A NETWORK MODEL FOR PRICE STABILIZATION

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Abstract: We consider a simple network model for economic agents where each can buy commodities in the neighborhood. Their prices may be initially distinct in any node. However, by assuming some rules on new prices, we show that the distinct prices will be converged to unique by iterating buy and sell operations. If we consider the price determination process as a kind of consensus problem, we can apply the stabilization proof to it. So we first present a naive protocol in which each agent always offers half of the difference between his own price and the lowest price in the neighborhood, called max price difference. Then, we consider game theoretic price determination in two ways, that is, by using different payoff functions. Finally, we propose a protocol in which each agent makes a bid uniformly distributed over the max price difference.

1 INTRODUCTION

Conventionally, the topics of price determination have been discussed in the context of microeconomics approach (J. E. Stiglitz, 1993). A famous model, supply and demand curves, has been used as an abstract, theoretical method which explains a price equilibrium. However, for example, there is no distance concept in the model. To know a detailed process to the equilibrium, we need more sophisticated model, e.g., multiagent approach, which gives us another insight into the price determination.

We construct a price determination model by applying the idea of stabilization to the multiagent approach. The self-stabilization (S. Dolev, 2000) has been originally studied as the recovery from transient faults in distributed systems. From any initial state, self-stabilizing algorithms eventually lead to a legitimate state without any aid of external actions. In particular, a self-stabilizing consensus algorithm is associated with the price determination because every agent eventually has the same value.

We show a network model consisting of nodes and edges as cities and their links to neighbors, respectively. Each node contains an agent which represents people in the city. Any interaction among agents is governed by micro-rules, that is, the agents who want to buy a commodity make bids to their neighboring nodes. Then, the agents who want to sell the commodity accept the highest bid, like an auction (V. Krishna, 2002). By iterating these rules, the prices will reach an equilibrium.

First, we present a naive protocol in which each agent always offers a fixed price without considering other bidders' strategies. Then, we analyze the stabilization time of the protocol for a special case. Next, we consider game theoretic price determination in two ways, that is, by using different payoff functions. Finally, we propose a protocol in which each agent offers a random price and show that it stabilizes with high probability.

2 MODEL

Our system can be represented by a connected network $G = (V,E)$, consisting of a set of n nodes V and edges E, where the nodes represent cities and a pair of neighboring nodes (cities) is linked by an edge. We assume that each node $i \in V$ has a commodity and its initial price may be different. Let N_i be a set of neighboring nodes of $i \in V$, and let $N_i^+ = N_i \cup \{i\}$. Let $P_i(t)$ be the commodity price in the node i at time t. It is also denoted by P_i if time t is not important. We say that the price $P_i(t)$ is *maximal* if $P_i(t) \geq P_j(t)$ for any

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$j \in N_i$. Each node $i \in V$ has exactly one representative agent a_i who always stays at i and can buy commodities in the neighborhood N_i, where the *buy operation* is executed as follows.

First, each agent a_i compares the commodity price $P_i(t)$ with $P_j(t)$ for $j \in N_i$. If node $j \in N_i$ has the cheapest commodity in N_i with $P_i(t) > P_j(t)$, the agent a_i wants to buy it from the node j. (Otherwise, that is, there is no node $j \in N_i$ with $P_i(t) > P_j(t)$, agent a_i wants to buy it in the self node i.) We call such $P_i(t) - P_j(t)$ a *max price difference*. Then, agent a_i submits a bid to node j containing some price in accordance with a protocol. After accepting bids from N_j, agent a_j *contracts* with exactly one agent who submitted the highest price. Then, a_j sells the commodity to the contracted agent and sets $P_j(t+1)$ to the highest price. We ignore the carrying time of commodities and focus on the change of prices. In this way, at every time, any price is updated if necessary.

We assume a *synchronous model*, that is, every agent periodically (for each *round*) exchanges messages and knows the states of neighboring agents.

3 NAIVE PROTOCOL

In this section, we consider a naive protocol, called **HalfBid**, in which each agent always offers half of the max price difference. Here, we focus on a star, the part of a network G, with a center node c.

HalfBid

- Each agent a_j makes a bid with an integer price

$$P_c(t) + \left\lceil \frac{P_j(t) - P_c(t)}{2} \right\rceil$$

  to node $c \in N_j^+$ which has the lowest-priced commodity in N_j^+. The agent a_c contracts with the neighboring a_j who has submitted the highest bid. That is, the commodity price at time $t+1$ is

$$P_c(t+1) := P_c(t) + \max_{j \in N_c} \left\lceil \frac{P_j(t) - P_c(t)}{2} \right\rceil$$

- If $P_c(t)$ is maximal and a_c accepts no bidding from N_c, the price at time $t+1$ will be cut to

$$P_c(t+1) := P_c(t) - \max_{j \in N_c} \left\lfloor \frac{P_c(t) - P_j(t)}{2} \right\rfloor$$

- If several agents make bids to node c with the same highest price, agent a_c contracts with one of them with equal probability.

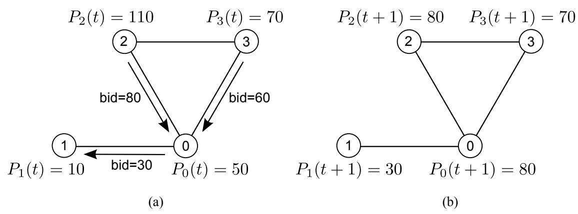

Figure 1: An illustration of naive protocol **HalfBid**.

Example 1. *Figure 1 shows an example of our network system consisting of 4 nodes* $V = \{0,1,2,3\}$. *At time* t, *the prices of commodities are* $(P_0(t), P_1(t), P_2(t), P_3(t)) = (50, 10, 110, 70)$ *as shown in Figure 1(a). Each agent* a_c *wants to buy the commodity if its price is lower than* $P_c(t)$, *i.e.,* $P_c(t) > \min_{j \in N_c} P_j(t)$. *Thus, agent* a_2 *makes a bid to node 0 with price* $50 + (110 - 50)/2$. *Likewise, agents* a_0 *and* a_3 *make bids to node 1 and node 0, respectively. Then,* a_2*'s bid and* a_0*'s bid are successful,* a_2 *(resp.* a_0*) makes a contract with* a_0 *(resp.* a_1*). At time* $t+1$, *the prices become* $(80, 30, 80, 70)$ *as shown in Figure 1(b). Since node 2 was maximal and no bid was submitted, the price is cut to 80 at time* $t+1$. □

Let $C_t \subseteq V$ be the set of nodes that have updated their prices from time t to $t+1$. Let the highest price be $P^{max}(t) = \max_{i \in C_t} P_i(t)$, and the lowest price be $P^{min}(t) = \min_{i \in C_t} P_i(t)$. The following lemma states that prices continue to move until every node has the same price.

Lemma 1. *The protocol* **HalfBid** *is deadlock-free. That is, there exist some nodes in* C_t *as long as the unique price is not determined.*

Proof sketch. The lemma is proved by contradiction. □

Lemma 2. *Let* $diff(t) = \max_{i \in C_t} P_i(t) - \min_{i \in C_t} P_i(t)$. *As long as* $C_t \neq \phi$, *we have*

$$diff(t) > diff(t+1).$$

Proof. First, we consider a node $i \notin C_t$. Since agent a_i does not make any bid to other nodes, there is no change in price. Thus, $\max_{j \in N_i} \lfloor \frac{P_i(t) - P_j(t)}{2} \rfloor = 0$ holds.

Next, suppose that a node has the maximum price in C_t. Since no neighboring nodes make bids to such a node, the price will be down at time $t+1$. If a node has the minimum price in C_t, there is a neighboring agent who makes a bid to the node. Thus, the price will be up at time $t+1$. Let $P^{max2}(t)$ be the second maximum price among the nodes in C_t. Then, the price will not exceed $P^{max}(t)$ at time $t+1$ because $P^{max2}(t)$ goes maximumly up only when it accepts an offer from $P^{max}(t)$. Even if it occurs, the increase is at most the half of the difference between them. Thus, we have

$$P^{max}(t) > P^{max2}(t+1), \; P^{max}(t) > P^{max}(t+1).$$

On the other hand, the node with $P^{min}(t)$ accepts a new bid and the price $P^{min}(t)$ goes up at time $t+1$. Let $P^{min2}(t)$ be the second minimum price among the nodes in C_t. Then, it maximumly decreases without any offers only when it is linked with the node with $P^{min}(t)$. Then, we have

$$P^{min}(t) \leq P^{min2}(t+1), \; P^{min}(t) < P^{min}(t+1).$$

Thus, $diff(t) > diff(t+1)$ holds. □

Theorem 1. *Our protocol will eventually stabilize the price.* □

In the sequel, we analyze the stabilization time of a path $(1,\ldots,n)$.

Theorem 2. *If network G is a path, the stabilization time of our* **HalfBid** *is* 2τ *rounds, where* τ *satisfies* $(\frac{3}{4})^{\tau}(\frac{1}{3})^{n/2+1}\binom{\tau}{n/2+1}^{n/2+1} = 1$.

Proof. We call the price difference between neighboring nodes a *gap*, and call the gaps as 1st gap, 2nd gap ... in the ascending order of the nodes. Let $d_i(t)$ be the difference of the i-th gap at time t, where t means every other time here. Then, we have the following recurrences.

$$d_i(t+1) = \frac{1}{4}d_{i-1}(t) + \frac{1}{2}d_i(t) + \frac{1}{4}d_{i+1}(t) \quad (1)$$

$$d_1(t+1) = \frac{1}{2}d_1(t) + \frac{1}{4}d_2(t) \quad (2)$$

$$d_h(t+1) = \frac{1}{2}d_h(t) + \frac{1}{4}d_{h-1}(t) \quad (3)$$

Let $S_{h-j}(t) = \sum_{i=j+1}^{h-j} d_i(t)$ and $S_h(0) = D$. Summing (1) from $i = 1$ to h by using (2) and (3) gives

$$S_h(t+1) = S_h(t) - \frac{1}{4}(d_1(t) + d_h(t)).$$

Since $d_1(t) + d_h(t) = S_h(t) - S_{h-1}(t)$, we have

$$S_h(t+1) = \frac{3}{4}S_h(t) + \frac{1}{4}S_{h-1}(t). \quad (4)$$

By using a generating function, $S_h(t)$ is given by

$$D(3/4)^t \sum_{k=0}^{h} (1/3)^k \binom{t}{k}$$
$$= D(3/4)^t \{(1+1/3)^t - (1/3)^{h+1}\binom{t}{h+1} - O(1/3^{h+2})\}$$
$$\leq D\left\{1 - \left(\frac{3}{4}\right)^t \left(\frac{1}{3}\right)^{n/2+1} \binom{t}{n/2+1}^{n/2+1}\right\}$$

because $h \approx n/2$. Hence, $S_h(t) = 0$ gives $(\frac{3}{4})^t(\frac{1}{3})^{n/2+1}\binom{t}{n/2+1}^{n/2+1} = 1$. Since it takes $2t$ rounds until convergence, the lemma follows. □

4 GAME THEORETIC PRICE DETERMINATION

In this section, we consider how to determine a bidding price in two ways by using Bertrand model (J. E. Stiglitz, 1993). Here, as customary, we use "Player" instead of "agent".

4.1 Payoff with Second Price

For simplicity, we consider a three-node path $(1,c)$ and $(c,2)$. Suppose that both Player 1 (i.e., agent a_1) at node 1 and Player 2 (i.e., agent a_2) at node 2 buy a commodity in the center node c at time $t+1$. Let q_1 and q_2 be the bidding prices of Player 1 and Player 2, respectively. Then, the payoff of Player 1 is $y = P_1(t) - q_1$. On the other hand, the payoff of Player 2 is $y = P_2(t) - q_2$ in the same situation.

As the price goes up, the payoffs of the players go down. In addition, Player 1 (resp. Player 2) regrets if the difference $q_1 - q_2$ (resp. $q_2 - q_1$) is large even if he gets the commodity. Next, if $q_1 = q_2$, both Player 1 and Player 2 get the commodity with probability 1/2. Thus, the payoff function of Player 1, $u_1(q_1,q_2)$, is defined as

$$u_1(q_1,q_2) = \begin{cases} (P_1 - q_1)/(q_1 - q_2) & q_1 > q_2 \\ (P_1 - q_1)/2 & q_1 = q_2 \\ 0 & q_1 < q_2 \end{cases}$$

The payoff function of Player 2, $u_2(q_1,q_2)$, is similarly defined.

Now we temporarily assume that q_1 and q_2 are continuous variables. Then,

$$\frac{\partial u_1}{\partial q_1} = (q_2 - P_1)/(q_1 - q_2)^2 = 0$$

That is, we obtain $q_2 = P_1$. Similarly,

$$\frac{\partial u_2}{\partial q_2} = (q_1 - P_2)/(q_2 - q_1)^2 = 0$$

That is, we obtain $q_1 = P_2$. Thus, $(q_1,q_2) = (P_2,P_1)$ is the Nash equilibrium. This is also true when q_1 and q_2 are integers.

4.2 Payoff without Second Price

In this section, we define the payoff function of Player 1 and that of Player 2 as follows.

$$u_1(q_1,q_2) = \begin{cases} P_1 - q_1 & q_1 > q_2 \\ (P_1 - q_1)/2 & q_1 = q_2 \\ 0 & q_1 < q_2 \end{cases}$$

$$u_2(q_1,q_2) = \begin{cases} P_2 - q_2 & q_1 < q_2 \\ (P_2 - q_2)/2 & q_1 = q_2 \\ 0 & q_1 > q_2 \end{cases}$$

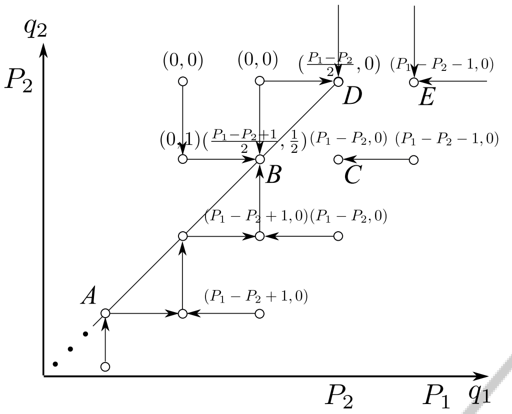

Figure 2: Payoffs without second price.

Without loss of generality, we assume $P_2(t) \leq P_1(t)$ and every price takes an integer value. By using the payoff functions, we have payoff values as shown in Figure 2.

Figure 2 illustrates a q_1q_2-space determined by the payoff functions. A pair of values in a bracket means (u_1,u_2) located at the intersection of the grid, corresponding to integer values of q_1 and q_2. For example, at D, a pair of bidding $(q_1,q_2) = (P_2,P_2)$ produces payoffs $(u_1,u_2) = (\frac{P_1-P_2}{2},0)$. The horizontal arrows indicate Player 1's move, while the vertical arrows indicate Player 2's move.

If $q_1 \neq q_2$ holds, the payoff of the smaller bid is 0. Thus, their offered prices must be equal like A. However, since Player 1 wants to increment its bid because his payoff would be twice. Then, Player 2 wants to raise his bid to the equal value to Player 1 because his payoff would be 0. In this way, Players 1 and 2 move from A to B. After the price has reached B, Players 1 and 2 have no incentive to move anymore if $(P_1-P_2+1)/2 \geq P_1-P_2$. The price moves to C if $(P_1-P_2+1)/2 < P_1-P_2$.
In summary, we obtain the following results.

(1) $P_1-P_2=0$**:** $(q_1,q_2) = (P_2-1,P_2-1),(P_2,P_2)$ are the Nash equilibria.

(2) $P_1-P_2=1$**:** $(q_1,q_2) = (P_2-1,P_2-1),(P_2,P_2-1),(P_2,P_2)$ are the Nash equilibria.

(3) $P_1-P_2=2$**:** $(q_1,q_2) = (P_2,P_2-1),(P_2,P_2)$, (P_2+1,P_2) are the Nash equilibria.

(4) $P_1-P_2 \geq 3$**:** $(q_1,q_2) = (P_2,P_2-1),(P_2+1,P_2)$ are the Nash equilibria.

5 RANDOMIZED PROTOCOL

Based on the consideration in Section 4, we propose a protocol, called **RandomBid**, using a price with the Bertrand model. Again, we focus on a star, the part of a network G, with a center node c.

RandomBid

- At time $t+1$, agent a_i offers a random integer price over the range $[P_c,P_i-1]$ to the neighboring node c with the minimum price $P_c(t)(< P_i(t))$.
- If agent a_c with maximal $P_c(t)$ does not accept any bid from N_c and its own offer is accepted by node i, the price $P_c(t)$ is decreased to the offered price.
- If several agents make bids to node c with the same highest price at time t, agent a_c contracts with one of them with equal probability.

The following lemma states that the agent with a maximal price will win the contract.

Lemma 3. *The agent a_i with a maximal price will win the contract with probability at least $\frac{P_i-P_j}{P_i-P_c}$, where agent a_j has the second maximal price.*

Proof. Since only agent a_i can submit a bid with range $[P_i-P_j]$, it can win the contract with agent a_c at the rate. Thus, the probability is at least $\frac{P_i-P_j}{P_i-P_c}$. □

The above lemma means that the agent a_i with a maximal price will win the contract with probability at least $1-(\frac{P_j-P_c}{P_i-P_c})^m$ after m rounds.

Theorem 3. *Our* **RandomBid** *will eventually stabilize the price with high probability.* □

6 CONCLUSIONS

In this paper we considered a new network model for the price stabilization. The model shows that the self-stabilization has a wide application to various areas. Our goal is to construct a good multiagent protocol which enables us to simulate a realistic social system. Then, we could analyze and estimate several economic phenomena.

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