Output Feedback Control for a Class of Nonlinear Delayed Systems

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Abstract. In this work, delays affecting either the output measurements or the input for a class of nonlinear systems are coped with. This problem is particulary challenging since time delays arise in variety of applications, such as systems communicating through (wireless) networks. Indeed when the controller is a remote one, delays must be taken into account, affecting both the input and the output of the system. We first present a set of cascade high gain observers for triangular nonlinear systems with delayed output measurement. A sufficient condition ensuring the exponential convergence of the observation error towards zero is given. This approach is then applied to design an output feedback control in the presence of input delay. These results are illustrated through numerical simulations.

1 Introduction

Systems communicating through wireless network are now quite common. Data transmissions such as output measurements or control laws are necessarily subject to delays inherent to the communication process. The aim of this paper is twofold. Delays affecting either the output measurements or the input for a class of nonlinear systems are coped with. This problem is particulary challenging since time delays affecting input or output measurements arise in a variety of applications. One can cite for example systems which are controlled by a remote controller. In these systems, the input or the output data are transmitted between the controller and the system throughout a communication system, which can be a wireless network. This network introduces a time-delay between the process and the controller. The design of controllers for such systems can be viewed as an output feedback design based on state prediction system. In the linear case, this problem has been solved by the well-known *Smith predictor* [1] and several predictive control algorithms [2], [3]. Recently, for the nonlinear case, a new kind of chained observers which reconstruct the state at different delayed time instants for *drift observable* systems has been presented in [4]. The authors showed, by using *Gronwall* lemma, that under some conditions on the delay, exponential convergence of the

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chained observers is ensured. These conditions have been relaxed in [5] by using an approach based on a first-order singular partial differential equation. On the other hand, in [6] a novel predictor for linear and nonlinear systems with time delay measurement has been designed. This predictor is a set of cascade observers. Sufficient conditions based on *linear matrix inequalities* are derived to guarantee the asymptotic convergence of this predictor. Concerning delays affecting the input of the system, very little attention has been paid to this subject. For relevant work, the reader is referred to [7] and the references therein.

In the present work, the design of nonlinear observers in the presence of delayed output measurement is first dealt with. To this purpose, we design a set of cascade high gain observers for nonlinear triangular systems by considering a time delay in the output measurement. We will show that the general high gain observer design framework developed in [8], [9], [10], to mention a few, for delay-free output measurements can be extended to systems with delayed output. More precisely, we propose to use a suitable *Lyapunov-Krasovskii functional* and a sufficient number of high gain observers, in order to guarantee the exponential convergence of the estimated state at time t towards the true state at time t, even if the output is affected by any constant and known delay. We will also give an explicit relation between the number of observers and the delay. Then in a second part, this observer is used to design a feedback controller based on a dual approach of high gain techniques [11].

The present paper is organized as follows : In section 2, we present the class of considered systems and the different assumptions. In the third one, we present the proposed observers and prove their convergence. Section 4 is devoted to the design of a feedback control law based on the previous observers. In the last section, we illustrate our results throughout simulations on academic examples.

2 Preliminaries and Notations

First some mathematical notations which will be used throughout the paper are introduced.
The euclidian norm on $\mathbb{R}^n$ will be denoted by $||.||$. The matrix X^T represents the transposed matrix of X. $e_s(i) = \underbrace{(0,\ldots,0,\overbrace{1}^{i^{th}},0,\ldots,0)}_{\text{s components}} \in \mathbb{R}^s, s \geq 1$ is the i^{th} vector of canonical basis of $\mathbb{R}^s$. The convex hull of $\{x,y\}$ is denoted as $\text{Co}(x,y) = \{\lambda x + (1-\lambda)y, 0 \leq \lambda \leq 1\}$. $\lambda_{min}(S)$ and $\lambda_{max}(S)$ are the minimum and maximum eigenvalues of the square matrix S.

In the first part of this paper, we consider the following class of nonlinear systems:

$$\begin{aligned} \dot{x} &= Ax + \phi(x,u) \\ y &= Cx(t-\tau) \end{aligned} \tag{1}$$

where

$$A = \begin{pmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \vdots \\ \vdots & 0 & \dots & 1 & 0 \\ \vdots & \vdots & \dots & \dots & 1 \\ 0 & \dots & \dots & \dots & 0 \end{pmatrix} \tag{2}$$

$$C = \begin{pmatrix} 1 & 0 & \dots & 0 \end{pmatrix} \tag{3}$$

$$\phi(x,u) = \begin{pmatrix} \phi_1(x,u) \\ \vdots \\ \phi_n(x,u) \end{pmatrix} \tag{4}$$

The term τ represents the measurement time delay, $x(t) \in \mathbb{R}^n$ is the vector state which is supposed unavailable. The output $y(t) \in \mathbb{R}$ is a linear function of the state x at time $t - \tau$. The input $u \in U$ where U is a compact set in $\mathbb{R}$. The functions $\phi_i, \quad i = 1, \dots, n$ are supposed smooth. This class represents the class of uniformly observable systems. It has been shown [8], [9] that these models concern a wide variety of systems, such as bioreactors…
Throughout the paper, we assume that the following hypotheses are satisfied.

$\mathcal{H}1$. The functions $\phi_i(x,u)$ are triangular in x, i.e $\frac{\partial \phi_i(x,u)}{\partial x_{k+1}} = 0$, for $k = i, \dots, n-1$
$\mathcal{H}2$. The functions $\phi_i(x,u)$ are globally Lipschitz, uniformly in u
$\mathcal{H}3$. The time delay τ is supposed constant and known.

3 Observer Design

In this section, we consider an arbitrary long time delay τ affecting the output measurement of system (1). The proposed nonlinear observer for system (1) is a set of m cascade high gain observers. Each one of them estimates a delayed state vector with sufficiently small delay $\frac{\tau}{m}$.
In order to present the proposed observer, we use the following convenient notations adopted from [4]:

$$x_j(t) = x(t - \tau + j\frac{\tau}{m})$$

where $j = 1, \dots, m$
Then the proposed observer can be written in the following form, for $j = 1, \dots, m$:

$$\begin{aligned} \dot{\hat{x}}_1 &= A\hat{x}_1 + \phi(\hat{x}_1) - \theta \Delta^{-1} S^{-1} C' C(\hat{x}_1(t - \frac{\tau}{m}) - x(t-\tau)) \\ \hat{y}_1 &= C\hat{x}_1(t - \frac{\tau}{m}) \\ \vdots &= \vdots \\ \dot{\hat{x}}_j &= A\hat{x}_j + \phi(\hat{x}_j) - \theta \Delta^{-1} S^{-1} C' C(\hat{x}_j(t - \frac{\tau}{m}) - \hat{x}_{j-1}(t)) \\ \hat{y}_j &= C\hat{x}_j(t - \frac{\tau}{m}) = C\hat{x}_{j-1}(t) \end{aligned} \tag{5}$$

where θ is a positive constant satisfying $\theta > 1$.

S is a symmetric positive definite matrix, solution of the following algebraic Lyapunov equation:

$$SA + A^T S - C^T C = -S \tag{6}$$

and Δ is a diagonal matrix which has the following form :

$$\Delta = Diag\left(1, \dots, \frac{1}{\theta^{i-1}}, \dots, \frac{1}{\theta^{n-1}}\right). \tag{7}$$

We will show that the vector $\hat{x}_j(t)$ estimates the delayed state $x_j(t)$, $j = 1, \dots, m-1$ and $\hat{x}_m(t)$ estimates $x(t)$.

Before proving the exponential convergence of the proposed chained observers, we consider the case when the delay τ is sufficiently small. Then only one high gain observer is required to estimate the state of system (1).

Lemma 1. *Consider the following observer:*

$$\begin{aligned} \dot{\hat{x}} &= A\hat{x} + \phi(\hat{x}, u) - \theta\Delta^{-1}S^{-1}C^T C(\hat{x}(t-\tau) - x(t-\tau)) \\ \hat{y} &= C\hat{x}(t-\tau) \end{aligned} \tag{8}$$

Then for sufficiently large positive θ, there exists a sufficiently small positive constant τ_1 such that $\forall \tau \leq \tau_1$, observer (8) *converges exponentially towards system* (1)*.*

Proof

First let us denote the observation error as $\tilde{x} = \hat{x} - x$.

Then we will have:

$$\dot{\tilde{x}} = A\tilde{x} + \phi(\hat{x}, u) - \phi(x, u) - \theta\Delta^{-1}S^{-1}C^T C\tilde{x}(t-\tau) \tag{9}$$

If we apply the relation

$$\tilde{x}(t) = \tilde{x}(t-\tau) + \int_{t-\tau}^{t} \dot{\tilde{x}}(s)ds \tag{10}$$

and the change of coordinates $\bar{x} = \Delta\tilde{x}$, system (9) can be rewritten in the following manner:

$$\dot{\bar{x}} = \theta(A - S^{-1}C^T C)\bar{x} + \Delta(\phi(\hat{x}, u) - \phi(x, u)) + \theta S^{-1}C' C \int_{t-\tau}^{t} \dot{\bar{x}}(s)ds. \tag{11}$$

In order to derive an upper bound τ_1 for the delay τ, to ensure the exponential convergence to zero of the error $\bar{x}$, we use the following *Lyapunov-Krasovskii functional* [12]:

$$W = \bar{x}^T S\bar{x} + \int_{t-\tau_1}^{t} \int_{s}^{t} ||\dot{\bar{x}}(\xi)||^2 d\xi ds. \tag{12}$$

This functional can be written after an integration by parts as follows (see [12] for more details):

$$W = \bar{x}^T S\bar{x} + \int_{t-\tau_1}^{t} (s - t + \tau_1)||\dot{\bar{x}}(s)||^2 ds \tag{13}$$

If we compute its time derivative, we obtain

$$\begin{aligned} \dot{W} \leq\; & \theta\bar{x}^T(A^T S + SA - 2C^T C)\bar{x} + 2\bar{x}^T S\Delta(\phi(\hat{x}) - \phi(x)) \\ & + 2\theta\bar{x}^T C^T C \int_{t-\tau}^{t} \dot{\bar{x}}(s)ds + \tau_1||\dot{\bar{x}}(t)||^2 - \int_{t-\tau_1}^{t} ||\dot{\bar{x}}(s)||^2 ds \end{aligned} \tag{14}$$

Using (6), we have

$$\dot{W} \leq -\theta\bar{x}^T S\bar{x} + 2\bar{x}^T S\Delta(\phi(\hat{x}) - \phi(x)) + -\theta\bar{x}^T C^T C\bar{x} + 2\theta\bar{x}^T C^T C\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds$$
$$+ \tau_1||\dot{\bar{x}}(t)||^2 - \int_{t-\tau_1}^{t}||\dot{\bar{x}}(s)||^2 ds. \tag{15}$$

Note that by using the mean value theorem [13], we can write

$$\Delta\left(\phi(\hat{x}) - \phi(x)\right) = \Delta\left(\sum_{i,j=1}^{n,n} e_n(i)^T e_n(j)\frac{\partial\phi_i}{\partial x_j}(\xi)\right)\Delta^{-1}\bar{x} \tag{16}$$

where $\xi \in \mathrm{Cov}(x, \hat{x})$.

Then we will have

$$2\bar{x}^T S\Delta(\phi(\hat{x}) - \phi(x)) = 2\bar{x}^T S\Delta(\sum_{i,j=1}^{n,n} e_n(i)^T e_n(j)\frac{\partial\phi_i}{\partial x_j})\Delta^{-1}\bar{x} \tag{17}$$

Using the triangular structure and the Lipschitz properties of the functions ϕ_i, and the fact that $\theta > 1$, we deduce that

$$||2\bar{x}^T S\Delta(\phi(\hat{x}) - \phi(x))|| \leq k_1 V \tag{18}$$

where $V = \bar{x}^T S\bar{x}$ and k_1 is a positive constant which does not depend on θ.
Using the following property :

$$2\theta\bar{x}^T C^T C\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds - \theta\bar{x}^T C^T C\bar{x}$$
$$= -\theta(Cx + C\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds)^T(Cx + C\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds)$$
$$+ \theta(\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds)^T C^T C(\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds). \tag{19}$$

This means that

$$2\theta\bar{x}^T C^T C\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds - \theta\bar{x}^T C^T C\bar{x} \leq \theta(\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds)^T C^T C(\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds) \tag{20}$$

From this, we will have

$$\dot{W} \leq -\theta V + k_1 V + \theta(\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds)^T C^T C(\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds)$$
$$+\tau_1||\dot{\bar{x}}(t)||^2 - \int_{t-\tau_1}^{t}||\dot{\bar{x}}(s)||^2 ds \tag{21}$$

Now, let us remark that if we use equation (11), it comes:

$$||\dot{\bar{x}}(t)||^2 \leq \theta^2 k_2[V + ||\int_{t-\tau}^{t}\dot{\bar{x}}(s)ds||^2] \tag{22}$$

where k_2 is also a positive constant which does not depend on θ.
Using this and equation (21), we will have:

$$\dot{W} \leq -\theta V + k_1 V + \theta I^T C^T C I + \tau_1 \theta^2 k_2 [V + ||I||^2] - \int_{t-\tau_1}^{t} ||\dot{x}(s)||^2 ds. \quad (23)$$

where $I = \displaystyle\int_{t-\tau}^{t} \dot{x}(s) ds$.
To prove the above lemma (1), it is sufficient to find conditions which guarantee the inequality $\dot{W} + \frac{1}{\sqrt{\theta}} W < 0$.
From (23), we can write

$$\dot{W} + \frac{1}{\sqrt{\theta}} W \leq -\theta V + k_1 V + \frac{V}{\sqrt{\theta}} + \theta I^T C^T C I + \tau_1 \theta^2 k_2 [V + ||I||^2] + \frac{\tau_1}{\sqrt{\theta}} \int_{t-\tau_1}^{t} ||\dot{x}(s)||^2 ds - \int_{t-\tau_1}^{t} ||\dot{x}(s)||^2 ds. \quad (24)$$

If we use the following *Jensen's* inequality :

$$\int_{t-\tau_1}^{t} ||\dot{x}(s)||^2 ds \geq \frac{1}{\tau_1} ||I||^2 \quad (25)$$

and if $\tau_1 \leq \sqrt{\theta}$, we have

$$\dot{W} + \frac{1}{\sqrt{\theta}} W \leq -(\theta - k_1 - \tau_1 \theta^2 k_2 - \frac{1}{\sqrt{\theta}}) V - (\frac{1}{\tau_1} - \theta - \tau_1 \theta^2 k_2 - \frac{1}{\sqrt{\theta}}) ||I||^2 \quad (26)$$

Then, we can say that lemma 1 is verified for

$$\begin{cases} \theta \geq \max\{2, (k_1 + k_2 + \frac{1}{\sqrt{2}})\} \\ \tau_1 = \frac{1}{\theta^2}. \end{cases} \quad (27)$$

To summarize Lemma 1, it gives the maximum delay supported by observer (8) which enables $\hat{x}(t) \to x(t)$, once θ has been fixed according to conditions (27). To cope with a larger measurement delay, we propose in next paragraph a procedure to estimate $x(t)$, based on a chain of high-gain observers: each observer will estimate the state at a given fraction of the output delay.

Cascade High Gain Observers. After proving that the convergence of the observer (8) requires a small delay, we will see that when the delay is arbitrary long, a set containing a sufficient number of cascade high gain observers (5) can reconstruct the states of system (1).

Theorem 1. *Let us consider system (1), then for any constant and known delay τ, there exist a sufficiently large positive constant θ and an integer m such that the observer (5) converges exponentially towards the system (1).*

Proof
The convergence of the cascade observer will be proved step by step :

Step 1: We consider the first observer in the chain:

$$\begin{aligned}\dot{\hat{x}}_1 &= A\hat{x}_1 + \phi(\hat{x}_1) - \theta\Delta^{-1}S^{-1}C^TC(\hat{x}_1(t-\frac{\tau}{m}) - x(t-\tau)) \\ \hat{y}_1 &= C\hat{x}_1(t-\frac{\tau}{m})\end{aligned} \tag{28}$$

We remark that $x(t-\tau) = x_1(t-\frac{\tau}{m})$ and consequently, if we choose θ sufficiently large, and by choosing the integer m such that $m \geq \theta^2\tau$, then $\hat{x}_1(t)$ converges towards $x_1(t) = x(t-\tau+\frac{\tau}{m}) = x(t-(m-1)\frac{\tau}{m})$.
Indeed, we are brought back to conditions of Lemma 1, since the delay to handle with is now $\frac{\tau}{m}$, which is assumed smaller than $\frac{1}{\theta^2}$.

Step j: at each step $(j = 2, \ldots, m)$, we estimate the delayed state $x(t-\tau+j\frac{\tau}{m})$ by using the following observer:

$$\begin{aligned}\dot{\hat{x}}_j &= A\hat{x}_j + \phi(\hat{x}_j) - \theta\Delta^{-1}S^{-1}C^TC(\hat{x}_j(t-\frac{\tau}{m}) - \hat{x}_{j-1}(t)) \\ \hat{y}_j &= C\hat{x}_j(t-\frac{\tau}{m}) = C\hat{x}_{j-1}(t)\end{aligned} \tag{29}$$

It is not difficult to see that by considering the observation error vector $\tilde{x}_j = x_j - \hat{x}_j$, if we add and subtract the term $\theta\Delta^{-1}S^{-1}C^TCx_{j-1}(t)$ in the previous equation, we obtain

$$\dot{\tilde{x}}_j = A\tilde{x}_j + \phi(\hat{x}_j) - \phi(x_j) - \theta\Delta^{-1}S^{-1}C^TC(\tilde{x}_j(t-\frac{\tau}{m}) - \tilde{x}_{j-1}(t)) \tag{30}$$

If we consider the following change of coordinates $\bar{x}_j = \Delta\tilde{x}_j$, we will have

$$\begin{aligned}\dot{\bar{x}}_j &= \theta(A - S^{-1}C^TC)\bar{x}_j + \Delta(\phi(\hat{x}_j) - \phi(x_j)) \\ &+ \theta S^{-1}C^TC\int_{t-\frac{\tau}{m}}^{t}\dot{\bar{x}}_j(s)ds - \theta S^{-1}C^TC\bar{x}_{j-1}.\end{aligned} \tag{31}$$

In order to prove by recurrence the convergence of the error $\bar{x}_j$, we suppose that the observation error $\bar{x}_{j-1}(t)$ converges exponentially towards zero.
Then we consider the following *Lyapunov-Krasovskii functional*

$$W_j = \bar{x}_j^TS\bar{x}_j + \int_{t-\frac{\tau}{m}}^{t}(s-t+\frac{\tau}{m})||\dot{\bar{x}}_j(s)||^2ds \tag{32}$$

Then its time derivative satisfies the following inequality:

$$\begin{aligned}\dot{W}_j \leq & -\theta\bar{x}_j^TS\bar{x}_j + 2\bar{x}_j^TS\Delta(\phi(\hat{x}_j) - \phi(x_j)) + -\theta\bar{x}_j^TC^TC\bar{x}_j - 2\theta\bar{x}_j^TC^TC\bar{x}_{j-1} \\ & +2\theta\bar{x}_j^TC^TC\int_{t-\tau}^{t}\dot{\bar{x}}_j(s)ds + \tau_1||\dot{\bar{x}}_j||^2 - \int_{t-\tau_1}^{t}||\dot{\bar{x}}_j(s)||^2ds.\end{aligned} \tag{33}$$

As in the proof of the lemma 1, we will also have:

$$\dot{W}_j \leq -(\theta - k_1')V_j + \theta I_j^TC^TCI_j - 2\theta\bar{x}_j^TC^TC\bar{x}_{j-1} + \tau_1||\dot{\bar{x}}_j||^2 - \int_{t-\tau_1}^{t}||\dot{\bar{x}}_j(s)||^2ds \tag{34}$$

where $V_j = \bar{x}_j^T S \bar{x}_j$, $I_j = \int_{t-\tau}^{t} \dot{\bar{x}}_j(s)ds$ and k'_1 is a positive constant which does not depend on θ and $k'_1 \geq k_1$.
Now, by using Young's inequality, we derive the following inequalities

$$||\dot{\bar{x}}_j||^2 \leq \tau_1 k'_2 \theta^2 (V_j + ||I_j||^2 + ||\bar{x}_{j-1}||^2) \tag{35}$$

$$-2\theta \bar{x}_j^T C^T C \bar{x}_{j-1} \leq \frac{1}{\sqrt{\theta}} V_j + \frac{\theta^2 \sqrt{\theta}}{\lambda_{min}(S)} ||\bar{x}_{j-1}||^2. \tag{36}$$

where k'_2 is a positive constant which does not depend on θ and $k'_2 \geq k_2$.
Choosing $\tau_1 = \frac{1}{\theta^2}$, and using (34), (35) and (36), we derive

$$\begin{aligned} \dot{W}_j + \frac{1}{\sqrt{\theta}} W_j \leq & -(\theta - k'_1 - \tau_1 \theta^2 k'_2 - \frac{2}{\sqrt{\theta}}) V_j - (\frac{1}{\tau_1} - \theta - \tau_1 \theta^2 k'_2 - \frac{2}{\sqrt{\theta}}) ||I_j||^2 \\ & + (\frac{\theta^2 \sqrt{\theta}}{\lambda_{min}(S)} + k'_2) ||\bar{x}_{j-1}||^2 \end{aligned} \tag{37}$$

Then, we can say that if

$$\left\{ \theta \geq 2 + k'_1 + k'_2, \quad \tau_1 = \frac{1}{\theta^2} \right. \tag{38}$$

we will have

$$\dot{W}_j \leq -\frac{1}{\sqrt{\theta}} W_j + (\frac{\theta^2 \sqrt{\theta}}{\lambda_{min}(S)} + k'_2) ||\bar{x}_{j-1}||^2 \tag{39}$$

Using the comparison lemma [14], we conclude that if $\bar{x}_{j-1}$ converges exponentially towards zero, then $\bar{x}_j$ converges also exponentially towards zero. Note that conditions (38), also ensure the convergence of the first observer $(j = 1)$, then we deduce, recursively, that all observation errors converge exponentially towards zero.

4 Output Feedback Controller Design

In previous section, we addressed an observer synthesis issue, when the output measurement is affected by any delay. Now we consider what can be thought of as a dual problem. The aim is to design a stabilizing feedback control law when the controller is a remote one, which inevitably leads to delays on the system input. To this end we consider the following class of nonlinear systems:

$$\begin{cases} \dot{x}(t) = Ax(t) + \phi(x(t)) + bu(t - \tau) \\ y(t) = Cx(t) \end{cases} \tag{40}$$

and we assume that hypotheses $\mathcal{H}_1$ to $\mathcal{H}_3$ are fulfilled.
We detail now a solution to the above problem that makes use of the previous results. In order to cope with the input delay, we use the above cascaded observer (5) to derive a prediction of state $x(t)$ used in the feedback control $u(t - \tau)$. For sake of simplicity, we suppose that we need only one observer to face this delay. The same reasoning as in previous section can be extended to deal with a larger delay, with cascaded observers.

Using the work developed in [11], the feedback control which stabilizes (40) can be expressed as:

$$u(t) = -\lambda^n b^T \bar{S} \Delta_\lambda \hat{x}(t+\tau) \tag{41}$$

where $\lambda > 0$ is a suitable tuning parameter like the parameter θ in the observer design and the matrix Δ_λ is defined as in (7) where θ is replaced by λ.

Using the results detailed in previous section, $\hat{x}(t+\tau)$ can be computed, see (5). Then this predicted state is used to compute eq. (41). As a consequence, this prediction cancels the effects of the delay affecting the transmission of the control law.

We give now a sketch of how to proceed. • Make the following variable change to obtain an estimation of the predicted state: $z(t) = \hat{x}(t+\tau)$. This is equivalent to $\hat{x}(t) = z(t-\tau)$.
Then the observer can be expressed as:

$$\dot{z}(t) = Az(t) + \phi(z(t)) + bu(t) - \theta \Delta_\theta^{-1} S^{-1} C^T \left(y(t) - z(t-\tau)\right) \tag{42}$$

where $u(t) = -\lambda^n b^T \bar{S} \Delta_\lambda z(t)$.
We are now brought back to the former problem of section 3. • The key point is to use the delayed control law $u(t-\tau)$ in the system dynamics, which corresponds to the real applied control, whereas we use $u(t)$ in the observer's dynamics.
•The reader is referred to [11] for a detailed proof of the stabilization of the system (40) and the convergence of the observer (42).

5 Example

To illustrate the obtained results, consider the following nonlinear system, affected first only by delayed measurements:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -2x_1(t) + 0.5\tanh(x_1(t) + x_2(t)) + x_1(t)u(t) \\ y(t) = x_1(t-\tau) \end{cases} \tag{43}$$

The input is $u(t) = 0.1\sin(0.1t)$. System (43) belongs to the considered class of triangular systems with Lipschitz nonlinearities (1).
The initial conditions for the system and for the observer have been chosen as $x(t) = \begin{pmatrix} 1 & -1 \end{pmatrix}^T, \quad \hat{x}(t) = \begin{pmatrix} 0 & 0 \end{pmatrix}^T, \quad \forall t \in [-\tau, 0]$.
Simulations have been performed using Matlab-Simulink, and a fourth order Runge-Kutta integration routine. The high gain parameter is set to $\theta = 2$, the control parameter is set to $\lambda = 2$. We show the efficiency of the stabilizing control law given in eq. (40) and (41), based on observer (42), on the example below:

$$\begin{cases} \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = -2x_1(t) + 0.5\tanh(x_1(t) + x_2(t)) + u(t-\tau) \\ y(t) = x_1(t) \end{cases} \tag{44}$$

The stabilization of the controlled state to zero is shown in figure 1.

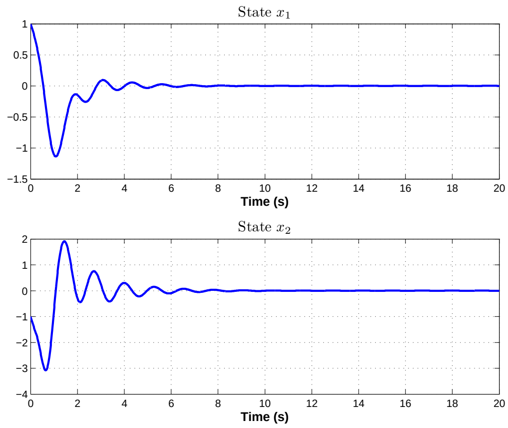

Fig. 1. Evolution of the controlled states.

6 Conclusions

In this paper, a novel predictor based on high gain observer has been presented. This observer can be applied to the class of nonlinear uniformly observable systems, subject to input or output delays arising from communication networks for example. The case of a variable delay can be considered on the basis of the presented work. The design of adaptive observers for nonlinear systems with delayed output and uncertain or unknown parameters is under investigation.

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