THE SIGNING OF A PROFESSIONAL ATHLETE

Reducing Uncertainty with a Weighted Mean Hemimetric for Φ – Fuzzy Subsets

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Abstract: In this paper we present a tool to help reduce the uncertainty presented in the decision-making process associated to the selection and hiring of a professional athlete. A weighted mean hemimetric for Φ – fuzzy subsets with trapezoidal fuzzy numbers (TrFN) as their elements, allows to compare candidates to the "ideal" player that the technical body of a team believes should be hired.

1 INTRODUCTION

Uncertainty is present in all the decision-making processes we face. In human resources selection, there is a great deal of uncertainty. Employers check references and apply a battery of tests to candidates, with the hope of making an appropriate choice to fill the vacancy.

In professional sports, the decision-making process associated with the hiring of an athlete involves facing a possible sporting and economic fiasco[1], due to strategic factors associated with the selected candidate and the magnitud of the contracts signed. A wrong decision could disrupt a championship and the future of a team. Therefore, a large number of variables from the different areas that can determine the success of an athlete, must be analyzed: technical, tactical, physical performance, medical, economic, psychological, social or other. To make things more complicated, selection criteria can be different between a coach that needs to fill a particular need, a general manager or executive who is interested on inmediate impact, or an owner who would like to sign someone who helps boost attendance(Young, 2008).

Win percentage, can be seen as a metric associated with performance of sports teams(Borghesi, 2008). In sports with a high production of statistics, the marginal production associated with the recruitment of a new team member can be easily studied. The research line that develops statistical methods for assessing the appropriateness of an athlete signing, has been deeply analized (Krautmann and Oppenheimer, 2002; Hendricks et al., 2003; Massey and Thaler, 2006). Nonetheless, it is difficult to allocate the precise fraction that a player contributes to the victory of a team[2].

In sports like football[3], there are very few variables stochastical in nature. Most of the characterization of a player is made through scouting reports, with assesments that are filled with subjective information.

The theory of fuzzy subsets created by (Zadeh, 1965), is the tool that allows us to mathematically model the uncertainty and seek solutions to the problems it presents. One of the issues explored is to determine the best among a group of candidates, when we are in the presence of uncertainty[4].

The evaluation of candidates in the presence of

[1] 14 english clubs entered administration and were effectively insolvent", as reported by (Sloane, 2006).

[2] Win Shares, an statistical method for baseball found in (James and Henzler, 2002), assigns 3 shares to each team win. Total win shares for the team are distributed to the team members by an analysis of their individual performance, their performance in the context of their home field and their performance relative to that of their league. In each league, every year is different, but the amount of games per season is constant over time. This allows to make comparisons between athletes who played at times when there was a preponderance of either the offensive or defensive. Also, comparisons between players of different field positions are possible.

[3] Soccer.

[4] As an example of this line of research, we can observe the work of (Chen and Wang, 2001) and its application to the search for the perfect home (Chen and Wang, 2007). The work developed by (Yang et al., 2005) and its application in databases is also very interesting. Even the International Olympic Committee has used a method based on the theory of fuzzy subsets for the selection of the venue of the 1st Summer Youth Olympic Games (IOC Panel of Experts, 2007).

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uncertainty is based on the experience of the evaluator, who has a key role in helping to reduce it. The use of fuzzy subsets in the human resources selection process, since the seminal works of (Gil-Aluja, 1987; Gil-Aluja, 1996), has been a profoundly studied line of research (Cannavacciuolo et al., 1994; Gil-Lafuente, 1999; Gil-Lafuente, 2000; Gil-Lafuente, 2001; Gil-Lafuente, 2005; Rojas-Mora and Gil-Lafuente, 2008).

We follow on this research with a weighted mean hemimetric for $\Phi -$ fuzzy sets, specially designed to evaluate the case when the assessment of a candidate's characteristic, modelled as a trapezoidal fuzzy number, reaches that of an "ideal" candidate, but doesn't exceed it.

The reasoning behind this condition is simple. In the process of selecting human resources, there are situations where we only need to know how much a candidate's characteristics needs to the reach the required level. By modeling with trapezoidal fuzzy numbers we find that the assessment given to the candidate may overlap in whole or in part the level set by the recruiter. Therefore, we will limit ourselves to measure the distance from the candidate's assessment to the "ideal" level, in areas where he has not reached it.

The rest of this paper is organized as follows; on section 2 a brief definition of fuzzy numbers is carried out. A hemimetric for trapezoidal fuzzy numbers and a weighted mean hemimetric for $\Phi -$ fuzzy sets is presented on section 3. An small example on how the numerical calculations are carried out is shown on section 4. Finally, section 5 comprises some conclusions.

2 FUZZY SUBSETS AND FUZZY NUMBERS

In situations of uncertainty, the theory of fuzzy subsets can model assessments that, on a particular topic, an expert gives out. These assessments take the form of fuzzy subsets.

Definition 2.1. A fuzzy subset $\tilde{A}$ is a set in which its elements may not follow the law of excluded middle that rules over boolean logic, i.e., their membership function can be mapped as:

$$\mu_{\tilde{A}} : X \to [0,1]. \tag{1}$$

In general, a fuzzy subset $\tilde{A}$ can be represented by a set of pairs consistents of the elements x of the universal set X and a grade of membership $\mu_{\tilde{A}}(x)$:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) \mid x \in X, \mu_{\tilde{A}}(x) \in [0,1]\}. \tag{2}$$

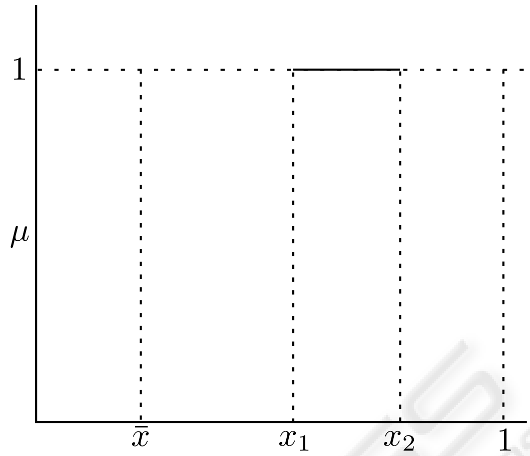

Figure 1: Fuzzy singleton and interval of confidence.

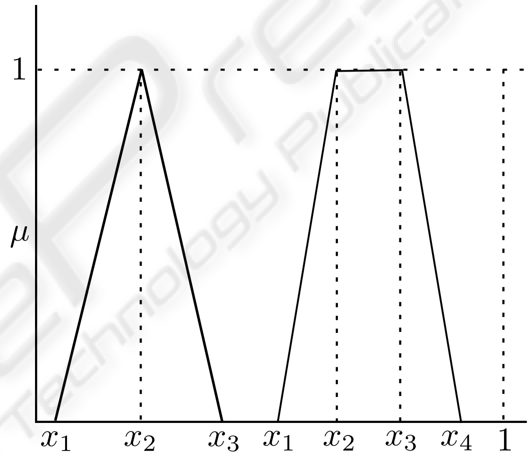

Figure 2: Triangular fuzzy number (TFN) and trapezoidal fuzzy number (TrFN).

Definition 2.2. A fuzzy number $\tilde{M}$ is a fuzzy subset for which:

1. $x \in \mathbb{R}$.
2. $\text{hgt}(\tilde{M}) = 1$, i.e., there is at least one element for which $\mu_{\tilde{M}}(x) = 1$.
3. $\lambda u + (1 - \lambda v) \in M_\alpha \forall u, v \in M_\alpha \wedge \alpha, \lambda \in [0,1]$, where M_α is the α-cut for $\tilde{M}$. This is, the convexity condition holds.

Remark. There is a fourth condition (Hanss, 2005) which states that in a fuzzy number $\tilde{M}$ there is exactly one $\bar{x} \in \mathbb{R}$ for which $\mu_{\tilde{M}}(\bar{x}) = 1$. However, as (Zimmermann, 2005, 57) explains, for computational simplicity there is a tendency, that we will follow, to avoid this condition, calling "fuzzy numbers" the fuzzy subsets that meet the first three conditions. Moreover, in fuzzy numbers used throughout this paper $x \in [0,1]$ (see figures 1 and 2).

In this work, assessments obtained from experts take the form of fuzzy numbers. Entropy of these assessments, namely the certainty the expert shows in his opinion, is observed in the area covered by fuzzy numbers. For the purposes of this study, we define four kinds of fuzzy numbers according to their entropy.

Definition 2.3. A fuzzy number whose support is a single point $\bar{x}$ is called a fuzzy singleton, and its membership function is:

$$\mu_{\tilde{M}}(x) = \begin{cases} 1 & \text{, if } x = \bar{x} \\ 0 & \text{, else.} \end{cases}$$

Definition 2.4. When there is not enough certainty to give an assessment as a singleton, but it can be given as an interval (x_1, x_2), we are defining an interval of confidence with membership function:

$$\mu_{\tilde{M}}(x) = \begin{cases} 1 & \text{, if } x_1 \leq x \leq x_2 \\ 0 & \text{, else.} \end{cases}$$

Definition 2.5. When there is a maximun of presumption (Kaufmann and Gupta, 1985, 1) in x_2, but the certainty linearly decreases to zero in x_1 and x_3, we are talking of a triangular fuzzy number (TFN) (x_1, x_2, x_3) with membership function:

$$\mu_{\tilde{M}}(x) = \begin{cases} 1 + \frac{x_2 - x}{x_2 - x_1} & \text{, if } x_1 \leq x \leq x_2 \\ 1 + \frac{x - x_2}{x_3 - x_2} & \text{, if } x_2 < x \leq x_3 \\ 0 & \text{, else.} \end{cases}$$

Definition 2.6. When the maximum of presumption covers the interval between x_2 and x_3, and then linearly decreases to zero in x_1 and x_4, we are defining a trapezoidal fuzzy number (TrFN) (x_1, x_2, x_3, x_4) with membership function:

$$\mu_{\tilde{M}}(x) = \begin{cases} 1 + \frac{x_2 - x}{x_2 - x_1} & \text{, if } x_1 \leq x < x_2 \\ 1 & \text{, if } x_2 \leq x \leq x_3 \\ 1 + \frac{x - x_3}{x_4 - x_3} & \text{, if } x_3 < x \leq x_4 \\ 0 & \text{, else.} \end{cases}$$

Remark. According to the definition of *LR*-type fuzzy numbers made by (Dubois and Prade, 1979; Dubois and Prade, 1988, 340) and explained by (Zimmermann, 2005, 64), any fuzzy number $\tilde{M}$ with $|\tilde{M}| \leq 4$ can be expressed as a TrFN, as shown on table 1.

Table 1: Fuzzy numbers equivalence.

Fuzzy Number	TrFN Equivalence
Fuzzy Singleton	$(\bar{x}, \bar{x}, \bar{x}, \bar{x})$
Interval of Confidence	(x_1, x_1, x_2, x_2)
TFN	(x_1, x_2, x_2, x_3)
TrFN	(x_1, x_2, x_3, x_4)

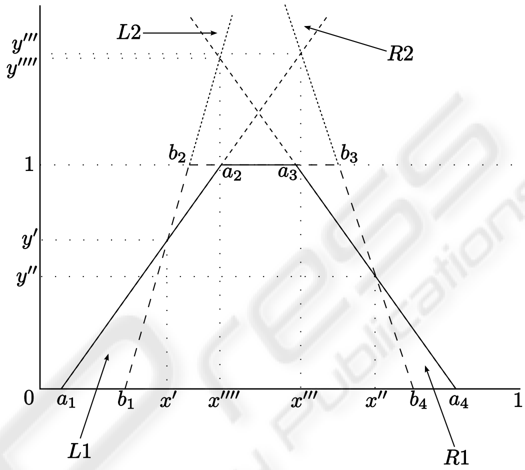

Figure 3: Distance between TrFN.

3 A WEIGHTED MEAN HEMIMETRIC FOR Φ – FUZZY SUBSETS

Let $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ be two fuzzy numbers in the sense of definition 2.6, and $D(\tilde{A}, \tilde{B})$ a distance function between them. For the purpose of this paper, this distance function implies a sort of "projection" of $\tilde{A}$ in $\tilde{B}$ in the four regions defined by L_1, L_2, R_1 and R_2 in figure 3.

Definition 3.1. For two TrFN $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$, there are four intersection points, $\{(x', y'), (x'', y''), (x''', y'''), (x'''', y'''')\}$ which can be found using the line equation:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1). \tag{3}$$

For the intersection between $\overline{a_1 a_2}$ and $\overline{b_1 b_2}$:

$$\begin{aligned} x' &= \frac{a_1 b_2 - b_1 a_2}{b_2 - a_2 - b_1 + a_1} \\ y' &= \frac{a_1 - b_1}{b_2 - a_2 - b_1 + a_1}, \end{aligned} \tag{4}$$

between $\overline{a_3 a_4}$ and $\overline{b_3 b_4}$:

$$\begin{aligned} x'' &= \frac{a_3 b_4 - b_3 a_4}{b_4 - a_4 - b_3 + a_3} \\ y'' &= \frac{b_4 - a_4}{b_4 - a_4 - b_3 + a_3}, \end{aligned} \tag{5}$$

between $\overline{a_1a_2}$ and $\overline{b_3b_4}$:

$$\begin{aligned} x''' &= \frac{a_2b_4-a_1b_3}{b_4-b_3+a_2-a_1} \\ y''' &= \frac{b_4-a_1}{b_4-b_3+a_2-a_1}, \end{aligned} \tag{6}$$

and between $\overline{a_3a_4}$ and $\overline{b_1b_2}$:

$$\begin{aligned} x'''' &= \frac{a_4b_2-a_3b_1}{b_2-b_1+a_4-a_3} \\ y'''' &= \frac{a_4-b_1}{b_2-b_1+a_4-a_3}. \end{aligned} \tag{7}$$

Definition 3.2. The mean quadratic distance (MQD) function for each region ζ of the set $Z = \{L_1, L_2, R_1, R_2\}$, is calculated by:

$$D_\zeta = \frac{\int_{\alpha_\zeta}^{\beta_\zeta} (b_\zeta - a_\zeta)^2 \, dy}{\beta_\zeta - \alpha_\zeta}, \tag{8}$$

where a_ζ is the equation of the line delimiting ζ on the left, expressed in terms of y, b_ζ is the equation of the line delimiting ζ on the right, expressed in terms of y, and $\{\alpha_\zeta, \beta_\zeta\} \in [0,1]$, $\alpha_\zeta \leq \beta_\zeta$, are the integration limits in y for ζ, by definition 3.1 and figure 3, except when $a \parallel b$, where $\alpha = 0$ and $\beta = 1$. For the purposes of this paper, we want to measure the average distance $\tilde{A}$ needs to be contained in $\tilde{B}$. For this reason, the area of $\tilde{A}$ that is already contained in $\tilde{B}$ will generate a MQD zero. As an example, the MQD from $\overline{y'a_2}$ to $\overline{y'b_2}$ in figure 3 is equal to zero.

The solution for L_1, R_1, R_2 and L_2 are equations (11), (12), (13) and (14), respectively.

Definition 3.3. The distance function $D(\tilde{A}, \tilde{B})$ between two TrFN $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ is:

$$D(\tilde{A}, \tilde{B}) = \sqrt{\frac{SD}{N}}, \tag{9}$$

where:

$$\begin{aligned} SD &= D_{L_1} + D_{L_2} + D_{R_1} + D_{R_2} \\ N &= 1(D_{L_1} > 0) + 1(D_{L_2} > 0) + 1(D_{R_1} > 0) + 1(D_{R_2} > 0). \end{aligned}$$

This distance function is a hemimetric because it satisfies the following conditions:

1. $D(\tilde{A}, \tilde{B}) \geq 0$
2. $D(\tilde{A}, \tilde{C}) \leq D(\tilde{A}, \tilde{B}) + D(\tilde{B}, \tilde{C})$
3. $D(\tilde{A}, \tilde{A}) = 0$.

This hemimetric fails to satisfy the indentity of indiscernibles, i.e., for any two TrFN $\tilde{A}, \tilde{B}$ with $\tilde{A} \subsetneq \tilde{B}$, $D(\tilde{A}, \tilde{B}) = 0$, even though $\tilde{A} \neq \tilde{B}$. Also, this hemimetric fails to satisfy the symmetry condition, i.e., for any two TrFN $\tilde{A}, \tilde{B}$ with $\tilde{A} \subsetneq \tilde{B}$, $D(\tilde{A}, \tilde{B}) \neq D(\tilde{B}, \tilde{A})$, because $\tilde{A} \subset \tilde{B}$, but $\tilde{B} \not\subset \tilde{A}$.

Table 2: Candidate players assessments and "ideal" player levels.

$\underset{\sim}{P}^1$	$\underset{\sim}{P}^2$	$\underset{\sim}{I}$
[0.5,0.6,0.6,0.8]	[0.4,0.4,0.5,0.5]	[0.7,0.7,1.0,1.0]
[0.4,0.4,0.7,0.7]	[0.8,0.8,0.8,0.8]	[0.8,0.9,1.0,1.0]
[0.7,0.8,0.9,1.0]	[0.5,0.7,0.9,1.0]	[0.6,0.7,0.8,0.9]

Definition 3.4. Let $\underset{\sim}{P} = \{\tilde{P}_1, \tilde{P}_2, \ldots, \tilde{P}_n\}$ and $\underset{\sim}{I} = \{\tilde{I}_1, \tilde{I}_2, \ldots, \tilde{I}_n\}$ be two Φ – fuzzy sets (Kaufmann and Gupta, 1985, 125), i.e., both are sets of fuzzy numbers, and $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ a vector of weights such that $\sum_{i=1}^{n} \omega_i = 1$ and $\omega_i \neq 0$. The weighted mean hemimetric (WMH) between $\underset{\sim}{P}$ and $\underset{\sim}{I}$ will be:

$$\delta\left(\underset{\sim}{P}, \underset{\sim}{I}\right) = \sum_{i=1}^{n} \omega_i D\left(\tilde{P}_i, \tilde{I}_i\right) \tag{10}$$

4 EXAMPLE

A football [5] team manager needs to find a new midfielder for his team. He would like to evaluate them in three different variables:

1. Vision: ability to, in advance, visualize the development of a play. The weight given to this variable is $\omega_1 = 0.5$.
2. Passing: ability to make a pass to the intended place and player. The weight given to this variable is $\omega_2 = 0.3$.
3. Transfer Fee: the ammount of money needed to bring this candidate to the team. The weight given to this variable is $\omega_1 = 0.2$.

He has two candidate players, P^1 and P^2, to choose from. From scouting reports and from his own experience, he has built, for each player, a Φ – fuzzy set of the assessments in each of the three variables. He has also constructed a model for the "ideal player" I to compare with the candidates (see table 2).

We will calculate the WMH from P^1 to I as an example:

$$\begin{aligned} D(\tilde{P}_1^1, \tilde{I}_1) &= \sqrt{\frac{0.02333 + 0 + 0 + 0.00333}{1 + 0 + 0 + 1}} \\ &= 0.11547 \end{aligned}$$

$$\begin{aligned} D(\tilde{P}_2^1, \tilde{I}_2) &= \sqrt{\frac{0.20333 + 0 + 0 + 0.02333}{1 + 0 + 0 + 1}} \\ &= 0.33665 \end{aligned}$$

[5] Soccer.

$$D_{L_1} = \begin{cases} \frac{b_2{}^2+b_1b_2-2a_2b_2-a_1b_2+b_1{}^2-a_2b_1-2a_1b_1+a_2{}^2+a_1a_2+a_1{}^2}{3} & \text{, if } a_1 \leq b_1 \wedge a_2 \leq b_2 \\ \frac{(b_1-a_1)^2}{3} & \text{, if } a_1 < b_1 \wedge a_2 > b_2 \\ \frac{(b_2-a_2)^2}{3} & \text{, if } a_1 > b_1 \wedge a_2 < b_2 \\ 0 & \text{, if } a_1 > b_1 \wedge a_2 > b_2. \end{cases} \quad (11)$$

$$D_{R_1} = \begin{cases} \frac{b_4{}^2+b_3b_4-2a_4b_4-a_3b_4+b_3{}^2-a_4b_3-2a_3b_3+a_4{}^2+a_3a_4+a_3{}^2}{3} & \text{, if } b_3 \leq a_3 \wedge b_4 \leq a_4 \\ \frac{(b_4-a_4)^2}{3} & \text{, if } b_3 > a_3 \wedge b_4 < a_4 \\ \frac{(b_3-a_3)^2}{3} & \text{, if } b_3 < a_3 \wedge b_4 > a_4 \\ 0 & \text{, if } b_3 > a_3 \wedge b_4 > a_4. \end{cases} \quad (12)$$

$$D_{R_2} = \begin{cases} \frac{b_4{}^2+b_3b_4-a_2b_4-2a_1b_4+b_3{}^2-2a_2b_3-a_1b_3+a_2{}^2+a_1a_2+a_1{}^2}{3} & \text{, if } a_1 \geq b_4 \\ \frac{(b_3-a_2)^2}{3} & \text{, if } a_1 < b_4 \wedge a_2 > b_3 \\ 0 & \text{, if } a_2 \leq b_3. \end{cases} \quad (13)$$

$$D_{L_2} = \begin{cases} \frac{b_2{}^2+b_1b_2-a_4b_2-2a_3b_2+b_1{}^2-2a_4b_1-a_3b_1+a_4{}^2+a_3a_4+a_3{}^2}{3} & \text{, if } a_4 \leq b_1 \\ \frac{(b_2-a_3)^2}{3} & \text{, if } a_3 < b_2 \wedge a_4 > b_1 \\ 0 & \text{, if } a_3 \geq b_2. \end{cases} \quad (14)$$

$$D(\tilde{P}^1_3, \tilde{I}_3) = \sqrt{\frac{0+0+0.01+0}{0+0+1+0}} = 0.1$$

$$\delta(\underset{\sim}{P}^1, \underset{\sim}{I}) = 0.5 \cdot 0.11547 + 0.3 \cdot 0.33665 + 0.2 \cdot 0.1 = 0.1783$$

The WMH from P^2 to I is:

$$\delta(\underset{\sim}{P}^2, \underset{\sim}{I}) = 0.16113.$$

As $\delta(\underset{\sim}{P}^1, \underset{\sim}{I}) > \delta(\underset{\sim}{P}^2, \underset{\sim}{I})$, the manager will prefer to sign the second candidate over the first one to fill the midfielder position.

5 CONCLUSIONS

We have presented a hemimetric for TrFN and a weighted mean hemimetric for $\Phi -$ fuzzy sets, both useful for human resources comparison in order to fill an available position.

The hemimetric for TrFN was designed to take into account the case where part or all of the first TrFN area is contained in the second. In human resources selection, once a particular feature of the candidate is within the requirements to fill the position, there is no need to calculate a distance. Therefore, the only thing we need to know is how far the candidate is to the level required in a particular feature.

By using $\Phi -$ fuzzy sets, we can extend this hemimetric to a large set of variables, in many different areas, each with a particular importance in the decision-making process. The flexibility given to the team manager allows him to overcome the natural uncertainty in his work and gives an a strong base to his decision.

The selection of human resources in professional sport is an activity that should balance the economic cost with the technical capability of the performers. A failure in either area can lead to a fiasco. The recovery of such problems is, by general rule, a task that takes a long time, except for teams with the greatest economic power. The shielding of a signing by such techniques as the presented is absolutely necessary given the present competitiveness of the sports world.

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