Robust Arbitrary Reference Command Tracking with Application to Hydraulic Actuators

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Abstract: In this paper a robust tracking controller is proposed in order to track arbitrary reference signals in the presence of same type disturbance signals. The robust tracking controller is based on the well known Internal Model Principle appropriately modified with a Hurwitz invariability technique. The controller parameters are computed using a finite step algorithm. Solvability conditions are derived. The proposed controller is successfully applied to a hydraulic actuator uncertain model including uncertain parameters arising from changes of the operating conditions and other physical reasons. Simulation results for all the expected range of the actuator model uncertainties are presented indicating the satisfactory performance of the robust controller in the presence of external disturbances.

1 INTRODUCTION

The problem of output tracking appears to be one of the most popular control design problems (see (Chen, 1984), (Horowitz, 1963), (Dorf and Boshop, 2001), (Goodwin et al., 2001), (Corless et al., 1984), (Takaba, 1998); (Yaesh and Shaked, 1991) and the reference therein). The problem of output tracking for both non uncertain and uncertain systems (robust tracking) is treated mainly using stabilizability techniques, e.g. Dorf and Boshop, 2001; Corless et al., 1984; Takaba, 1998. For robust tracking a variety of approaches, to the most optimal or adaptive, has been proposed in (Corless et al., 1984), (Takaba, 1998), (Yaesh and Shaked, 1991), (Skarpetis et al., 2006a,b), and (Skarpetis et al., 2007).

The problem of robust tracking appears to be of major interest in the design of controllers for hydraulic actuators. This type of actuators is widely used in many applications like manufacturing, robotics, constructions and avionics. The dynamics of fluid power are inherently uncertain. So, robust control strategies are indispensible if one wishes to guarantee safety and reliability of hydraulic actuators (see Skarpetis et al., 2007; Karpenko and Shapehri, 2005; Koumboulis et al., 2006a and b; Koumboulis et al., 1998; Kliffken, 1997; Kliffken and Heinrich, 2001; and the references therein). Robust asymptotic tracking techniques like those in Skarpetis et al., 2006a-b; Skarpetis et al., 2007 and robust PI-PID design techniques like those in Koumboulis et al., 2006a and b; Huang and Wang, 2000; Ho and Huang, 2003; Musch and Steiner, 1995; Ge et al., 2002; Toscano, 2005; Garcia et al., 2004; Koumboulis, 2005; and Koumboulis, 1999; perform satisfactory in many industrial hydraulic plants.

In this paper a robust tracking controller is proposed in order to satisfy asymptotic command following for arbitrary reference signals. The design technique is based on the well known Internal Model Principle (Goodwin et al., 2001), appropriately extended using Hurwitz invariability for the augmented system including the error dynamics. An arbitrary reference model that produces desired reference signals is used in the controller structure and the overall closed loop robust stability is guaranteed under sufficient conditions. The robust tracking controller appears to guarantee satisfactory performance under the influence of external disturbance signals.

The present results are successfully applied to control the position of a hydraulic actuator model involving uncertain parameters arising from changes of the operating conditions (temperature, pressure, entrained air or water) as well as physical uncertainties (loss in the effective area of the actuator piston seal due to wear (Karpenko and Shapehri, 2005)). Solvability conditions are established. An analytic finite step algorithm for the computation of the robust controller parameters is

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proposed. Following the algorithm, first, robust stability regions are determined. Second, the metaheuristic optimization algorithm proposed in (Koumboulis and Tzamtzi, 2007) is applied, inside these regions, to fulfil performance criteria. The effectiveness of the controller is illustrated through simulations for various values of the model uncertain parameters. The present results appear to be simple and easily applicable.

2 PRELIMINARY RESULTS

Consider the linear time-invariant SISO system with non linear uncertain structure described by

$$\begin{aligned} \dot{x}(t) &= A(q)x(t) + b(q)u(t) + d(q)w(t) \\ y(t) &= c(q)x(t) \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}$ is the input and $y(t) \in \mathbb{R}$ is the output and $w(t) \in \mathbb{R}$ is external disturbance. $A(q) \in [\wp(q)]^{n\times n}$, $b(q) \in [\wp(q)]^{n\times 1}$, $d(q) \in [\wp(q)]^{n\times 1}$ and $c(q) \in [\wp(q)]^{1\times n}$ are function matrices depending upon the uncertainty vector $q = \begin{bmatrix} q_1 & \cdots & q_l \end{bmatrix} \in \mathbb{Q}$ ($\mathbb{Q}$ denotes the uncertain domain). The set $\wp(q)$ is the set of nonlinear functions of q. The uncertainties $q_1,\ldots,q_l$ do not depend upon the time. With regard to the nonlinear structure of $A(q)$, $b(q)$, $d(q)$ and $c(q)$ no limitation or specification is considered (i.e. boundness, continuity).

Consider the case where the reference signal $y_r(t)$ is the output of a linear model described by

$$\dot{x}_r(t) = A_r x_r(t) \quad ; \quad y_r(t) = c_r x_r(t) \tag{2}$$

where $y_r(t) \in \mathbb{R}$, $x_r(t) \in \mathbb{R}^{r\times 1}$ and where

$$A_r = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -d_r & -d_{r-1} & -d_{r-2} & \cdots & -d_1 \end{bmatrix},$$
$$c_r = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$

Consider the vector $x_{r,0}$ denoting arbitrary initial conditions for system (2). Clearly, it holds that

$$y_r^{(r)}(t) + \sum_{i=1}^{r} d_i y_r^{(r-i)}(t) = 0 \tag{3}$$

The disturbance is assumed to be of the same type as the reference signal, i.e.

$$w^{(r)}(t) + \sum_{i=1}^{r} d_i w^{(r-i)}(t) = 0 \tag{4}$$

Define the tracking error

$$\varepsilon(t) = y(t) - y_r(t) \tag{5}$$

Differentiating the error r-times, we get

$$\begin{aligned} \varepsilon^{(r)}(t) &= c(q)x^{(r)}(t) - y_r^{(r)}(t) = \\ &= c(q)x^{(r)}(t) + \sum_{i=1}^{r} d_i y_r^{(r-i)}(t) \end{aligned} \tag{6}$$

or equivalently

$$\begin{aligned} &\varepsilon^{(r)}(t) + \sum_{i=1}^{r} d_i \varepsilon^{(r-i)}(t) = \\ &c(q)x^{(r)}(t) + c(q)\sum_{i=1}^{r} d_i x^{(r-i)}(t) \end{aligned} \tag{7}$$

Define the variables

$$\begin{aligned} z(t) &= x^{(r)}(t) + \sum_{i=1}^{r} d_i x^{(r-i)}(t) \\ \tilde{u}(t) &= u^{(r)}(t) + \sum_{i=1}^{r} d_i u^{(r-i)}(t) \end{aligned} \tag{8}$$

According to (4), (7) and (8) the following augmented system is defined:

$$\frac{d}{dt}\tilde{x}(t) = \tilde{A}(q)\tilde{x} + \tilde{b}(q)\tilde{u}(t) \tag{9}$$

$$\tilde{x}(t) = \begin{bmatrix} \varepsilon(t) & \varepsilon^{(1)}(t) & \cdots & \varepsilon^{(r-1)}(t) & z(t) \end{bmatrix}^T$$
$$\tilde{A}(q) = \left[\begin{array}{c|c} A_r & e_r c(q) \\ \hline 0_{n\times r} & A(q) \end{array}\right], \tilde{b}(q) = \begin{bmatrix} 0_{r\times 1} \\ b(q) \end{bmatrix},$$
$$e_r = \begin{bmatrix} 0_{(r-1)\times 1} \\ 1 \end{bmatrix}.$$

Consider the static state feedback control law

$$\tilde{u}(t) = f\tilde{x}(t) = f_1\tilde{\varepsilon}(t) + f_2 z(t) \tag{10}$$

where $\tilde{\varepsilon}(t) = \begin{bmatrix} \varepsilon(t) & \varepsilon^{(1)}(t) & \cdots & \varepsilon^{(r-1)}(t) \end{bmatrix}^T$.

The robust output command tracking is formulated as follows (Chen, 1984; Goodwin, Graebe and Salgado, 2001): the output of the uncertain system (1) follows the output of the reference system (2) while the tracking error (5) decreases asymptotically to zero. This is satisfied using a static state feedback control law of the form (10) guaranteeing robust stability of the polynomial

$$\tilde{p}_{cl}(s,q,f) = \det\left[sI_{r+n} - \tilde{A}(q) - \tilde{b}(q)f\right] \quad (11)$$

The control law (10) can be expressed in terms of the original systems using the differential equation:

$$\begin{aligned} u^{(r)}(t) + \sum_{i=1}^{r} d_i u^{(r-i)}(t) &= \sum_{i=1}^{r} f_{1,i}\varepsilon^{(i-1)}(t) + \\ + f_2\left(x^{(r)}(t) + \sum_{i=1}^{r} d_i x^{(r-i)}(t)\right) & \end{aligned} \quad (12)$$

where $f_{1,i}\ (i = 1,\dots,r)$ are the elements of f_1. Eq. (12) is realized in state space form as (see Figure 1):

$$\begin{aligned} &\dot{x}_c(t) = A_c x_c(t) + b_c\varepsilon(t),\ v(t) = c_c x_c(t) \\ &u(t) = v(t) + f_2 x(t) \end{aligned} \quad (13)$$

$$A_c = \begin{bmatrix} -d_1 & 1 & 0 & \cdots & 0 \\ -d_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -d_r & 0 & 0 & \cdots & 0 \end{bmatrix}, b_c = \begin{bmatrix} f_{1,r} \\ f_{1,r-1} \\ \vdots \\ f_{1,1} \end{bmatrix},$$

$$c_c = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}$$

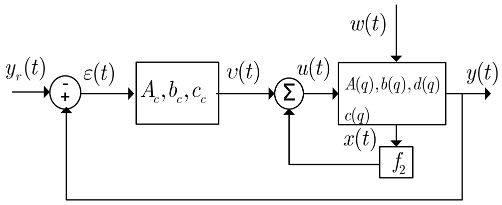

Figure 1: Closed loop structure.

3 SOLVABILITY CONDITIONS

The polynomial (11) can be rewritten as

$$\begin{aligned} \tilde{p}_{cl}(s,q,f) = \det\left[sI_r - A_r\right]\det\left[sI_n - A(q)\right] \\ - f\,\mathrm{adj}\left[sI_{n+r} - \tilde{A}(q)\right]\tilde{b}(q) \end{aligned} \quad (14)$$

Define:

$$\tilde{a}(q) = \begin{bmatrix} 1 & \tilde{a}_1(q) & \cdots & \tilde{a}_{r+n}(q) \end{bmatrix} \quad (15)$$

where $\tilde{a}_i(q)$ ($i = 1,\dots,r+n$) are the coefficients of the polynomial $\det[sI_r - A_r]\det[sI_n - A(q)]$. Also define the polynomial matrix

$$\begin{aligned} \tilde{P}(s,q) &= \Omega(q)[s^{\mu(q)} \cdots \ s^0]^T = \\ &\mathrm{adj}\left[sI_{n+r} - \tilde{A}(q)\right]\tilde{b}(q) \end{aligned} \quad (16)$$

where $\mu(q) \le n + r$ is the maximum degree of the polynomial matrix $\mathrm{adj}\left[sI_{n+r} - \tilde{A}(q)\right]\tilde{b}(q)$ and

$$\begin{aligned} \Omega(q) &= \left[\omega_0(q) \cdots \omega_{\mu(q)}(q)\right]; \\ \omega_i(q) &= \left[\tilde{\omega}_{i,1}(q) \cdots \tilde{\omega}_{i,n+r}(q)\right]^T \end{aligned} \quad (17)$$

According to definitions (15) and (17) the augmented closed loop characteristic polynomial (14) can equivalently be expressed as follows:

$$\tilde{p}_{cl}(s,q,f) = [s^{n+r} \ \cdots \ s^0]A^{**}(q)\begin{bmatrix} 1 \\ f^T \end{bmatrix} \quad (18)$$

where

$$A^{**}(q) = \begin{bmatrix} \tilde{a}^T & -\tilde{\Omega}^T \end{bmatrix},$$

$$\tilde{\Omega}(q) = \begin{bmatrix} 0_{(n+r)\times(n+r-\mu(q))} & \Omega(q) \end{bmatrix} \quad (19)$$

Based on the above definitions and the results in (Wei and Barmish, 1989), (Koumboulis and Skarpetis, 1996) and (Koumboulis and Skarpetis, 2000) the following theorem is presented.

Theorem 1. The problem of robust output command tracking for the uncertain system (1) and for arbitrary signals produced by the reference model (2), is solvable, via the controller (13), if the following conditions are satisfied

(i) The elements of $A^{**}(q)$ are continuous functions of q for every $q \in \mathbb{Q}$

(ii) There exists $(n + r + 1) -$row submatrix of $A^{**}(q)$, let $A^{*}(q)$ which is positive antisymetric.

Proof: According to the definition of the problem presented in Section 2, the problem of robust output command tracking for the uncertain system (1) via the controller (13) is solvable if the polynomial (14) is robustly stable. According to the results in (Wei and Barmish, 1989), (Koumboulis and Skarpetis, 1996) and (Koumboulis and Skarpetis, 2000) the uncertain polynomial is Hurwitz invariant if conditions (i) and (ii) of Theorem 1 are satisfied. ■

In the following theorem necessity is studied.

Theorem 2. For the problem of robust output command tracking for the uncertain system (1) and for arbitrary signals produced by the reference model (2), via the controller (13), it is necessary for the roots of the polynomial $c(q)\mathrm{adj}[sI_n - A(q)]b(q)$ not to be unstable roots of $\det\left[sI_r - A_r\right]$ for every $q \in \mathbb{Q}$.

Proof: The polynomial (14) can be rewritten as follows

$$\begin{aligned}\tilde{p}_{cl}\left(s,q,f\right) &= \det\left[sI_r - A_r\right]\det\left[sI_n - A(q)\right] + \\ &-f_1\,\mathrm{adj}\left[sI_r - A_r\right]c(q)\mathrm{adj}\left[sI_n - A\left(q\right)\right]b\left(q\right) + \\ &-f_2\,\mathrm{adj}\left[sI_n - A\left(q\right)\right]b(q)\det\left[sI_r - A_r\right]b\left(q\right)\end{aligned}$$

From the above relation it is clear that if $\sigma(q) \in \mathbb{C}$ is a root of $c(q)\mathrm{adj}[sI_n - A(q)]b(q)$, being unstable for at least one $q \in \mathbb{Q}$. Then its value for this q must not be eigenvalue of $\det\left[sI_r - A_r\right]$. ■

The condition of Theorem 2 is related to the uncontrollable part of the augmented system $(\tilde{A}(q),\tilde{b}(q))$. This condition is useful in choosing the model of the reference signal.

Remark 1. The class of the systems that satisfy condition (ii) of Theorem 1, can be widen, if, instead of $A^{**}(q)$ the matrix $A^{**}(q)T$ is considered where T is an appropriate invertible and independent from q matrix.

For the definition of positive antisymetric matrices see Wei and Barmish, 1989; Koumboulis and Skarpetis, 1996; and Koumboulis and Skarpetis, 2000). An analytic algorithm for the computation of an f preserving Hurwitz invariability can be found in the aforementioned papers.

4 ROBUST CONTROL FOR POSITION TRACKING OF A HYDRAULIC ACTUATOR

4.1 Actuator Model

Consider a double acting servo valve and piston actuator shown in Figure 2. The linearized differential equations that describe the actuator – valve dynamics can be formulated as follows (Karpenko and Shapehri, 2005):

$$\dot{x}_p(t) = \upsilon_p(t) \tag{20}$$

$$\dot{\upsilon}_p(t) = \frac{1}{m}\left[AP_L(t) - b\upsilon_p(t) - F_L(t)\right] \tag{21}$$

$$\dot{P}_L(t) = \frac{4\beta}{V}\left[K_f x_v(t) - K_{tp}P_L(t) - A\upsilon_p(t)\right] \tag{22}$$

where υ_p is the piston velocity, x_p is the piston position, P_L is the hydraulic pressure across the actuator piston, F_L is the external load disturbance and x_v is the spool valve displacement. The parameters A, m, β, b and V are: the piston surface area, the mass of the load, the effective bulk modulus of the hydraulic fluid, the viscous damping coefficient and the total volume of hydraulic oil in the piston chamber and the connecting lines, respectively. The coefficients K_f and K_{tp} arise from the linearization of the servo valve load flow and the leakage flow.

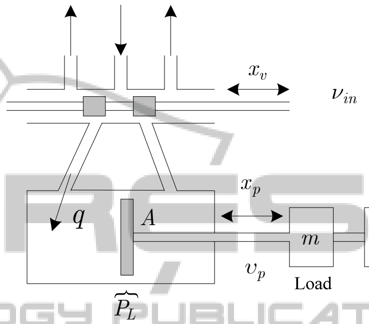

Figure 2: Valve and piston schematic.

The valve displacement is usually produced by a solenoid (electrohydraulic valve) actuated by the input voltage $\nu_{in}(t)$ of the solenoid. The transfer function of a solenoid can be approximated by the servo valve spool position gain denoted by k_v. Using (20)-(22) the following linear system with uncertain structure is derived in state space form:

$$\dot{x}(t) = A_0(q)x(t) + B_0(q)\nu_{in}(t) + D_0F_L(t) \tag{23}$$

$$y(t) = C_0x(t) \tag{24}$$

$$x(t) = \begin{bmatrix} x_p(t) & \upsilon_p(t) & P_L(t)\end{bmatrix}^T$$

$$C_0 = \begin{bmatrix}1 & 0 & 0\end{bmatrix}$$

$$A_0(q) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -b/m & A/m \\ 0 & -4q_1A/V & -4q_1q_2/V \end{bmatrix},$$

$$B_0(q) = \begin{bmatrix} 0 \\ 0 \\ 4q_1q_3k_v/V \end{bmatrix}, D_0 = \begin{bmatrix} 0 \\ -1/m \\ 0 \end{bmatrix}$$

The parameter $q_1 = \beta$ is an uncertain parameter since the effective bulk modulus of the hydraulic fluid changes due to temperature, pressure and entrained air or water fluctuations. The parameter $q_2 = K_{tp}$ changes due to migration of the system's operating point and the parameter $q_3 = K_f$ changes due to migration of the system operating point and to loss in the effective area of the actuator piston seal, due to wear (Karpenko and Shapehri, 2005). The vector $q = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix} \in \mathbb{Q}$ is the uncertain vector and $\mathbb{Q}$ is the domain of uncertainty. The nominal values of the system parameters are shown in Table 1 and the expected range of variations of the uncertain system parameters is shown in Table 2 (Karpenko and Shapehri, 2005).

Table 1: Nominal Values for the Hydraulic Actuator's Parameters.

	Definition	Nominal Values
V	volume of hydraulic oil in the piston chamber	$468 \,/\, 100^3 \ m^3$
A	piston surface area	$633 \,/\, 1000^2 \ m^2$
β	effective bulk modulus	$689 \times 10^6 \ Pa$
K_{tp}	total flow pressure coefficient	$0 \ \ m^3 \,/\, Pa - s$
b	viscous damping coefficient	$1000 \, Nm^{-1}s$
m	load mass	$12 \, Kg$
k_v	servo valve spool position gain	$0.0406 \times 10^{-3} \ \ m \,/\, V$
K_f	servo valve gain	$1.02 \ \ m^2 \,/\, \sec$

Table 2: Expected Range of Variations of the Uncertain Parameters.

Symbol	Minimum Values	Nominal Values	Maximum Values
$\beta(Pa)$	550×10^6	689×10^6	895×10^6
$K_{tp}\left(\dfrac{m^3}{Pa\, s}\right)$	0	0	9.5×10^{-11}
$K_f\left(\dfrac{m^2}{\sec}\right)$	1.02	1.02	1.76

4.2 Robust Tracking Controller

In this subsection a robust tracking arbitrary controller for asymptotic tracking of the piston position will be designed. According to (2) the reference output model is derived for $r = 2$ to be:

$$\dot{x}_r(t) = A_r x_r(t),\ y_r(t) = c_r x_r(t),\ x_{r,0} = \begin{bmatrix} x_{r,01} \\ x_{r,02} \end{bmatrix}$$

where $A_r = \begin{bmatrix} 0 & 1 \\ -d_2 & -d_1 \end{bmatrix}$ and $c_r = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

According to definitions of Section 2 the following augmented system is introduced

$$\frac{d}{dt}\tilde{x}(t) = \tilde{A}(q)\tilde{x} + \tilde{b}(q)\tilde{u}(t)$$

where

$$\tilde{x}(t) = \begin{bmatrix} \varepsilon(t) & \varepsilon^{(1)}(t) & z(t) \end{bmatrix}^T$$

$$\tilde{A}(q) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ -d_2 & -d_1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -b/m & A/m \\ 0 & 0 & 0 & -4q_1 A/V & -4q_1 q_2/V \end{bmatrix},$$

$$\tilde{b}(q) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 4q_1 q_3 k_v / V \end{bmatrix}.$$

Apply the static state feedback law: $\tilde{u} = f\tilde{x}$ with $f = \begin{bmatrix} f_{11} & f_{12} & f_{21} & f_{22} & f_{23} \end{bmatrix}$. The aforementioned controller can be produced by the original input signal $u(t)$ using the following state space form:

$$\dot{x}_c(t) = A_c x_c(t) + b_c e(t),\ v(t) = c_c x_c(t)$$

where $Ac = \begin{bmatrix} -d_1 & 0 \\ -d_2 & 0 \end{bmatrix}, b_c = \begin{bmatrix} f_{1,2} \\ f_{1,1} \end{bmatrix}, c_c = \begin{bmatrix} 1 & 0 \end{bmatrix}$ and $u(t) = v(t) + f_2 x(t)$ where $f_2 = \begin{bmatrix} f_{21} & f_{22} & f_{23} \end{bmatrix}$.

The augmented system closed loop characteristic uncertain polynomial is:

$$p_{cl}(s, q_1, q_2, f) = s^5 + \gamma_0(q,f)s^4 + \gamma_1(q,f)s^3 + \gamma_2(q,f)s^2 + \gamma_3(q,f)s^1 + \gamma_4(q,f) \quad (25)$$

where

$$\gamma_0(q,f) = d_1 + \frac{4mq_1(q_2 - f_{23}k_u q_3) + bV}{mV},$$

$$\gamma_1(q,f) = \frac{1}{mV}(4A^2 q_1 - 4Af_{22}k_u q_1 q_3 + 4bq_1(q_2 - f_{23}k_u q_3) + 4d_1 m q_1(q_2 - f_{23}k_u q_3) + bd_1 V + d_2 mV),$$

$$\gamma_2(q,f) = \frac{1}{mV}(4A^2 d_1 q_1 - 4A(f_{21} + d_1 f_{22})k_u q_1 q_3 + 4d_2 m q_1(q_2 - f_{23}k_u q_3) + b(4d_1 q_1(q_2 - f_{23}k_u q_3) + d_2 V)),$$

$$\gamma_3(q,f) = \frac{1}{mV}(4q_1(bd_2(q_2 - f_{23}k_u q_3) + A^2 d_2 - A(f_{12} + d_1 f_{21} + d_2 f_{22})k_u q_3)),$$

$$\gamma_4(q,f) = -\frac{4A(f_{11} + d_2 f_{21})k_u q_1 q_3}{mV}.$$

According to (15) and (17) define

$$\tilde{a}(q) = \begin{bmatrix} 1 & \tilde{a}_0 & \tilde{a}_1 & \tilde{a}_2 & \tilde{a}_3 & 0 \end{bmatrix} \quad (26)$$

$$\tilde{a}_0 = d_1 + \frac{b}{m} + \frac{4q_1 q_2}{V},$$

$$\tilde{a}_1 = \frac{4A^2 q_1 + 4bq_1 q_2 + 4d_1 m q_1 q_2 + bd_1 V + d_2 mV}{mV}$$

$$\tilde{a}_2 = \frac{4A^2 d_1 q_1 + 4bd_1 q_1 q_2 + 4d_2 m q_1 q_2 + bd_2 V}{mV},$$

$$\tilde{a}_3 = \frac{4d_2 q_1(A^2 + bq_2)}{mV}$$

$$\left[-\tilde{\Omega}(q)\right]^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{\omega}_{25} \\ 0 & 0 & 0 & \tilde{\omega}_{34} & \tilde{\omega}_{35} \\ 0 & 0 & \tilde{\omega}_{43} & \tilde{\omega}_{44} & \tilde{\omega}_{45} \\ 0 & \tilde{\omega}_{52} & \tilde{\omega}_{53} & \tilde{\omega}_{54} & \tilde{\omega}_{55} \\ \tilde{\omega}_{61} & 0 & \tilde{\omega}_{63} & 0 & 0 \end{bmatrix} \quad (27)$$

where $\tilde{\omega}_{61} = \tilde{\omega}_{52} = \tilde{\omega}_{43} = \tilde{\omega}_{34} = -\frac{4Ak_u q_1 q_3}{mV}$,

$\tilde{\omega}_{25} = -\frac{4k_u q_1 q_3}{V}$, $\tilde{\omega}_{53} = \tilde{\omega}_{44} = -\frac{4Ad_1 k_u q_1 q_3}{mV}$,

$\tilde{\omega}_{35} = -\frac{4k_u(b + d_1 m)q_1 q_3}{mV}$, $\tilde{\omega}_{55} = -\frac{4bd_2 k_u q_1 q_3}{mV}$,

$\tilde{\omega}_{63} = \tilde{\omega}_{54} = -\frac{4Ad_2 k_u q_1 q_3}{mV}$,

$\tilde{\omega}_{45} = -\frac{4k_u(bd_1 + d_2 m)q_1 q_3}{mV}$,

According aforementioned definitions the augmented closed loop characteristic polynomial (25) can equivalently be expressed as follows:

$$\tilde{p}_{cl}(s,q,f) = \begin{bmatrix} s^5 & s^4 & s^3 & s^2 & s^1 & s^0 \end{bmatrix} A^{**}(q) \begin{bmatrix} 1 & f_{11} & f_{12} & f_{21} & f_{22} & f_{23} \end{bmatrix}^T \quad (28)$$

where

$$A^{**}(q) = \begin{bmatrix} \tilde{a}^T & -\tilde{\Omega}^T \end{bmatrix} \quad (29)$$

Let

$$T = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Choose the following 6×6 row submatrix of $A^{**}(q)T$,

$$A^*(q) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \phi_{21} & \phi_{22} & 0 & 0 & 0 & 0 \\ \phi_{31} & \phi_{32} & \phi_{33} & 0 & 0 & 0 \\ \phi_{41} & \phi_{42} & \phi_{43} & \phi_{44} & 0 & 0 \\ \phi_{51} & \phi_{52} & \phi_{53} & \phi_{54} & \phi_{55} & 0 \\ 0 & 0 & 0 & \phi_{64} & 0 & \phi_{66} \end{bmatrix} \quad (30)$$

where $\phi_{21} = \tilde{a}_0$, $\phi_{31}(q) = \tilde{a}_1$, $\phi_{41} = \tilde{a}_2$, $\phi_{51} = \tilde{a}_3$, $\phi_{22} = -\tilde{\omega}_{25}$, $\phi_{32} = -\tilde{\omega}_{35}$, $\phi_{42} = -\tilde{\omega}_{45}$, $\phi_{52} = -\tilde{\omega}_{55}$, $\phi_{33} = -\tilde{\omega}_{34}$, $\phi_{43} = -\tilde{\omega}_{44}$, $\phi_{53} = -\tilde{\omega}_{54}$ $\phi_{44} = -\tilde{\omega}_{43}$, $\phi_{54} = -\tilde{\omega}_{53}$, $\phi_{64} = -\tilde{\omega}_{63}$, $\phi_{55} = -\tilde{\omega}_{52}$, $\phi_{66} = -\tilde{\omega}_{61}$.

Theorem 3. The problem of robust output command tracking for the uncertain system (1) via the controller (13) is always solvable.

Proof: Condition (i) can easily be verified. The matrix $A^*(q)$ is positive antisymetric. It can be constructed using the five positive up augmentations (ϕ_{66}, ϕ_{55}, ϕ_{44}, ϕ_{33} and ϕ_{22} are positive numbers for all the values of the uncertainties):

$$\Phi_1(q) \to \Phi_2(q) \to \Phi_3(q) \to \Phi_4(q) \to \to \Phi_5(q) \to A^*(q)$$

where $\Phi_1(q) = \phi_{66}$, $\Phi_2(q) = \begin{bmatrix} \phi_{56} & 0 \\ 0 & \phi_{66} \end{bmatrix}$,

$$\Phi_3(q)=\begin{bmatrix}\phi_{44} & 0 & 0\\ \phi_{54} & \phi_{55} & 0\\ \phi_{64} & 0 & \phi_{66}\end{bmatrix},$$

$$\Phi_4(q)=\begin{bmatrix}\phi_{33} & 0 & 0 & 0\\ \phi_{43} & \phi_{44} & 0 & 0\\ \phi_{53} & \phi_{54} & \phi_{55} & 0\\ 0 & \phi_{64} & 0 & \phi_{66}\end{bmatrix},$$

$$\Phi_5(q_1,q_2)=\begin{bmatrix}\phi_{22} & 0 & 0 & 0 & 0\\ \phi_{32} & \phi_{33} & 0 & 0 & 0\\ \phi_{42} & \phi_{43} & \phi_{44} & 0 & 0\\ \phi_{52} & \phi_{53} & \phi_{54} & \phi_{55} & 0\\ 0 & 0 & \phi_{64} & 0 & \phi_{66}\end{bmatrix}.$$

The vector $\overline{c}(q)=\Phi_1(q)$ is a Hurwitz invariant core since the associate polynomial of $\overline{c}(q)$ ($[\overline{c}(q)]^T$) is positive Hurwitz invariant. Hence, condition (ii) of Theorem 1 is satisfied. For reference and disturbance signals of sinusoidal form, condition (iii) of Theorem 1 is also satisfied. ■

5 COMPUTATION OF THE CONTROLLER PARAMETERS

Using a reference input of the form $y_r(t)=0.02Sin[0.2t]$ ($d_1=0, d_2=0.04$, $x_{r,01}=0, x_{r,02}=0.004$) the tracking controller will be computed using the following algorithm

Step 1 (Construction of the augmentation matrices): The core of $A^*(q)$ is $\overline{c}(q)=\Phi_1(q)$. From $\Phi_1(q)$ using five up positive augmentations the matrix $\Phi_6(q)=A^*(q)$ is constructed. Let $\tau_1=1$.

Step 2 (Determination of the region of $\varepsilon_1>0$ for which $\Phi_2(q)\begin{bmatrix}\varepsilon_1 & 1\end{bmatrix}^T$ is positive Hurwitz invariant): According to the form of the associated polynomial it is observed that robust stability is guaranteed $\forall\varepsilon_1>0$. Let $\tau_2=\begin{bmatrix}\varepsilon_1 & 1\end{bmatrix}$ and choose the stability region of ε_1 to be : $\varepsilon_1\in[0.25,0.55]$.

Step 3 The polynomial $\Phi_3(q)\begin{bmatrix}\varepsilon_2 & \tau_2\end{bmatrix}^T$ is robustly stable $\forall\varepsilon_2>0$. Let $\tau_3=\begin{bmatrix}\varepsilon_2 & \varepsilon_1 & 1\end{bmatrix}$ and choose the stability region of ε_2 to be: $\varepsilon_2\in[0.2,0.3]$.

Step 4: The polynomial $\Phi_4(q)\begin{bmatrix}\varepsilon_3 & \tau_3\end{bmatrix}^T$ is positive Hurwitz invariant inside the selected region $\varepsilon_3\in[0.01,0.015]$.Let $\tau_4=\begin{bmatrix}\varepsilon_3 & \varepsilon_2 & \varepsilon_1 & 1\end{bmatrix}$.

Step 5 : The polynomial $\Phi_5(q)\begin{bmatrix}\varepsilon_4 & \tau_4\end{bmatrix}^T$ is positive Hurwitz invariant inside the region $\varepsilon_4\in[6\times10^{-9},7\times10^{-9}]$.Let $\tau_5=\begin{bmatrix}\varepsilon_4 & \varepsilon_3 & \varepsilon_2 & \varepsilon_1 & 1\end{bmatrix}$.

Step 6: The polynomial $\Phi_6(q)\begin{bmatrix}\varepsilon_5 & \tau_5\end{bmatrix}^T$ is positive Hurwitz invariant inside the selected region $\varepsilon_5\in[0.0006,0.0008]$.

Step 7: Using a search algorithm in the stability regions specified in steps 2-6 and for all values of the uncertain parameters, the following values for $\varepsilon_1,\varepsilon_2,\varepsilon_3,\varepsilon_4,\varepsilon_5$ are derived: $\varepsilon_1=0.5$, $\varepsilon_2=0.25$, $\varepsilon_3=0.012$, $\varepsilon_4=6\times10^{-9}$ and $\varepsilon_5=0.0007$.

Step 8 (Derivation of the gain vector): The gain vector $\tilde{f}$ that robustly stabilizes the associated polynomial of $A^{**}(q)T$ ($A^{**}(q)T\tilde{f}^T$) is $\tilde{f}=\begin{bmatrix}\varepsilon_5 & \varepsilon_4 & \varepsilon_3 & \varepsilon_2 & \varepsilon_1 & 1\end{bmatrix}$ and consequently the gain vector that robustly stabilizes the associate polynomial of $A^{**}(q)$ is

$$T\tilde{f}^T=\begin{bmatrix}\varepsilon_5 & -1 & -\varepsilon_1 & -\varepsilon_2 & -\varepsilon_3 & -\varepsilon_4\end{bmatrix}^T$$

or equivalently the vector

$$T\tilde{f}^T/\varepsilon_5=\begin{bmatrix}1 & -\frac{1}{\varepsilon_5} & -\frac{\varepsilon_1}{\varepsilon_5} & -\frac{\varepsilon_2}{\varepsilon_5} & -\frac{\varepsilon_3}{\varepsilon_5} & -\frac{\varepsilon_4}{\varepsilon_5}\end{bmatrix}^T$$

Finally using the above relation the respective values from Step 7 of the algorithm and relation (28) the controller parameters are computed to be

$$f_{11}=-1428.57, f_{12}=-714.286,$$
$$f_{21}=-357.143, f_{22}=-17.1429,$$
$$f_{23}=-8.57143*10^{-6}$$

6 SIMULATION RESULTS

Using Table 1 and 2 and for a reference signal and external disturbance as in Figures 3 and 4, the closed loop performance is illustrated in Figures 5 – 7 and the control signal is illustrated in Figure 8.

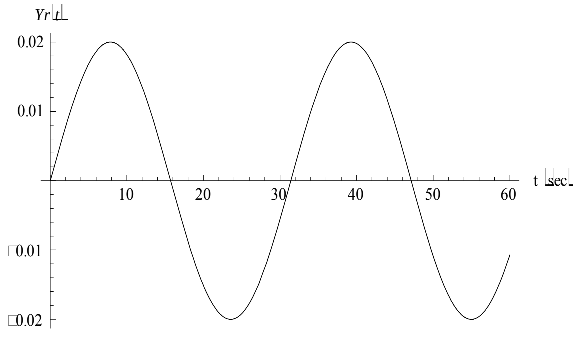

Figure 3: Reference signal.

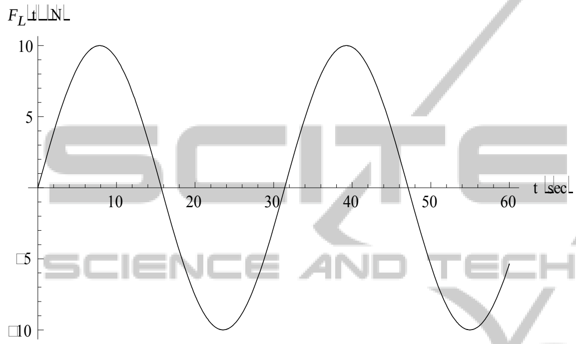

Figure 4: External Disturbance.

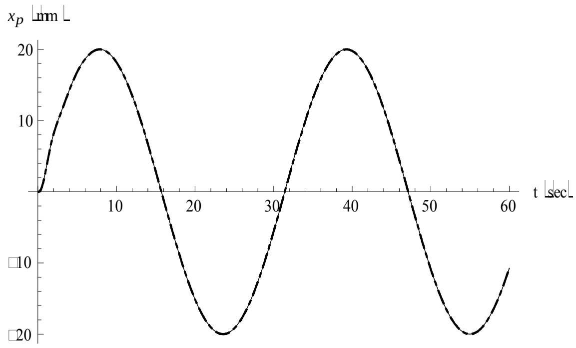

Figure 5: The piston position.

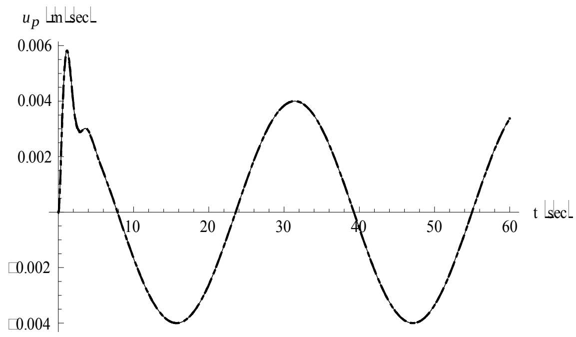

Figure 6: The piston velocity.

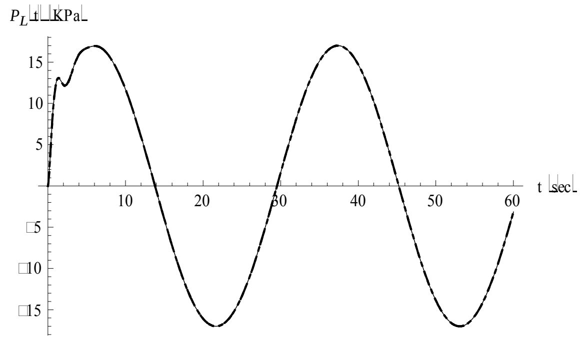

Figure 7: Hydraulic Pressure.

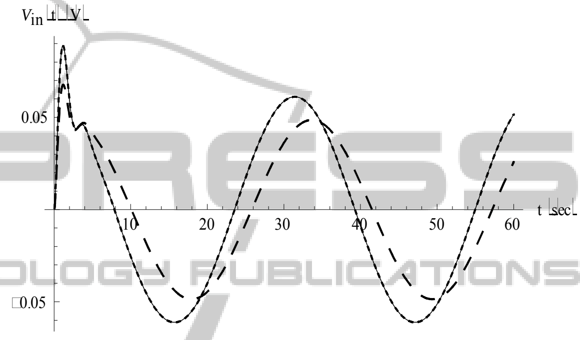

Figure 8: Input voltage of the solenoid.

(Dotted line, Solid line, Dashed line is for the minimum, a intermediary, and the maximum value of q, respectively).

7 CONCLUSIONS

A Robust tracking controller has been designed for arbitrary reference and disturbance signals. Sufficient conditions have been derived and a finite step algorithm has been proposed for fast and easy computation of the controller parameters. The results are successfully applied to a hydraulic actuator.

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