APPROXIMATION OF 5-LIMIT JUST INTONATION
Computer MIDI Modeling in Negative Systems of Equal Divisions of the Octave

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Keywords: Intonation, just, music, score, tuning.

Abstract: The article matter is related with music processing by MIDI protocol during computer modeling of fixed scales with non-traditional equal temperaments. Are touched negative temperaments, which based on closed series of fifths, compressed relative conventional tuning. Is marked, that such systems, can better approach to just intonation. They give sensation out of tuning during listening to music performed by scores with mistaken using of accidentals, which inaccessible in a conventional temperament system. Is given a subprogram example of the automatic Pitch Bend change in MIDI protocol for modeling of negative system of equal divisions of the octave.

1 INTRODUCTION

1.1 Purpose of the Paper

Author imparts about applying of negative systems of equal divisions of the octave and how they was obtained by means of MIDI protocol.

1.2 5-limit Just Intonation (5LJI) and Natural Scale (NS)

5LJI is an ideal infinite graded system of fixed tuning which provide:

1. for all pairs of grades, correlation of sound frequencies between grades of each pair with rational number factorable to primes no more 5 by value;
2. for each grade existence of all other grades, with anyone ratios of frequencies, which satisfy the condition 1.

Everywhere in this paper a music which allow 5LJI is supposed. Compositions by J. S. Bach can be such examples (Asmussen, 2001).

NS is a set of sounds of defined frequencies with correlations, which proper to natural numbers. Frequency of the 1-st element of NS is lowest, 2-nd in 2 times higher of lowest, 3-rd in 3 times higher of lowest, and so on.

In 5LJI is possible to build not full NS from any grade. In such ones there will be no elements with numbers which contain prime factors more 5 by value. Numbers 7, 11, 13, 14, … will be absent.

1.3 System of Equal Divisions of the Octave (EDO), Evaluations and Initial Values

The EDO system is graded finite fixed tuning ensuring:

1. for each grade possbility or existence of other grades, with interval of a perfect octave up and/or down;
2. for all intervals of a perfect octave existence of an identical quantity of grades inside each octave;
3. for all pairs of an identical interval between grades of each pair.

Conventional 12EDO system has so called semitone between adjacent grades. 1/100 of semitone named cent, is adopted as a basis of evaluations in this paper. The values cited without verifying calculations, are obtained in the software Scala (Op de Coul, 2004).

1.4 Negative EDO System

R. H. M. Bosanquet has offered a method of deriving of other EDO systems from an accepted for initial system 12EDO (E. T. at Bosanquet): «…Let a regular system of fifths start from *c*. If they

Khramov M. (2008).
APPROXIMATION OF 5-LIMIT JUST INTONATION - Computer MIDI Modeling in Negative Systems of Equal Divisions of the Octave.
In *Proceedings of the International Conference on Signal Processing and Multimedia Applications*, pages 181-184
DOI: 10.5220/0001932001810184

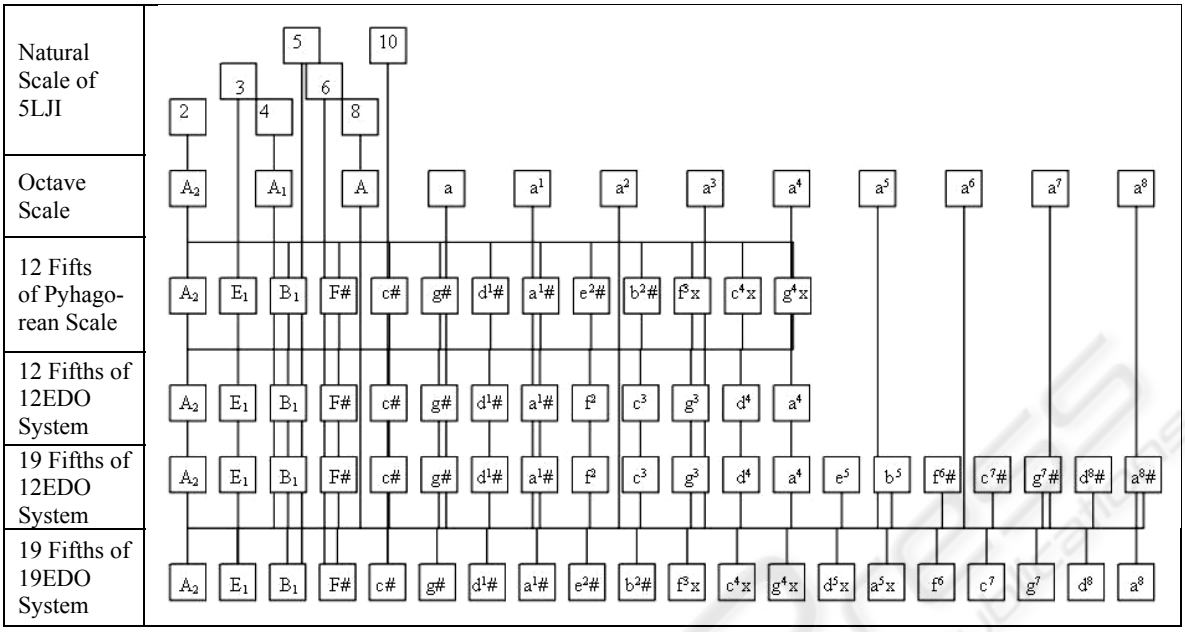

Figure 1: Scheme of Building of 19EDO System from Note A of Subcontraoctave (A_2) as Origin.

are positive, then at each step the pitch rises further from E. T. It can only return to *c* by sharpening an E. T. note.

Suppose that *b* is sharpened one E. T. semitone, so as to become *c*; then the return may be effected at the first; *b* in 5 fifths, at the second *b* in 17 fifths, at the third *b* in 29 fifths; and so on. Thus we obtain the primary positive systems…

If the fifths are negative, the return may be effected by depressing *c#* a semitone in 7, 19, 31... fifths; we thus obtain the primary negative systems…» (Bosanquet, 1875)

The Figure 1 illustrates application of this method from note A_2 as origin.

Initial system is created by uniform compressing of a series of 12 perfect fifths with a small interval, which is known as comma of Pythagoras (CP). In a scale of Pythagoras (SP) are used fifths absolutely coincident with interval between 2-nd and 3-rd elements of NS or perfect fifth.

The octave of all EDO systems is exactly equal to an interval between 2-nd and 4-th elements of NS or perfect octave. 12 fifths of SP exceed 7 octaves on CP by size 23.46 cent.

Uniform compression of all 12 fifths SP with the goal to eliminate CP gives 12 fifths of the most spreaded 12EDO system. A fifth of this system approximates perfect fifth with an error -1.955 cent, and is adopted as initial for a further building.

19 fifths of a 12EDO system exceed 11 octaves by one semitone of a 12EDO system. After uniform compression (for elimination of this semitone) all 19 fifths of a 12EDO system, are obtained 19 fifths of a 19EDO system.

A fifth of 19EDO system has departure -5.263 cent from an initial value and approximates perfect one with an error -7.218 cent.

A fifth of the next after 19EDO negative 31EDO system approximates perfect one with an error -5.181 cent.

The useful paradox of negative EDO systems: the impairment of approximation of perfect fifth of an initial 12EDO system, may automatically improve approximation of major third of 5LJI (interval between 8-th and 10-th elements of NS on Figure 1).

«… For if we take 4 negative fifths up, we have a third with negative departure (-4δ) which can approximately represent the departure of the perfect third. Thus *c#* is either the third to *a*, or four fifths up from *a*, in accordance with the usage of musicians…» (Bosanquet, 1875).

1.5 Approximation 5LJI in 19/31EDO Systems and Simplicity of Transposing of Music, from 12EDO System to Them

The attractiveness of such approximation becomes noticeable from a Table 1.

Table 1: Comparison of Three Variants of 5LJI Approximation by EDO systems.

5LJI			Approximation					
System / Interval	NS		12EDO		19EDO		31EDO	
	El-ts	Cents	Cents		Cents		Cents	
			Size	Abs. error	Size	Abs. error	Size	Abs. error
Major Third	10/8	386.314	400.000	13.686	378.947	7.367	387.096	0.782
Perfect Fifth	3/2	701.955	700.000	1.955	694.737	7.218	696.774	5.181
Octave	4/2	1200.000	1200.000	0.000	1200.000	0.000	1200.000	0.000
The average absolute error of approximation				5.214		4.862		1.988

Table 2: Modeling of 19EDO and 31EDO Systems by Evaluation for All Possible Notes of PB of Protocol MIDI.

Note Designation		System Grade from Origin at Cn										
Name	Accidental	12EDO, initial			19EDO				31EDO			
		Number	PB	Cents (Pitch)	PB	Cents		Number	PB	Cents		Number
						Depart.	Pitch			Depart.	Pitch	
G	x	9	8192	900.000	4959	-78.947	821.053	13	6210	-48.387	851.613	22
	#	8	8192	800.000	6467	-42.105	757.895	12	7135	-25.806	774.194	20
	n	7	8192	700.000	7976	-5.263	694.737	11	8060	-3.226	696.774	18
	b	6	8192	600.000	9485	31.579	631.579	10	8985	19.355	619.355	16
	bb	5	8192	500.000	10994	68.421	568.421	9	9910	41.935	541.935	14
:	:	:	:	:	:	:	:	:	:	:	:	:
A	x	11	8192	1100.000	4527	-89.474	1010.526	16	5946	-54.839	1045.161	27
	#	10	8192	1000.000	6036	-52.632	947.368	15	6871	-32.258	967.742	25
	n	9	8192	900.000	7545	-15.789	884.211	14	7796	-9.677	890.323	23
	b	8	8192	800.000	9054	21.053	821.053	13	8720	12.903	812.903	21
	bb	7	8192	700.000	10563	57.895	757.895	12	9645	35.484	735.484	19

19EDO system is less preferable, as in polyphony of instruments, rich by upper overtones (harpsichord, piano), it can give noticeable sensation out of tuning. It can not arise in same system, for timbres of flute for example. The most probable reason of it, is in a too large error of perfect fifth.

The Table 2 demonstrates simplicity of the correspondences between all possible designations of the notes and numbers of grades of these systems. In it the accidental *n* indicates a lack or cancellation of operation of other ones, i. e. neutral pitch of the note.

Application of additional accidentals actually is not required and direct using of scores for 12EDO system is possible for playing in 19/31EDO ones.

2 SUBJECT OF RESEARCH

2.1 Possibility of Computer Modeling of 19/31EDO Systems

Practically each modern computer has a sound synthesizer, supporting protocol MIDI. Such synthesizer has a set of channels.

Each channel is polyphony 12EDO system with independent control, providing changed Pitch Bend (PB).

PB works similarly to accidental, but shifts pitches of all notes in the channel, as sounding, as well consequent. Therefore its application for retuning of the musical pieces from a 12EDO system to other ones, requires in each channel only one voice. A polyphony in each channel also is possible, but the octaves and unisons only are admissible between voices.

Any grades of 19EDO and 31EDO systems can be obtained from suited 12EDO grades equipped by an appropriate PB value from the Table 2.

PB value for each note can be added to software Sibelius 4 (Eastwood and Others, 2005), allowing to transpose conventional scores to MIDI versions:

```
//(c)Mykhaylo Y. Khramov, 2006
switch (NameOfNote)
{
case ("Gx"){PB = "-48¢~B 66,48";}
case ("G#"){PB = "-26¢~B 95,55";}
case ("Gn"){PB = "-3¢~B 124,62";}
case ("Gb"){PB = "+19¢~B 25,70";}
case ("Gbb"){PB = "+42¢~B 54,77";}
:
```

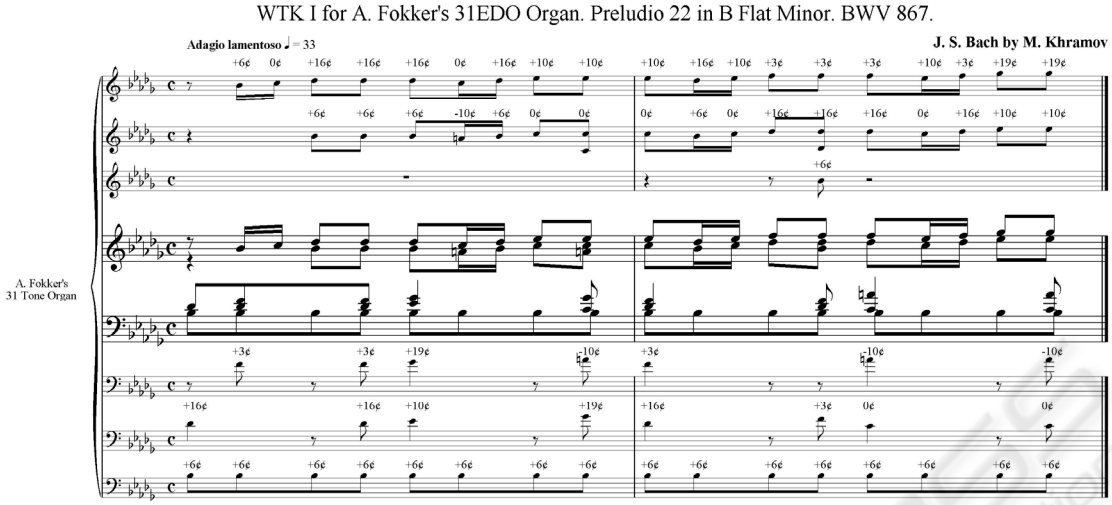

Figure 2: Prepared MIDI Version of Score.

```
case ("Ax"){PB = "-55¢~B 58,46";}
case ("A#"){PB = "-32¢~B 87,53";}
case ("An"){PB = "-10¢~B 116,60";}
case ("Ab"){PB = "+13¢~B 17,68";}
case ("Abb"){PB = "+35¢~B 45,75";}
default {PB =  "±00¢~B 0,64";}
}
return PB;
```

Such simple subprogram allows to retune automatically the musical pieces and to get audible models of their sounding. Some from them are freely offered for listening to and discussion (Khramov, 2004, 2006).

On Figure 2, the subprogram has added above each note in ossias the text, which is seen from an initial quotation marks up to a tilde, and is hidden, since a tilde and up to a completing quotation marks. The visible text shows to observer the departure of each note pitch, and hidden one transmits appropriate PB to a MIDI device. It calls a required departure of each note pitch for a selected 31EDO

2.2 Modeling of Negative Systems, Indicates Mistakes in Scores

If in a context 12EDO major third A-C# for example is in score mistakenly designated as A-Db, the playing by this score in 31EDO will cause obvious sensation out of tuning, completely inaccessible during the playing in a system 12EDO. The respelling of mistakenly notation removes sensation out of tuning also during the playing in 31EDO.

The verification of scores by hearing plays the important role in the process of transposing of 12EDO scores for the performance of them in positive by Bosanquet (22EDO, 53EDO) systems (Khramov, 2008).

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