The Application of Evolutionary Algorithm for the Linear Dynamic System Modelling

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Abstract: The approach to dynamic system modelling in the linear differential equations form is presented. The given approach fits the identification problems with the system output observations sample and the input sample even if the output data is distorted by a noise. The structure and parameters identification problem is reduced to a global optimization problem, so that every solution consists of the model structure and its parameters. This allows searching the analytical model in the ordinary differential equation form with any limited order. The analytical model delivers a special benefit in its further use in the control and behaviour estimation problem.

1 INTRODUCTION

There are many different approaches to make a model of the dynamic system. The identification task itself depends on the given structure and the parameters estimation special technique. Also, the practice need tends one to make the model in an analytical form so it would be easier to find out a control function or predict the system behaviour with different input functions or initial points. We can approximate the system output and use the special technique to define its unit step function reaction or we can make the model in a dynamic form. The model that was built as an approximation with a function base is not as useful and flexible as a dynamic model. Moreover, the task would be reduced to the enumerative technique for the different combination of functions, while we do not know, for example, the order of equation or multiplicity of characteristic equation roots. In the article (Janiczek, Janiczek, 2010) we can see an identification method in terms of fractional derivatives and the frequency domain. The information about the plant is taken from the given frequency domain and not from the output observations that could be distorted. Also, special control and regulation methods are required to the model in fractional derivatives. We can use stochastic difference equations (Zoteev, 2008), and build a model using the output observations, observations of the reaction on the step excitation. This approach is partially parameterized, i.e., the order and functional relation between the system state and previous states are unknown. In (Parmar, Prasad, Mukherjee, 2007), the dynamic system approximation with the second order linear differential equations is examined. The coefficients are determined with the genetic algorithm. In this paper, there is the description of the structure and parameters identification task solving by means of the reduction of the identification task to a real value optimization problem with the modified evolutionary strategies method. The algorithm workability and usefulness are demonstrated on the real identification problem.

The rest of the paper is organised in the following way. In Section 2 we describe the problem statement of the system structure and parameters estimation, in Section 3 the modified hybrid evolutionary strategies algorithm for the ordinary differential equation identification is described, in Section 4 we fulfill modelling the chemical reaction with described approach, and in Conclusion we summarise our results.

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2 STRUCTURE AND PARAMETERS ESTIMATION PROBLEMS

Let us have a sample $\{y_i, u_i, t_i\}$, $i = \overline{1,s}$, where s is its size, $y_i \in R$ are dynamic system output measurements at t_i, and $u_i = u(t_i)$ are control measurements. It is also known, that the system is linear and dynamic, so it can be described with the ordinary differential equation (ODE):

$$a_k \cdot x^{(k)} + a_{k-1} \cdot x^{(k-1)} + \ldots + a_0 \cdot x = b \cdot u(t), \quad x(0) = x_0. \tag{1}$$

Here x_0 is supposed to be known. In the case of the transition observation, we can put forward a hypothesis about initial point: the system output is known at initial time and the derivative values can be set to zero, because usually the system observation starts in its steady state. In general, the initial point can be approximated. Using the sample data we need to identify parameters and the system order m, which is assumed to be limited, so $m \le M, M \in N$. M is a parameter that is set by the researcher. This value limits the structure of the differential equation, i.e., it limits the ODE order. It is also assumed that there is an additive noise $\xi : E(\xi) = 0, D(\xi) < \infty$, that affects the output measurements:

$$y_i = x(t_i) + \xi_i. \tag{2}$$

Without information on the system order, we would not be able to solve the identification task, but because of the maximum order limitation, the task can be partially parameterized. The maximum order is supposed to be chosen a priori. It would specify the optimization problem space dimension.

Without loss of the generality, let the leading coefficient of ODE be the constant equal to 1, so that

$$x^{(k)} + \frac{a_{k-1}}{a_k} \cdot x^{(k-1)} + \ldots + \frac{a_0}{a_k} \cdot x = \frac{b}{a_k} \cdot u(t) \tag{3}$$

or

$$x^{(k)} + \tilde{a}_k \cdot x^{(k-1)} + \ldots + \tilde{a}_1 \cdot x = \tilde{b} \cdot u(t). \tag{4}$$

Then we can seek the solution of the identification task as a linear differential equation with the order m,

$$\hat{x}^{(m)} + \hat{a}_m \cdot \hat{x}^{(m-1)} + \ldots + \hat{a}_1 \cdot \hat{x} = \hat{a}_0 \cdot u(t), \quad \hat{x}(0) = x_0, \tag{5}$$

where the vector of equation parameters $\hat{a} = (0, \ldots, 0, \hat{a}_m, \ldots, \hat{a}_1, \hat{a}_0)^T \in R^n$, $n = M + 1$, delivers an extremum to the functional

$$I(a) = \sum_{i=1}^{N} \left| y_i - \hat{x}(t_i) \right|_{\hat{a}=a} \to \min_{a \in R^n}. \tag{6}$$

In general case, the solution $\hat{x}(t)$ is evaluated with a numerical integration method, because the control function has no analytical from, rather is given algorithmically. We prefer the criterion (6) instead of quadratic criteria because of its robustness. For the correct numerical scheme realization, let us have a coefficient restriction for equation (3), $|a_k| > 0.05$. Otherwise, this parameter is going to be equal to zero, so $a_k = 0, m = m - 1$. That condition prevents extra computational efforts of the numerical evaluation scheme and is necessary for the local optimization algorithm effecting on the system structure.

Now let us consider the specific modelling issue. The identification of linear differential equations system is connected with the optimization problem for the system of equations:

$$a^i_{k_i} \cdot x_i^{(k)} + \ldots + a^i_0 \cdot x_i = \sum_{\substack{j=1 \\ j \ne i}}^{n_o} b^i_j \cdot x_j + b^i_0 \cdot u_i(t), \tag{7}$$

where $x_i, i = \overline{1,n}$, is an observed system output; n_o is the number of outputs.

Equation (7) shows that the system is considered not in general way and every system output depends on other outputs but not on their derivatives. Also, there is only one control input for every equation. This can be easily extended to the case with many control inputs.

The identification problem for the system with equation (7) is important and an ability to solve it could be useful. And it is clear, that the functional (6) can be transformed into the functional

$$I(a) = \sum_{j=1}^{n_o} \sum_{i=1}^{N} \left| y_i^j - \hat{x}_j(t_i) \right|_{\hat{a}=a} \to \min_a \tag{8}$$

for the given systems by means of the same transformation that was made for a single output system.

3 MODIFIED HYBRID EVOLUTIONARY STRATEGIES ALGORITHM FOR ORDINARY DIFFERENTIAL EQUATION IDENTIFICATION

The reason why the modification of an evolutionary strategies algorithm was used is that the identification problem leads to solving the multimodal optimization task. The goal of the given approach is the identification of the parameters and the structure simultaneously. The system structure and its parameters can be represented by one vector. The criterion (8) for this vector is complex and sensitive to the vector components changed by stochastic search operators. This is why we have to develop the specific modification for the global optimization technique.

As a method to seek the task (8) solution, the hybrid modified evolutionary strategies (Schwefel, 1995) method was chosen. Let every individual be represented with tuple

$$H_i = \left\langle op^i, sp^i, fitness(op^i) \right\rangle, i = \overline{1, N_I},$$

where

$op_j^i \in R,\ j = \overline{1,k}$, is the set of objective parameters of the differential equation;

$sp_j^i \in R^+,\ j = \overline{1,k}$, is the set of strategic parameters;

N_I is the population size;

$fitness(x): R^k \to (0,1],\ fitness(x) = \frac{1}{1+I(x)}$ is the fitness function.

As the selection types, proportional, rank-based and tournament-based selections were chosen. The algorithm produces one offspring from two parents and every next population have the same size as previous. Recombination types are intermediate and discrete. The mutation of every offspring's gene happens with the chosen probability p_m. If we have the random value $z = \{0,1\}, P(z=1) = p_m$, which is generated for every current objective gene and its strategic parameter then

$$op_i^{offspring} = op_i^{offspring} + z \cdot N(0, sp_i^{offspring});$$

$$sp_i^{offspring} = \left| sp_i^{offspring} + z \cdot N(0,1) \right|,$$

where $N(m, \sigma^2)$ is the normally distributed random value with the mean m and the variance σ^2.

We suggest a new operation that could increase the efficiency of the given algorithm. For every individual, the real value is rounded down to the nearest integer. This provides searching for solutions with near the same structure.

Also for N_1 randomly chosen individuals and for N_2 randomly chosen objective chromosomes we make N_3 iterations of local search with the step h_l to determine the better solution. This is the random coordinate-wise optimization.

4 MODELLING OF THE CHEMICAL REACTION

We need to develop a model in the form of differential equations for the chemical reaction. The process of the hexadecane disintegration is considered, the concentration of the output products are measured. Using the sample of measurements we can build up a model with the modified evolutionary strategies algorithm.

The settings of the algorithm were chosen according to the previously conducted numerical experiments with some randomly generated systems where their use has given the best performance.

The disintegration of the hexadecane gives the following products: the spirits and carbonyl compounds. The initial point is known. There is no control input in this identification problem. We set the maximum order for the first equation to 10. The 50 runs of the algorithm gave us some different solutions that are shown in Table 1.

As we can see, the found parameters and system structure forms the first order differential equation, and that fact does not contradict the hypothesis (Romanovskii, 2006), which states that disintegration chemical reactions can be presented as first order linear differential equation.

Knowing the structure of the equations we can identify the system itself. The given optimization procedure is a stochastic algorithm, that is why the best solution from the 20 runs was taken.

Table 1: The hexadecane disintegration model.

Models and the error (I)
$4.05 \cdot x' + 0.9 \cdot x = 1,\ I = 0.3022$
$1.05 \cdot x' + 0.4 \cdot x = 1,\ I = 0.2834$
$2.1 \cdot x' + 0.55 \cdot x = 1,\ I = 0.1822$
$-1.05 \cdot x''' - 0.15 \cdot x'' - 6.85 \cdot x' - 0.9 \cdot x = 1,\ I = 0.227$
$-3.4 \cdot x' - 0.45 \cdot x = 0,\ I = 0.202$

The solution can be represented in the matrix form

$$A = \begin{pmatrix} -0.1671 & 0.7630 & -0.3625 \\ 0.0413 & -0.3428 & 0.115 \\ 0.0026 & 0.405 & -0.327 \end{pmatrix},$$

$$B = \begin{pmatrix} 0.3477 \\ 0 \\ 0 \end{pmatrix}.$$

The system outputs and the sample are shown on figures 1, 2 and 3 for hexadecane, spirits and carbonyl compounds respectively.

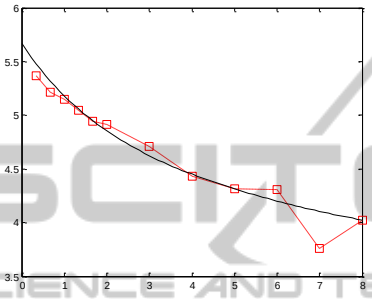

Figure 1: Hexadecane concentration measurements and the model output.

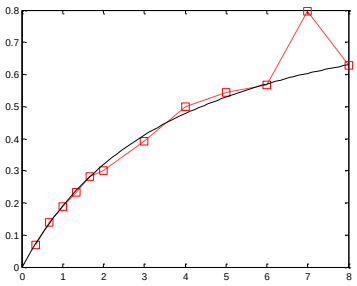

Figure 2: Spirits concentration measurements and the model output.

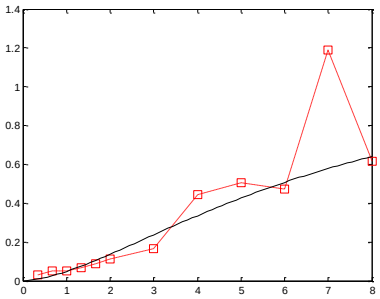

Figure 3: Carbonyl compounds concentration measurements and the model output.

As we can see on figures, the measurement at the point $t = 7$ is an abnormal measurement, but it did not effect on the model.

5 CONCLUSIONS

In this paper, the method of the ordinary differential equation structure and parameters identification was described. Within the proposed approach the system structure and its parameters are automatically determined. The suggested modifications of the evolutionary strategies algorithm increase the accuracy of model and allow solving two tasks at the same time. The identification problem for hexadecane disintegration reaction was considered. Numerical experiments have demonstrated the proposed approach usefulness.

The further investigation should be concentrated on the estimation of the performance of algorithm with the different local optimization and mutation parameters.

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