INTERPOLATORY ADAPTIVE SUBDIVISION FOR MESH LOD EDITING

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Abstract: We propose an adaptive interpolatory scheme for subdivided triangle meshes that is compliant with the modified butterfly subdivision and can be used effectively and efficiently in selective editing of meshes. Our scheme is developed upon the RGB subdivision, an adaptive scheme that is based on the factorization of the one-to-four triangle split pattern. We introduce the concept of *topological angle* and related operators to efficiently navigate and edit an adaptively subdivided mesh. On the basis of this new scheme, we present an interactive application that allows a user to freely edit the Level of Detail of a model starting at a base mesh.

1 INTRODUCTION

Subdivision surfaces are becoming more and more popular in computer graphics and CAD, with primary applications in the entertainment industry (Zorin and Schröder, 2000). The Loop scheme is very popular for subdividing triangular meshes for its C^2 smoothness. However, it may be not suitable to some modeling contexts, such as videogames, because it displaces vertices by introducing a sort of "shrinking" and "oversmoothing" on the surface. This may give rise to unpleasant warping effects that the designer cannot control. Details can be better preserved with an interpolatory scheme, such as the modified butterfly scheme (Zorin et al., 1996).

Most often subdivision is applied up to a certain level and, in many cases, different parts of the mesh should be refined at different levels of detail. Whenever a model is constrained to a certain budget of polygons, higher LOD should be used in the proximity of joints and in detailed areas. LOD editing is a customary task in Continuous Level Of Detail (CLOD) for free-form mesh modeling (Lübke et al., 2002), but its extension to subdivided meshes is not straightforward. The *RGB Subdivision* was introduced in (Puppo and Panozzo, 2008) to support such a task. It is an adaptive scheme for triangle meshes, which is based on the iterative application of local refinement and coarsening operators and it is compliant with the Loop subdivision.

Here, we extend such a scheme to interpolatory subdivision and we explicitly address the interactive editing of LOD. We propose a new set of operators to navigate a mesh, which are based on the notion of *topological angle* on an adaptively subdivided mesh. On this basis, we develop new efficient algorithms for stencil computation. We also present an interactive application, developed upon our new scheme, that allows a user to dynamically adjust LOD through brush tools.

2 RELATED WORK

In this section, we will review only those works related to adaptive subdivision of triangle meshes. The interested reader can refer to (Warren and Weimer, 2002) for a general textbook on subdivision surfaces. *Red-green triangulations* (Bank et al., 1983) are popular in the common practice to obtain conforming adaptive meshes from hierarchies generated from one-to-four triangle split. Variants of red-green triangulations were proposed in (Pakdel and Samavati, 2007; Zorin et al., 1997) in a modeling context to comply with either the Loop, or the butterfly subdivision. Apparently, such mechanisms were not designed to support interactive editing, but rather for "one shot" operations. Some other proposals exist to factorize the one-to-four triangle refinement scheme into atomic local operations, with the aim to support editing of adaptive subdivisions (Seeger et al., 2001; Velho, 2003). The $\sqrt{3}$ subdivision (Kobbelt, 2000)

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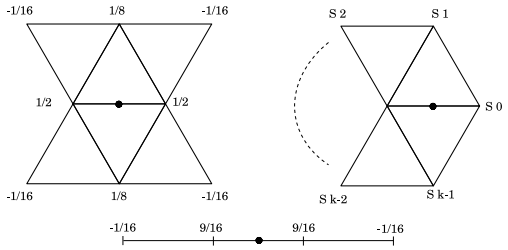


Figure 1: Stencils for the modified butterfly subdivision scheme: stencil for splitting an internal edge that has both endpoints at regular vertices (top-left); stencil for splitting an internal edge incident at an extraordinary vertex (top-right); stencil for splitting a boundary edge (bottom). The coefficients s_i are: for $k > 5$, $\frac{1}{k}\left(\frac{1}{4} + \cos\frac{2i\pi}{k} + \frac{1}{2}\cos\frac{4i\pi}{k}\right)$; for $k = 3$, $s_0 = \frac{5}{12}$, $s_{1,2} = -\frac{1}{12}$; for $k = 4$, $s_0 = \frac{3}{8}$, $s_2 = -\frac{1}{8}$, $s_{1,3} = 0$.

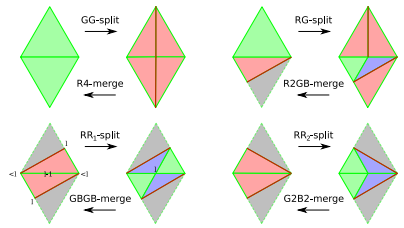


Figure 2: Edge split and edge merge operators. Labels denote the level of vertices and edges.

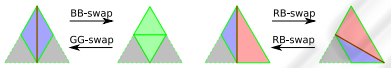


Figure 3: Edge swap operators.

and the 4-8 subdivision (Velho and Zorin, 2001) schemes are based on alternative subdivision patterns. They are naturally adaptive, being both based on local conforming operators, and they both address the correct relocation of vertices at the cost of some over-refinement.

3 BACKGROUND

3.1 Triangle Meshes

A *triangle mesh* is a triple $\Sigma = (V, E, T)$ where: V is a set of points in 3D space, called *vertices*; T is a set of triangles having their vertices in V and such that any two triangles of T either are disjoint, or share exactly either one vertex or one edge; E is the set of edges of the triangles in T. Standard topological incidence and adjacency relations are defined over the entities of Σ. We will assume to deal always with *manifold* meshes either with or without boundary.

3.2 Modified Butterfly Subdivision

The modified butterfly subdivision scheme, proposed in (Zorin et al., 1996), is based on the one-to-four triangle split pattern and it is an interpolatory scheme converging to a C^1 limit surface. The position of a new vertex inserted by subdivision is computed as a weighted average of the positions of vertices in a stencil in the neighborhood of the split edge. The stencils for the standard cases, together with the weights used to compute the average, are shown in Figure 1. For the sake of brevity, we do not report here additional stencils for edges in the proximity of boundary or crease edges. The algorithms that we provide in Section 5 are sufficient to fetch vertices of such stencils as well.

4 RGB TRIANGULATIONS

RGB triangulations have been defined in (Puppo and Panozzo, 2008) and provide a progressive mechanism for the refinement of a mesh through operators for local modification. The essential idea is to provide operators that modify a mesh by either introducing or deleting one vertex at a time and that can be used to factorize the one-to-four triangle split pattern and its reverse. Given a base mesh, we assign level zero to all its vertices, edges and triangles, and color green to all its edges and triangles. The color and level of triangles and edges in a subdivided mesh is defined inductively by the application of the eleven local operators depicted in Figures 2 and 3.

Edge split operators take an edge e at level l and split it by inserting a new vertex, at level $l+1$, at the midpoint of e. This induces the simultaneous bisection of triangles t_0 and t_1 incident at e, which may come in the following variants (see Figure 2):

- **GG-split:** t_0 and t_1 are both green. The bisection of each triangle t_0 and t_1 at the midpoint of e generates two red triangles at level l. Each such triangle will have: a green edge at level l, a green edge at level $l+1$ and a red edge at level l.
- **RG-split:** t_0 is green and t_1 is red. Triangle t_0 is bisected as above. Bisection of t_1 generates a blue triangle at level l and a green triangle at level $l+1$.
- **RR-split:** t_0 and t_1 are both red. Triangles t_0 and t_1 are both bisected as t_1 in the previous case. There are two variants: **RR$_1$-split** and **RR$_2$-split**,

which can be recognized by the cycle of colors on the boundary of the diamond formed by t_0 and t_1.

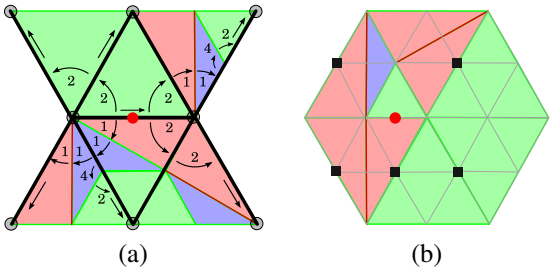
Edge merge operators reverse edge split and can be applied to triangles incident at vertices of valence four. The same cases depicted in Figure 2 occur (modifications apply right-to-left in this case): **R4-merge** inverts GG-split; **R2GB-merge** inverts RG-split; **GBGB-merge** inverts RR_1-split; **G2B2-merge** inverts RR_2-split. Edge swap operators take a quadrilateral formed by a pair of adjacent triangles and swap its diagonal. They are defined as follows (see Figure 3):

- **BB-swap** takes a pair of blue triangles at level l, which are adjacent along a red edge at level l, and produces a pair of green triangles at level $l+1$;
- **GG-swap,** which inverts BB-swap, takes a pair of adjacent green triangles t_0 and t_1 at level $l > 0$ if one of them has all three vertices at level l, and produces a pair of blue triangles adjacent along a red edge;
- **RB-swap** takes a pair formed by a red and a blue triangle at the same level l of subdivision, which are adjacent along a red edge, and produces another red-blue pair of triangles at level l.

An edge e at level $l \geq 0$ can split if and only if it is green and its two adjacent triangles t_0 and t_1 are both at level l. Red edges can just swap. Combinations of merge and swap operators can remove a vertex v at level $l > 0$, without removing other vertices, if and only if all vertices adjacent to v are at level $\leq l$. With the above operators at hand, a mesh can be refined and coarsened progressively.

5 RGB SUBDIVISION WITH THE MODIFIED BUTTERFLY

Given the above framework for refining and coarsening meshes, it follows that any RGB triangulation will contain just vertices that also appear in the (virtual and infinite) family of meshes generated through recursive one-to-four triangle split. In order to obtain an adaptive subdivision scheme compliant with the modified butterfly, we must guarantee that the position in 3D space assigned to each vertex is the same in the RGB subdivision and in a standard modified butterfly subdivision. In order to do this, it is sufficient to guarantee that, during a split operation, we always retrieve the vertices in the standard stencil and their positions. Since a RGB mesh is adaptive, given an edge e at level l to split, two opposite situations may occur:



(a) (b)

Figure 4: (a) Some triangles in the stencil have been refined beyond the level of the splitting edge. (b) Some triangles in the stencil are at a level lower than that of the splitting edge (marked with a red bullet).

- Some triangles of the stencil have been refined at levels $> l$. In the area spanned by the stencil, such refinement can be of arbitrarily many levels, and it is necessary to navigate the mesh, starting at e, to fetch the vertices of the stencil (see Figure 4a).
- Some triangles of the stencil are at a level $< l$. This means that some vertices in the stencil may not belong to the current mesh (see Figure 4b). In this case, we perform all the necessary splits to introduce the required vertices, and we mark such additional vertices as "overrefined". When the entire refining process is completed (not just a single split, but rather a batch of subdivision operations) a cleanup is performed, in which we apply coarsening operations to remove all vertices that result overrefined with respect to current LOD.

In the following subsections, we develop the necessary techniques and algorithms to identify the stencils.

5.1 Topological Angle Definition

We assign a *topological width* to the angles of every triangle in an RGB triangulation using the following rules (see Figure 5):

1. *Green Triangle*: each angle has a topological width of 2;
2. *Red Triangle*: the angle opposite to the red edge has topological width of 2; the angle opposite to the edge at the highest level has topological width of 1; and the remaining angle has a topological width of 3;
3. *Blue Triangle*: The angle opposite to the red edge has topological width of 4; the other two angles have topological width of 1.

An angle with topological width of 6 is said to be *flat*. Such values are not related to geometrical values, except when all green triangles are equilateral: only in

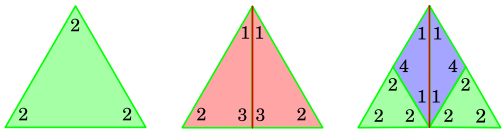


Figure 5: Topological angles: the angle value is assigned to each vertex in each triangle.

that case, a topological width of 1 corresponds to 30 degrees. We do not need to store angle widths in the data structure since they can be extracted using colors of triangles and edges, and the level of vertices.

We now prove some invariants on angles that will be useful for mesh navigation.

Lemma 5.1. *If an edge e is split into two edges e_0 and e_1 by adding a vertex v, both angles formed by e_0 and e_1 are flat.*

Proof. The only possible ways to split an edge are depicted in Figure 2. By comparing the triangles in such figures with the definitions of angles given above, and depicted in Figure 5, it is readily seen that in all cases the sum of angles on each side of the pair e_0e_1 is 6. □

Lemma 5.2. *The width of a topological angle between a pair of edges is invariant upon editing operations on the mesh.*

Proof. Consider a pair of edges e and e' incident at v and one of the two angles they form at v. It is sufficient to analyze editing operations that affect triangles spanned by such an angle. For each such triangle t, there are three possible cases, which are readily verified by comparing Figures 2 and 5: If the editing operation neither splits t with an edge incident at v, nor merges t with an adjacent triangle around v, then the angle of t at v is unchanged; If the angle of t at v is split into two angles, then sum of widths of such angles is equal to the width of the angle of t at v before split (this occurs in split and swap operations); If t is merged with another triangle t' adjacent to it around v, by deleting their common edge, then either e and e' are merged into a single edge (this occurs in merge operations only), or the width of angle at v of the new triangle is equal to the sum of widths of angles of t and t' at v (this occurs in merge and swap operations). □

Lemma 5.3. *No matter how an edge e is subdivided into a chain of edges $e_0,\ldots,e_k$, angles between two consecutive edges e_{i-1} and e_i, $i=1,\ldots,k$ are flat.*

Proof. The proof follows from the above two lemmas by noting that every split produces flat angles and such angles are invariant upon subsequent editing operations. □

5.2 Encoding and Navigating a RGB Triangulation

We now provide a set of primitive operations that allow us to move in a RGB triangulation, which will be used to fetch the vertices of stencils. We define switch operators similar to those proposed in (Brisson, 1993), plus two new operators, called **rotate** and **move**, that are specific for the RGB triangulation.

All the operators use a unique identifier, that we call *pos*, of position in the triangulation. The identifier contains a vertex v, an edge e incident at v, and a face f bounded by e. Given a *pos* **p**, we will denote by **p.v**, **p.e** and **p.f** its related vertex, edge and face, respectively.

1. **p.switchVertex()**: moves to a *pos* having the same edge and face of **p**, and the other vertex of **p.e** with respect to **p.v**.
2. **p.switchEdge()**: moves to a *pos* having the same vertex and face of **p**, and the other edge incident at both **p.v** and **p.f** with respect to **p.e**.
3. **p.switchFace()**: moves to a *pos* having the same vertex and edge of **p**, and the other face incident at **p.e** with respect to **p.f**.
4. **p.rotate(i)**: executes an alternate sequence of **p.switchEdge()** and **p.switchFace()** operators until a topological angle of width i has been scanned.
5. **p.move(l)**: executes an alternate sequence of **p.switchVertex()**, **p.rotate(6)** and **p.switchFace()** operators until a vertex with level $\leq l$ is reached. This primitive stops as soon as the vertex is reached.

The invariance lemmas proved in the previous section guarantee that, starting at a splitting edge **p.e** at level l, we can navigate the mesh by moving to adjacent triangles of the stencil at level l (through a **p.rotate(2)** operation) and we can follow chains of edges until we reach the other end of an edge at level l (through a **p.move(l)** operation).

A standard topological data structure for triangle meshes (representing at least vertices and triangles) is sufficient to support the operations described above. Note that the management of RGB triangulations does not need to store any hierarchy. It is just sufficient to add the following fields to the standard data structure: for each vertex: its level of insertion (one byte); for each edge (if represented in the data structure): its color and level (one byte); for each triangle: its color and level (one byte). Therefore, the overhead with

respect to a standard topological data structure is negligible.

5.3 Algorithms to Identify the Stencils

Using the previous primitives, it is easy to fetch all vertices of a stencil. The algorithm takes as input a *pos* **p**, where **p.e** identifies the edge to split. In the regular case, two vertices of the standard butterfly stencil are immediately available (i.e., the endpoints of the splitting edge). The remaining vertices of stencil are fetched navigating the mesh through the algorithm described in the following.

Algorithm 1: **fetchRegularStencil**(**Pos.** pos)).

```
 1: list<Vertex> stencil;
 2: Pos pos2 = pos;
 3: int maxlevel = maxVertexLevel(pos);
 4: pos2.switchVertex();
 5: stencil.add(pos.v);
 6: stencil.add(pos2.v);
 7: splitEdgesIfNeeded(pos.v,maxlevel,STD);
 8: splitEdgesIfNeeded(pos2.v,maxlevel,STD);
 9: pos2 = pos;
10: pos2.switchFace();
11: fetchHalf(pos,stencil);
12: fetchHalf(pos2,stencil);
13: return stencil;
```

Algorithm 2: **fetchHalf**(**Pos.** pos, **list**<**Vertex**> stcl).

```
 1: Pos p;
 2: p = pos;
 3: p.rotate(2);
 4: p.move(maxlevel);
 5: stcl.add(p.v);
 6: p = pos;
 7: p.rotate(4);
 8: p.move(maxlevel)
 9: stcl.add(p.v);
10: p = pos;
11: p.switchVertex();
12: p.rotate(4);
13: p.move(maxlevel)
14: stcl.add(p.v);
```

Function maxVertexLevel(p) returns the highest level of the two vertices incident at **p.e**. Function splitEdgesIfNeeded(v,l,type) checks if the neighbor vertices of the vertex v at level l are present. If not, it performs recursive split operations to add them. The function looks at the incident edges and splits every edge of level $< l$. In case an edge is red, since it cannot split directly, it is necessary to split the lowest green edge of every red triangle incident at it, and apply a BB-swap next. vertices checked depend on the value of parameter *type,* which is related to the type of stencil analyzed: in case this value is STD, only the four neighbors found by pivoting around v with rotations of width 2 and 4, respectively, are checked; in case the value is BOUND, just the neighbor along the boundary is checked; in case the value is EXTRA, all neighbors of v are checked. Function fetchHalf finds the upper/lower vertices in the stencil. The identification of the stencil in the extraordinary case is similar. The extensions of the previous algorithms to boundary cases is straightforward. The complexity of fetching a stencil is bounded from above by the length of chains of edges, i.e., by the maximum number of times an edge at level l adjacent to the splitting edge has been subdivided. In the worst case, this number can be linear in the size of the mesh, but this occurs only in pathological situations where the mesh is highly refined only around a vertex and it abruptly degrades to the base level elsewhere. In practical cases, chains are usually quite short. So, these algorithms can be considered to run in constant time on average.

6 INTERACTIVE EDITING OF LOD

On the basis of our RGB subdivision, we have developed an interactive application that allows a user to start with a base mesh and edit its LOD by using two tool brushes to increase and decrease detail locally, according to her/his needs. Our application is a prototype implemented as a plugin for *MeshLab*, an open source tool for processing, editing and visualizing 3D triangular meshes (Meshlab, 2008). A beta version of the software can be currently downloaded from `http://ggg.disi.unige.it/rgbtri/`.

We have tested our tool on a number of models representing various objects. Most objects were described with base meshes in the order of 10^2-10^4 faces, which have been selectively refined up to sizes of order 10^6. In Figure 6 we present some results and we show some comparisons between the butterfly and the Loop RGB subdivisions. The mesh refined with the Loop subdivision results smoother than the mesh refined with the butterfly subdivision, but the shape is severely warped near the eyes, nose and chin.

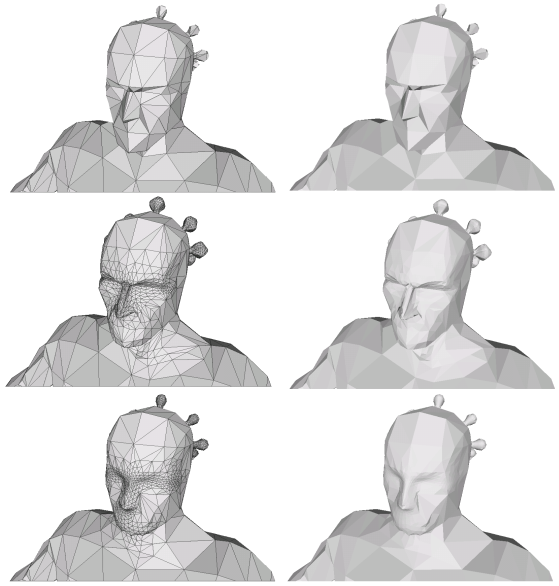


Figure 6: The base mesh (top); the same mesh refined on the eyes, nose and hair tufts at level 3, using the the butterfly scheme (middle); the same mesh refined in the same way with the Loop scheme (bottom).

7 CONCLUSIONS

The RGB subdivision scheme has several advantages over both classical and adaptive subdivision schemes, as well as over CLOD models: it supports fully dynamic selective refinement while remaining compliant with standard schemes; it is better adaptive than previously known schemes based on the one-to-four triangle split pattern; it does not require hierarchical data structures; mesh editing can be implemented efficiently by plugging faces inside the mesh, according to rules encoded in lookup tables, thus avoiding cumbersome procedural updates.

A similar approach can be undertaken also to develop hybrid tri-quad adaptive meshes for the selective refinement of quad meshes. These extensions are the subject of our current and future work. We believe that this approach to adaptive subdivision may give valid substitutes or complements to standard subdivision for solid modelers and simulation systems. Combined with reverse subdivision techniques, it may also offer a valid alternative to CLOD models for free-form objects in computer graphics.

A crucial feature to support modeling is the ability to edit the position of vertices of the control mesh and propagate this consistently on the subdivided mesh. This should be easy on the butterfly RGB subdivision: when a vertex of the control mesh is moved, its effects are propagated through the network of edges to vertices having that vertex in their mask. The navigation primitives that we have defined in Section 5.2, can be used effectively to this purpose. In the future, we plan to develop these features and to integrate our scheme in the Blender (Blender, 2008) solid modeler, which offers an open source platform that can be extended by external plugins.

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