Descriptive Analysis of Image Data: Basic Models

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Abstract. The paper is devoted to the foundations, general methodology, the axiomatic and formal structures of the Descriptive Theory for Image Analysis (DTIA) providing a methodology, mathematical and computational techniques for automation of image analysis and estimation (IAE). The main purpose of theoretical apparatus of the DTIA is structuring of the variety of methods, operations and representations being used in IEA. The final goal of the DTIA is automated image mining: a) automated selection of techniques and algorithms for image recognition, estimation, and understanding; b) automated testing of the raw data quality and its suitability for solving the image recognition problem. The DTIA provides mathematical fundamentals for image mining. The axiomatics and formal structures of Descriptive Theory of Image Analysis provide the ways and means to represent and to describe images for its analysis and estimating. The main contributions of axiomatics are Descriptive Image Models: its definitions, classification, properties, interrelations, and conditions of generation.

1 Introduction

Automation of image processing, analysis, estimating and understanding is one of the crucial points of theoretical computer science having decisive importance for applications, in particular, for diversification of solvable problem types and for increasing the efficiency of its solving.

The role of an image as an analysis and estimation object is determined by its specific and inalienable informational properties. Image is a mixture and a combination of initial (raw, “real”) data and its representation means, of computational and physical nature and models of objects, events and processes to be represented via an image.

The specificity, complexity and difficulties of image analysis and estimation (IAE) problems stem from necessity to achieve some balance between such highly contradictory factors as goals and tasks of a problem solving, the nature of visual perception, ways and means of an image acquisition, formation, reproduction and rendering, and mathematical, computational and technological means allowable for the IEA.

We may consider that the main contradiction is related to the “pictorial nature” of an image and the “formal” (symbolic) foundations of IAE: it is well known that to

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take an advantage from data representation an image form is necessary to reduce the latter to a “non-image” form.

The paper is devoted to the axiomatics and formal constructions of the Descriptive Theory for Image Analysis (DTIA) [2], providing a methodology, mathematical and computational techniques for automation of IAE.

2 The Descriptive Theory for Image Analysis

Taking as a strategic goal the automated image mining it is necessary to provide image analysis professionals and final users with the following opportunities:

• automated design, test and adaptation of techniques and algorithms for image recognition, estimation and understanding;

• automated selection of techniques and algorithms for image recognition, estimation and understanding;

• automated testing of the raw data quality and suitability for solving the image recognition problem;

• standard technological schemes for image recognition, estimation, understanding and retrieval.

Automation of image-mining is possible by complex application techniques for image analysis, understanding and recognition.

The goal of image analysis (IA) is extraction from an image of the information, which could be a base for intellectual decision making concerning objects, situations and scenes represented in the image.

Computer vision which is based on the techniques and means of mathematical IA is functionally a technical analogue of living organisms vision (ideally – of human vision). The IA provides reducing of an image to a recognizable form. i.e. constructing of a formal description – a model –of an image. The main subproblems are segmentation (clustering an image to nonintersecting fragments), selection of features characterizing an image structure and content, computation of characteristics values useful for an image model synthesis and the synthesis of the model.

Image recognition includes setting up and solving of pattern recognition problems for cases when a raw data is given as separate images, sets of images and image models. The recognition results are determining some class membership of an image under recognition, its fragments or some objects in an image or clustering of the objects under recognition into nonintersecting subsets (classes).

The main problems of IA are:

1. Image matching for classification with an image, a set of images and a series of images.
2. Searching an image for some regularity/irregularity/object/token/fragment/primitive of arbitrary or prescribed type/form.
3. Clustering of an image set.
4. Image segmentation (for homogeneous regions, groups of objects, selection of features).
5. Automatic selection of image descriptors (primitives, specific objects, feature objects, logical and spatial relations) useful for an image model synthesis.
6. Image reduction to a recognizable form (constructing an image model).

7. Image analysis problem decomposition and synthesis.
8. Problems 1-7 for dynamical images with a complex background and with considering the ways of an image acquisition, formation and representation.

In solving these subproblems an automated/interactive IA system is faced with the specific subproblems as follows: a) extract meaningful 2-D grouping of intensity-location-time values via identification of groups of image entities – pixels by means of similarity of intensity value or by similarity discontinuity or similarity of change or constancy over time; typical groupings are edges, regions, and flow vectors; b) infer 3-D surfaces, volumes, boundaries, shadows, occlusion, depth, color, motion using the groupings of pixels and their characteristics; c) group information into unique physical entities; d) transform image-centered representations into world-centered representations; e) label entities depending on system goals and world model; f) infer relations among entities; g) construct a consistent internal description.

Image understanding is considered as an emulation of human visual capabilities. In particular, as the derivation of high-level (abstract) information from an image or series of images and more specifically as the derivation of knowledge on 3-D world from 2-D images and the construction of the description of 3-D scene represented as 2-D image (s). The result of an image understanding is a symbolic description of the image in terms of its elements, relations between them and the image properties. The description should ensure the decision making in a real 3-D environment (recognition of 3-D objects, automated navigation, etc.). The image understanding process is implemented by combining the results of image processing, analysis and recognition with the knowledge on a scene.

In IA is used a wide spectrum of mathematical techniques from algebra, geometry, discrete mathematics, mathematical logics, probability theory, mathematical statistics, calculus, as well as the techniques of mathematical theory of pattern recognition, digital signal processing, and physics (in particular, optics).

The transition to practical, reliable and efficient automation of image-mining is directly dependent on introducing and developing of theoretical means for IAE.

The natural way to overcome the above mentioned contradiction between "pictorial nature" of an image and the "formal" (symbolic) foundations of IAE is to introduce pattern recognition oriented image models and necessary means and techniques for reduction of an image to a recognizable form without loss of an image specificity. The careful study of the challenge revealed the opportunity to solve it via a theory establishing reasonable ties between an image nature, IAE applications, pattern recognition philosophy, image representations and models, IAE transforms, and corresponding information technologies.

In a whole the success of IAE depends mainly on the success of an image reduction to a recognizable form, which could be accepted by appropriate image analysis/recognition algorithms. It appeared that an image reduction to a recognizable form is a crucial issue for IA applications, in particular, for qualified decision making on the base of image mining. The main tasks and problems of an image reduction to a recognizable form are:
1. Formal description of images: 1) study and construction of image models; 2) study and construction of image representations by multiple models.
2. Description of image classes reducible to a recognizable form: 1) introduction of new mathematical settings of an image recognition problem; 2) establishing and study

of links between multiple model representation of images and image metrics; 3) study and use of image equivalencies.

3. Development, study and application of an algebraic language for description of the procedures of an image reduction to a recognizable form.

The development of the sought for mathematical theory is going in the direction of its algebraisation based on the Algebraic Approach to Pattern Recognition and Classification Problems [8], its specialization for a case of a raw data represented in an image form – DTIA [2], [5] and on development of image algebras and of descriptive image algebras (DIA). The DTIA provides a variety of mathematical and computational techniques for IAE automation.

The DTIA provides an opportunity to solve the problems connected with the development of formal descriptions for an image as a recognition object as well as the synthesis of procedures for an image recognition and understanding. The analysis of the problems is based on the investigation of inner structure and content of an image as a result of the procedures "constructing" it from its primitives, objects, descriptors, features and tokens

This approach to an image characterization is an operational one. It implies that a process of IAE as a whole (including an image formal description synthesis, analysis, estimating and recognition) – a problem solving trajectory – may be considered as a sequence/combination of some transforms combined with computation of some sets of intermediate and final (decision making) estimates. The transforms are defined on image equivalence classes. The latter, by definition, are defined in a descriptive manner – by some set of basic prototypes and a corresponding set of generative transforms being functionally complete in respect to an equivalence class of allowable transforms.

The IAE process is implemented in the form of a trajectory – a sequence of allowable transforms. It represents a route in the space of image formal descriptions. The latter play a dual role being at the same time the objects and the results of the IAE transforms. The description space is a hierarchical one and it includes image models of different types. The models could correspond to different morphological and scale levels of an image representation. They may represent different aspects of an image properties and characteristics and may be realized as multilevel, multi-aspect and multi-model image descriptions. It allows one to choose and change a degree and aspects of an image description detailed elaboration for an applied problem at hand.

The main purpose of theoretical apparatus of the DTIA is structuring of the variety of methods, operations and representations being used in IEA. The final goal of the DTIA is automated image mining: a) automated selection of techniques and algorithms for image recognition, estimation, and understanding; b) automated testing of the raw data quality and its suitability for solving the image recognition problem.

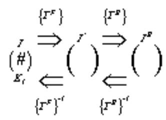
The DTIA provides mathematical fundamentals for image mining:

• specialization of Zhuravlev's Algebra for an image recognition case;
• standardization of image analysis and recognition problems representation;
• standardization of a descriptive language for image analysis and recognition procedures;
• means for applying common mathematical apparatus for operations over image analysis and recognition algorithms and over image models.

The DTIA is based on:

• descriptive model of image recognition procedure;
• special mathematical setting of an image analysis problem;
• image reduction to a recognizable form;
• algebraization of image mining;
• generative principle and bases of transforms and models;
• plurality of image models – multi-model representations of images;
• introduction of a knowledge into image-mining processes.

The corner-stone of the DTIA is a descriptive model of image recognition procedures (Figure 1).



$$J, J^*, J^R; T^F, T^R, T^F : J \Rightarrow J^*, T^R : J^* \Rightarrow J^R; \{J\} = \bigcup_j K_j.$$

Fig. 1. Descriptive model of image recognition procedures.

{J} - a set of ideal images; {J^*} - a set of observable images; {J^R} - a set of images – results of the recognition process; {T^F}- a set of allowable transforms securing an image formation; {T^R} - a set of allowable transforms securing an image recognition; {K_i} –an image equivalence class.

In pattern recognition problems a raw data represented in an image form is ill-structured and “non-formalized”. So, it is impossible as a rule to apply pattern recognition algorithms to the data directly, i.e. it is impossible to set up and solve an image recognition problem as a standard pattern recognition problem. It is necessary to reduce an image to a recognizable form to overcome this circumstance – to create a formal description of an image (image model), accepting a recognition operator, and as a consequence to get an opportunity to use metrics for estimation of images/fragments of images proximity.

The preliminary condition of image mining algebraization is development of formal systems for image representation and transformation satisfying to the following conditions: a) each object is a hierarchical structure constructed by a set of operations of an image algebra applied to a set of elements of images; b) the objects are points, sets, models, operations, morphisms; c) each transform is a hierarchical structure constructed by a set of operations of image algebra on the set of basic transforms.

The DTIA provides construction and application of two types of such formal systems - special versions of algebras - image algebras [7] and descriptive image algebras [4].

The application of the generative principle is based on the generalized inductive definition (defining a class of objects by a set of initial objects and a set of rules for obtaining new objects of the class from the defined objects) and on the U,Grenander’s concept of combinatorial regular structures [1].

An image (or its complex object) description is constructed in the form of a hierarchical structure including “more simple” objects. It allows to use and represent explicitly an information included in an image. Thus, it is possible to get unlimited variety of descriptions using only limited amount of atomic (primitive) elements and limited amount of generative (combining) rules and applying as many times as necessary the latter to initial and generated objects.

So, a formalized description of an image is determined by a set of objects extracted from an image, linked by structural relations and constructed by allowable generative transforms. Exploitation of the generative principle and basises of transforms and models provides for decomposition of a problem into primitive tasks, establishing of the correspondence between basic primitive tasks and basic primitive transforms and combining of basic algorithms and models.

The processes of information extraction from images use different kinds of knowledge: on subject domain, on a problem nature and specificity, on physics and geometry of the scene, on logical, mathematical and physical laws and conditions controlling an object of imaging, on the ways and means used for an image acquisition, registration and formation and some others. The knowledge is applied for image models construction (selection of image primitives and tokens, of aspects, types and levels of formalization) and for construction of a recognition process models and for controlling it (hypothesis generation on the final information, selection of heuristic transforms, stopping rule). The knowledge is mainly limited by semantic and contextual information and by a set of logical and physical conditions).

DTIA main tools are Descriptive Image Algebras (DIA) [4], [6], Descriptive Image Models (DIM) and Multiple Model and Multi-aspect representation of images by Generating Descriptive Trees [5].

DIA is a mathematical language developed for description, comparison and standardization of algorithms for image analysis, processing and recognition. It provides an opportunity to obtain flexibility and standardization in development and implementation of image-mining algorithmic schemes.

Image-mining problems, objects and transforms are set up by hierarchical structures constructed via application of DIA operations to a set of primitive tasks, image primitives and basic transforms.

It is possible to vary methods for a subtask solution using image analysis operations as elements of DIA and preserving an overall scheme of an image-mining technology.

DIM are mathematical objects providing representation of information carried by an image and by an image legend (context) in a form acceptable for a recognition algorithm. Image legend may contain information on a subject domain, a scene, illumination, sensors, image acquisition and formation system, an observer position and other "useful" semantic and helpful information. The DIM were introduced for defining an image recognition process [2] in such a way that the reducing of an image to a recognizable form could be implemented as transformation of a raw image into an image model acceptable for a recognition algorithm.

3 The State of the Art in Image-Mining

The state of the art in image-mining is characterized by the following trends: Plurality and Fusion, Multiple Classifiers and Multi-model Representations.

The DTIA suggests mathematical and methodological bases for answering the image-mining challenge.

The most important – critical points of an applied image-mining problem solving are as follows:
1. Precise setting of a problem.
2. Correct and "computable" representation of raw and processed data for each algorithm at each stage of processing.
3. Automated selection of an algorithm:
• decomposition of the solution process for main stages;
• indication of the points of potential improvement of the solution ("branching points");
• collection and application of problem solving experience;
• selection for each problem solution stage of basic algorithms, basic operations and basic models (operands);
• classification the basic elements.
4. Performance evaluation at each step of processing and of the solution:
• analysis, estimation and utilization of the raw data specificity;
• diversification of mathematical tools used for performance evaluation;
• reduction of raw data to the real requirements of the selected algorithms.

The further development of the DTIA should provide necessary means for overcoming the critical points.

4 The Axiomatics

This part concerns with the axiomatics and formal structures of DTIA providing the ways and means to represent and to describe images for its analysis and estimating. The main contributions are DIM: its definitions, classification, properties, interrelations, and conditions of generation.

As it is known it is impossible to set up and solve an image recognition problem as a standard pattern recognition problem – standard pattern recognition algorithms don't accept images as input data. Thus, to overcome this circumstance it is necessary to reduce an image under recognition to a recognizable form – to construct a formal description of an image (image model) allowable for a recognition operator.

Our main goal is to setup the construction of image models population – image model space, i.e. to setup its structure, types of elements and classes of allowable transforms. DIM are used for standardization of raw data for recognition algorithms. GDT is an instrument for classification and representation of any information which can be used for image model constructing. GDT can be employed to make more convenient the selection and construction of image models. In particular, GDT can be used for creating and combining some new partial image representations (multi-model representations). It is also could be used for creation of a new type of multiple classifiers providing application of different recognition operators to different multi-aspect partial image models and combining the results.

4.1 Descriptive Image Models

DIM is a mathematical object providing representation of information carried by an image and by an image legend (context) in a form acceptable for a recognition algorithm. The DIM notion was used for introducing a model for an image recognition procedure: a step of image reduction to a recognizable form consists in the construction of some image models for initial images, while the models are acceptable for a recognition algorithm.

The following classes of DIM were introduced: P-models (Parametric Models), G-models (Generating Models), T-models (Transformation or Procedure Models) and I-models (initial images as are). Now on the base of general principles of DITA outlined in the first part of paper we introduce the axioms and basic definitions for DIM.

Axiom 1: It is possible to put into one to one corresponds an aggregate of sets ($\{\tilde{I}_0\}$, $\{\tilde{t}\}$, $\{\tilde{M}\}$) to any **image** I, where $\{\tilde{I}_0\}$-a set of initial data (see below the lemma), $\{\tilde{t}\}$-a set of transforms applied to the set of initial data, and $\{\tilde{M}\}$-a set of results of transforms applying to initial data.

The scheme 1 illustrates the axiom 1:

$$\{t\} : (I \simeq \{\tilde{I}_o\}) \Rightarrow \{\tilde{M}\} \tag{1}$$

Let us consider each of the above mentioned sets in details.

Lemma: A set of initial data $\{\tilde{I}_0\}$ consists of two sets $\{\tilde{I}'\}$ and $\{\tilde{B}\}$:

1) $\{\tilde{I}'\}$ is a set of an image realizations $I' \in \{\tilde{I}'\}$ for an image *I,* representing specified object or scene, such that $I' = \{(x, f(x))\}_{x \in D_f}$ is a set of points x belonging to definition domain of image realization D_f, and the set of values $f(x)$ is defined in each point of definition domain D_f;
2) A set $\{\tilde{B}\}$ is an image semantic and context information.

The definition domain of image realization is a subset of n-dimensional discrete space $\mathbb{Z}^n$ (for 2-D image n=2).

Definition 1: **I-model** of an image is any element I' of a set of image realizations $\{\tilde{I}'\}$.

Let us consider a set of transforms $\{\tilde{t}\}$.

Axiom 2: A set of transforms $\{\tilde{t}\}$ is specified by a set of structuring elements $\{\tilde{S}\}$, by a set of generating rules $\{\tilde{R}\}$ and by 3 subsets of transforms $\{\tilde{t}_T\}$, $\{\tilde{t}_P\}$, $\{\tilde{t}_G\}$: 1) procedural transforms $\{\tilde{t}_T\}$, 2) parametric transforms $\{\tilde{t}_P\}$, 3) generating transforms $\{\tilde{t}_G\}$.

Definition 2: Structuring element S$\in \{\tilde{S}\}$ is a two-dimensional spatial object, providing by convolution with an image its defragmentation into fragments. The later allows getting the image descriptors by application of corresponding transforms. A structuring element is specified by some parameters including numerical and geometrical characteristics.

Definition 3: Procedural transform $t_T \in \{\tilde{t}_T\}$ **with arity *r*** over images $\{I_i\}_{1...r}$ is an operation applied to images $\{I_i\}_{1...r}$, providing conversion of initial images into other images, or into an image or into image fragments.

In our axiomatics a procedural transform $t_T \in \{\tilde{t}_T\}$ with arity *r* over image I-models $\{I'_i\}_{1...r}$ is an operation applied to an image I-models $\{I'_i\}_{1...r}$ providing conversion of an image I-models into I-models of other images, of an image or of image fragments. The operands of this operation are I-models of one initial image, or I-models of different initial images.

Definition 4: Parametric transform $t_P \in \{\tilde{t}_P\}$ over image *I* is an operation applied to image *I* providing transforms of initial image into a numerical characteristic *p*, which correlates with the properties of geometrical objects, brightness characteristics and configurations formed by regular repeating of geometrical objects and brightness characteristics of initial image.

The set of image realizations as well as an image semantic and context information can be used for numerical characteristic *p* calculation.

Definition 5: Generating transform $t_G \in \{\tilde{t}_G\}$ is a transform permitting to generate some partial representation reflecting specific properties (features, distinctive features, particular qualities) of analyzed image. The definition of representation is given below.

The examples of such transforms are functions representing curves, a function of disjunction, a function of conjunction, an image code function.

Definition 6: Generating rule R is a rule of an image model construction using an image and generating transforms defining a strict sequence of operations applied to an image for its model construction. The definition of image model is given below.

Definition 7: Generating transform t_G is **correct** for a given image if and only if there are generating rules insuring construction of a generating image model by a generating transform t_G and the image. The definition of generating model is given below.

Definition 8: An image representation $\Re(I)$ is a formal scheme for getting standardized formal description of forms, surfaces and point configurations formed an image and relationships between them.

Definition 9: An image model $M(I)$ is a formal image description generating by a realization of an image representation.

Definition 10: Realization of an image representation is a process of applying of image representation to image realizations of initial image with concrete values of parameters of transforms included to representation.

Definition 11: A correct image model is an element of a set of allowable image models generated by realizing image representation consisting of a set of transforms $\{\tilde{t}\}$ on the set of initial data $\{\tilde{I}_0\}$.

Theorem: An element *m* of a set $\{\tilde{M}\}$ of results of transforms $\{\tilde{t}\}$ applying to a set of initial data $\{\tilde{I}_0\}$ is **a correct image model**.

The prove is based on definitions 7-11.

The scheme 2, 3 illustrate interrelations of the image representations and image models.

$$\{\tilde{\Re}(I)\} = \{\tilde{t}, \tilde{S}\} : \{I\} \Rightarrow \{\tilde{M}\} \quad (2)$$

$$\{\Re(I)\}(p) = \{t_1, t_2, \ldots t_n, \tilde{S}'\}(p) : \{I'\} \in \{I\} \Rightarrow M_1 \in \{\tilde{M}\} \quad (3)$$

Definition 12: T-representation $\Re_T(\tilde{\rho})$ of an image *I (*$\tilde{\rho}$ is parameters of procedure transformations and structuring elements*)* is specified by an aggregate of sets $(\{I'\} \subset \{\tilde{I}'\}, \{B\} \subset \{\tilde{B}\}, \{S\} \subset \{\tilde{S}\}, \{t_T\} \subset \{\tilde{t}_T\})$, where a set of procedure transforms $\{t_T\} \subset \{\tilde{t}_T\}$ and a set of structuring elements $\{S\} \subset \{\tilde{S}\}$ are applied to a set of I-models $\{I'\} \subset \{\tilde{I}'\}$. An image semantic and context information $\{B\} \subset \{\tilde{B}\}$ can also be used. A set of all possible T-representations is denoted as $\{\tilde{\Re}_T(\tilde{\rho})\}$.

$$\begin{aligned} \Re_T(\tilde{p}) &= \{\{I'\}, \{B\}, \{S\}, \{t_T\}\} \\ \Re_T(\tilde{p}) &= \{S, t_T\}(\tilde{p}) : \{I', B\} \end{aligned} \quad (4)$$

Statement 1: Any T-representation $\Re_T(\tilde{\rho}) \in \{\tilde{\Re}_T(\tilde{\rho})\}$ of image generates a set of **image T-models** $\{M^{\rho_0}{}_T\}$ by setting values of parameters $\tilde{\rho}$ of procedural transforms and structuring elements. A set of all possible image T-models is $\{\tilde{M}_T\}$.

Statement 2: Any image T-model $M_T \in \{\tilde{M}_T\}$ generates a new image realization, that is an image I-model $M_I \equiv I' \in \{\tilde{M}_T\} \equiv \{\tilde{I}'\}$.

Example. Let $\{I'\}$ is a set of image *I* realizations. A 3D-image may consist of a set of electronic image realizations $\{I'\}$. A 2D-image may be described by a set of image fragments. Let $\{\tilde{t}_T\} = \{t_j(\tilde{\alpha}_j)\}_{1\ldots r}$ is a set of procedural transforms over $\{I'\}$, $\tilde{\alpha}_j$ - parameters of procedure transforms t_j. Then T-representation is specified by an aggregate of sets $(\{I'\}, \{t_j(\tilde{\alpha}_j)\}_{1\ldots r})$. In this case procedural transforms are applied to images, but structuring elements are not used, that is $\{S\} = \varnothing$. This image T-representation generates a set of image T-models $M_T(I) = \{t_j(\tilde{\alpha}^0_j\}_{1\ldots r}(\{I'\})$ by setting values of parameters of procedural transforms: operations are applied to image realizations in parallel and sequential modes.

Definition 13: P-representaion $\Re_P(\tilde{\rho})$ of an image *I (*$\tilde{\rho}$ is parameters of parametric transforms*)* is specified by an aggregate of sets $(\{I'\} \subset \{\tilde{I}'\}, \{B\} \subset \{\tilde{B}\}, \{t_P\} \subset \{\tilde{t}_P\})$, where a set of parametric transforms

$\{t_P\} \subset \{\tilde{t}_P\}$ is applied to a set of I-models $(\{I'\} \subset \{\tilde{I}'\})$. An image semantic and context information $\{B\} \subset \{\tilde{B}\}$ can also be used. A set of all possible P-representations is $\{\tilde{\Re}_P(\tilde{\rho})\}$.

$$\Re_P(\tilde{p}) = \{\{I'\}, \{B\}, \{t_P\}\}$$
$$\Re_P(\tilde{p}) = \{t_P\}(\tilde{p}) : \{I', B\} \quad (5)$$

Statement 3: Any P-representation $\Re_P(\tilde{\rho}) \in \{\tilde{\Re}_P(\tilde{\rho})\}$ of an image generates a set of **image P-models** $\{M^{\rho_0}{}_P\}$ by setting values of parameters $\tilde{\rho}$ of parametric transforms. A set of all possible image P-models is $\{\tilde{M}_P\}$.

Example: Let I', I'' are I-models of initial image *I*. A color image can be stored in format of RGB (I') and in format of grey level image (I''). Let $\{t_P\} = \{f_1(\tilde{\alpha}_1), f_2(\tilde{\alpha}_2), \ldots f_n(\tilde{\alpha}_n)\}$, where $f_1, f_2, \ldots f_{n_1}$ are functions of feature calculation over an image I-model $I' \subset \{\tilde{I}'\}$, and $f_{n_1+1}, f_{n_1+2}, \ldots f_n$ are functions of feature calculation over an image I-model $I'' \subset \{\tilde{I}'\}$, $\tilde{\alpha}_1, \tilde{\alpha}_2, \ldots, \tilde{\alpha}_n$ are parameters of functions of feature calculation. Then a P-representation is specified by an aggregate of sets $(\{I', I''\}, \{f_1(\tilde{\alpha}_1), f_2(\tilde{\alpha}_2), \ldots f_n(\tilde{\alpha}_n)\})$. P-representation generates a set of P-models of image *I* by setting values of parameters: $M_P(I) = (f_1(\tilde{\alpha}_1^0)(I'), f_2(\tilde{\alpha}_2^0)(I'), \ldots, f_{n_1}(\tilde{\alpha}_{n_1}^0)(I'), f_{n_1+1}(\tilde{\alpha}_{n_1+1}^0)(I''), f_{n_1+2}(\tilde{\alpha}_{n_1+2}^0)(I''), \ldots, f_n(\tilde{\alpha}_n^0)(I''))$ is a vector of numeric features.

Definition 14: G-represenation $\Re_G(\tilde{\rho})$ of an image *I* ($\tilde{\rho}$ is parameters of generating transforms, generating rules and structuring elements) is specified by an aggregate of sets $\{I'\} \subset \{\tilde{I}'\}, \{S\} \subset \{\tilde{S}\}, \{B\} \subset \{\tilde{B}\}, \{t_G\} \subset \{\tilde{t}_G\}, \{\tilde{R}\}$. A set of all possible G-representations is $\{\tilde{\Re}_G(\tilde{\rho})\}$.

$$\Re_G(\tilde{p}) = \{\{I'\}, \{S\}, \{B\}, \{t_G\}, \{R\}\}$$
$$\Re_G(\tilde{p}) = \{R, t_G, S\}(\tilde{p}) : \{I', S\} \quad (6)$$

Statement 4: Any G-representation $\Re_G(\tilde{\rho}) \in \{\tilde{\Re}_G(\tilde{\rho})\}$ of an image generates a set of **image G-models** $\{M^{\rho_0}{}_G\}$ by setting values of parameters of generating transforms, generating rules and structuring elements. A set of all possible image G-models is $\{\tilde{M}_G\}$.

The interrelations of all introduced notions are represented on scheme in figure 2.

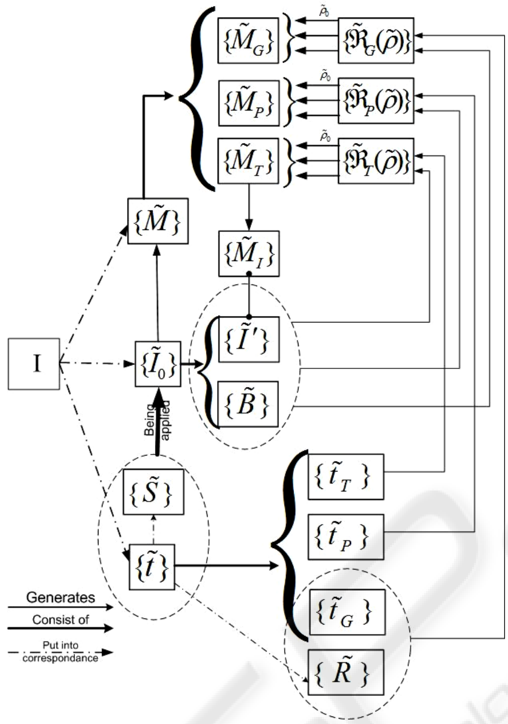


Fig. 2. Interrelations of introduced notions.

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