ( $2-5 \%$ ) if considered individually and not as a string. A set of two or more consecutive anchor points and their corresponding invariants are considered to be a "string" (e.g., the set $(i, i+1, i+2, \ldots)$ on one fragment and the set $(j, j+1, j+2, \ldots)$ on another fragment). We allow for a small error $\%(2-5 \%)$ in the values of the invariants, to declare matching edges in that string search. The sets ( $i, i+1, i+$ $2, \ldots$ ) and $(j, j+1, j+2, \ldots)$ of anchor points are declared as matching if $\left|I^{\prime}(j)-I(i)\right|<0.05 \cdot I(i)$ for every pair in the two sets. In case that there are
missing points, for example, in Figure 1, $L_{4} \neq L_{4^{\prime}}$, then the distance $L_{5^{\prime}}$ will be discarded and the distance between points $4^{\prime}$ and $6^{\prime}$ will be calculated, then $L_{4}=L_{4^{\prime} 6^{\prime}}$. Generally speaking, if we encounter an "unmatched" segment, we will always jump over one point and check its next anchor point. As this process is recursive we can deal with more than one missing point.

The rigid transformation $T=\{[R], t\}$ can be recovered from three pairs of matched points, or estimated from more matched anchor points using a least square error (LSE) estimation method (Umeyama, 1991).

Once the transformation parameters are found, the fragments are mapped into the same coordinates system by undoing the rigid transformation and the fragments are aligned through their common curve


Figure 3: With/without global optimization.

### 2.3 Global Optimization

The mending described in the previous section and shown in Figure 2 is a pair wise mending process. It is conceivable that alignment errors will accumulate, rendering the reconstruction result less than satisfactory (See right hand side of Figure 3). For vessels that are axially symmetric, this problem can be solved by adopting the surface of revolution as a global constraint. The surface of revolution is obtained by going through the following steps:

1. Extract the "profile curve" (Willis, Orriols et al. 2003).
2. Obtain the symmetry axis (revolution axis).
3. Rotate the profile curve about the axis.
4. Generate a rotation surface.
5. Use the surface as a global optimization "basis".

Steps 1 and 2 are shown in Figure 4. The lower rim and upper rim are obtained by fitting a circle to the fragment border segments. Of all the fragment border segments, the one with the smallest fitting error is the rim. M is a point on the rim. T is the tangent vector of the upper rim at M . We can find a plane $\alpha$ that is orthogonal to T and containing M .


Figure 4: Obtain profile curve and symmetric axis.
The cross-section curve c is the profile curve.
Steps 3 and 4 are shown in Figure 5. Here we rotate the profile curve about the revolution axis to generate a set of curves which constitute the rotation surface. This rotation surface is used in our mending process as a basis shape, where the fragments are not only aligned to each other, but are also aligned to this surface. With this global optimization constraint, the result is improved as shown on the left hand side of Figure 3.

Note that before a basis shape is found, fragments with no upper and lower rims are set aside, and only fragments that do possess both rims are found and possibly mended using the pair wise invariants approach. After we obtain a mended fragment with both upper and lower rims, we extract the profile curve and the rotation surface (steps 1-5). We then improve on the mending for all fragments using the basis shape as a global constraint.

## 3 EXPRIMENTS

Excavated shards are scanned using a Konica Minolta Vivid 910 3D scanner. The scanned raw data (discrete 3D points) is represented as triangle mesh.


Figure 5: Generate a rotation surface.

We define the residual error for each fragment as the mean point-to-point distance map between the mended fragments and the original unbroken jar. The mean distance is normalized against the resolution of the scanner. In all our experiments, the 19 vessels were scanned prior to breaking them into fragments, with the latter being also scanned individually.

The described method is tested on 313 fragment pieces coming from 19 different ceramic objects. Each fragment is scanned separately.

The mending process for one object is shown in Figure 2 with all the anchor points showing for two mended pieces on the vessel. A total of 245 out of 313 fragments were mended properly to their correct vessels as shown in Figure 6. There were 68 pieces that could not be mended (See "The Remaining Pieces" subplot on the upper left of Figure 6) due to insufficient anchor points.

The entire mending process took 59 seconds on an Intel i7 processor PC with 18 GB memory for the mending process, and approximately 60 minutes for the scanning. This is to be compared to 12 hours for the stitching done by the experts.
$80 \%$ fragments were properly mended with distance map errors at or below the scanner resolution, i.e. the residual errors normalized by the scanner resolution were all below or close to 1 .

## 4 CONCLUSIONS

We present a methodology to mend fragments into vessels based on anchor points on fragment borders. This work is part of a collaborative project for which the main objective is to develop and utilize novel computer vision technology to assist in the reconstruction of ceramic artifacts recovered from an excavation site. The work has focused on the use of one aspect (fragments borders) amongst many embedded in the fragments. This, in conjunction with many other aspects such as markings, texture, or surface information, could be collectively used as enabling technology helping in the mending process. This is particularly important if the extracted anchor points in the paper are absent due to complete erosion of the fragments borders on abutting fragments, which would limit the success of such a method. The whole project as an application of computer vision in archaeology is unique as an enabling technology for timely analysis, interpretation, and presentation of history evidence. It is also considered as a great need by the U.S. Department of the Interior National Park Service.


Figure 6: Mended objects.

## ACKNOWLEDGEMENTS

This work is supported by the National Science Foundation IRIS Division under grant \#0803670

## REFERENCES

Bratko, I. 1990. Prolog Programming for Arificial Intelligence. London, UK, Addison Wesley.
Kühnel, W. 2006. Differential geometry : curves - surfaces - manifolds. Providence, R.I., American Mathematical Society.
Papaioannou, G. and E. A. Karabassi 2003. "On the automatic assemblage of arbitrary broken solid artefacts." Image and Vision Computing 21(5): 401 412.

Rodriguez, W., M. Last, et al. 2004. "3-Dimensional curve similarity using string matching." Robotics and Autonomous Systems 49(3-4): 165-172.
Ucoluk, G. and I. H. Toroslu 1999. "Automatic reconstruction of broken 3-D surface objects." Computers \& Graphics 23(4): 573-582.
Umeyama, S. 1991. "Least-Square Estimation of Transformation Parameters Between Two Point Patterns." IEEE Transactions on Pattern Analysis And Machine Intelligence 13(4): 376-380.
Willis, A., X. Orriols, et al. 2003. Accurately Estimating Sherd 3D Surface Geometry with Application to Pot Reconstruction. CVPR Workshop: ACVA. Madison, WI, USA, IEEE Computer Society.

