

# PLAUSIBLE MOTION SIMULATION: INCHWORM VS. ROLLER

Juha Holopainen

*Department of Computer Science, University of Kuopio  
PO Box 1627, 70211 Kuopio, Finland*

Mauno Rönkkö

*Department of Computer Science, University of Kuopio  
PO Box 1627, 70211 Kuopio, Finland*

**Keywords:** Particle systems, plausible motion simulation, intelligent control, emergent dynamics.

**Abstract:** In this paper, we study the use of a specific particle system in motion simulation. The novelty of the particle system is that, with it, everything is formalized in a systematic and uniform manner using just six matrices. We illustrate use of the particle system by presenting a challenging environment where two different entities, Inchworm and Roller, are tested and their behavior is examined. As the main contribution, we demonstrate here that, despite its simplicity, the particle system supports plausible motion simulation for organic and mechanistic motion. In addition, as the entities are modeled in a systematic and uniform manner, they are reusable.

## 1 INTRODUCTION

Objects and substances displaying emergent dynamics are often simulated with particle systems (Eberhardt et al., 1996; Wejchert and Haumann, 1991; Witkin, 1997; Wojtan et al., 2006).

Here, we present a particle system (Rönkkö, 2006b) which captures a wide range of dynamics without being too complex. This is achieved by formalizing the components in a systematic and uniform manner using matrices, as discussed in Section 2.

Due to the formalization, the particle system is modular, and the components can be reused in different settings. Furthermore, because the interaction of the components arises as emergent dynamics, the components automatically adapt to the new settings.

As the main contribution, we demonstrate here with two active entities that the particle system supports plausible motion simulation for both organic and mechanistic motion.

The two entities are Inchworm and Roller, which are introduced in Sections 3 and 4 respectively. Inchworm is an organic entity displaying life-like motion. Roller, in turn, is a mechanical entity, displaying classical mechanics. The entities were tested on different terrains. The terrain which revealed the most interesting dynamics, along with the test runs, is presented in Section 5. The implications and the plan for further development are then discussed in Section 6.

## 2 THE PARTICLE SYSTEM

In the particle system (Rönkkö, 2006b), there is a fixed and finite number of particles. They are unit balls with uniformly distributed, constant unit mass.

The particles are modeled using six matrices. The matrices  $\mathbf{p}$ ,  $\mathbf{v}$ ,  $\mathbf{d}$ , and  $\mathbf{m}$  capture the position, velocity, damping coefficient, and memory, respectively. The matrices  $\mathbf{b}$  and  $\mathbf{c}$  indicate which particles are bound to each other or collide with each other. The particles are identified by the rows of the matrices. The values of  $\mathbf{p}_i$ ,  $\mathbf{v}_i$ , and  $\mathbf{m}_i$  are three-dimensional vectors, and  $\mathbf{d}_i$  is a real valued number. Lastly,  $\mathbf{b}$  and  $\mathbf{c}$  are square matrices, where any  $\mathbf{b}_{ij}$  and  $\mathbf{c}_{ij}$  is either 0 or 1.

The dynamics of all the particles are governed by a model of motion. It is a numerical approximation of the trivial model of motion with forward error correction. Let  $t$  denote a fixed time step,  $B$  denote a bond function,  $C$  denote a collision function,  $R$  denote a model specific reaction function, and  $M$  denote a model specific memory update function. Then, the motion of a particle  $i$  is computed:

$$\begin{aligned}\mathbf{p}'_i &= \mathbf{p}_i + t\mathbf{v}'_i \\ \mathbf{v}'_i &= \mathbf{d}_i\mathbf{v}_i + B_i(\mathbf{p}) + C_i(\mathbf{p}) + R_i(\mathbf{p}, \mathbf{v}, \mathbf{m}) \\ \mathbf{m}'_i &= M_i(\mathbf{p}, \mathbf{m})\end{aligned}$$

Here, the updated position,  $\mathbf{p}'_i$ , is computed using the updated velocity,  $\mathbf{v}'_i$ , which is computed using af-

fecting forces. The binding forces are computed with respect to the initial positions, denoted by  $\mathbf{p}^0$ , as:

$$B_i(\mathbf{p}) = \sum_j \mathbf{b}_{ij}(\mathbf{p}_j - \mathbf{p}_i) \left(1 - \frac{\|\mathbf{p}_j^0 - \mathbf{p}_i^0\|^2}{\|\mathbf{p}_j - \mathbf{p}_i\|^2}\right)$$

Similarly, the collision forces are computed as:

$$C_i(\mathbf{p}) = \sum_j \mathbf{c}_{ij}(\mathbf{p}_j - \mathbf{p}_i) \min\left\{0, 1 - \frac{4}{\|\mathbf{p}_j - \mathbf{p}_i\|^2}\right\}$$

In short, the binding forces try to maintain the initial distance between the particles designated by  $\mathbf{b}$ . The collision forces push apart any overlapping particles that are allowed to collide by  $\mathbf{c}$ . Two particles overlap, when their distance is less than the sum of their radii.

When modeling, we first assign particles to the components. We then define the initial values for  $\mathbf{p}$ ,  $\mathbf{m}$ ,  $\mathbf{d}$ ,  $\mathbf{b}$ , and  $\mathbf{c}$ . The matrix  $\mathbf{v}$  is zero initially. Lastly, we define the functions  $R$  and  $M$ . The simulation is then an iterative computation of the model of motion.

In the following, we assume  $t = 0.01$  as the time step. Also, we use a conditional expression  $\langle P \rangle$ , which returns 1 if the predicate  $P$  holds and 0 otherwise.

### 3 INCHWORM

Inchworm, presented originally in (Rönkkö, 2006b), is similar to Arctic Caterpillar (Miller, 1988). It has a body and two leg particles. The body is composed of 60 particles, forming a muscle that operates in two phases: it contracts and relaxes, as captured in Figure 1. When contracting Inchworm grasps on to the road with its front leg, and when relaxing, it grasps on to the road with its tail leg. Contraction and relaxation are repeated sequentially, lasting the same number of iteration rounds. For this purpose, Inchworm uses a memory. In the sequel, we identify a particle  $i$  with:

$$\begin{aligned} \text{body}(i) &= \langle 0 \leq i < 60 \rangle \\ \text{legs}(i) &= \langle 60 \leq i < 62 \rangle \\ \text{road}(i) &= \langle 62 \leq i \rangle \end{aligned}$$

Consider a body particle  $i$  of Inchworm, that is  $\text{body}(i) = 1$ , and any particle  $j$ . Then, the model of the body is:

$$\begin{aligned} \mathbf{p}_i^0 &= (15, 4, -0.3i + 5) \\ \mathbf{m}_i^0 &= (0, 0, 0) \\ \mathbf{d}_i &= 0.95 \\ \mathbf{b}_{ij} &= (1 - \text{road}(j)) \cdot \langle 0 < \|\mathbf{p}_j^0 - \mathbf{p}_i^0\| < 2.5 \rangle \\ \mathbf{c}_{ij} &= \text{road}(j) + (1 - \text{road}(j)) \langle 2.5 < \|\mathbf{p}_j^0 - \mathbf{p}_i^0\| \rangle \\ R_i(\mathbf{p}, \mathbf{v}, \mathbf{m}) &= 0.1 \langle 0 < \mathbf{m}_{(i,1)} \rangle \langle 15 \leq i < 45 \rangle \cdot (\mathbf{p}_{53} - \mathbf{p}_{61}) \\ M_i(\mathbf{p}, \mathbf{m}) &= (\mathbf{m}_{(i,1)} + 1, 0, 0) - \langle 2000/999 < \mathbf{m}_{(i,1)} \rangle \cdot (0, 0) \end{aligned}$$

Consider a leg particle  $i$  and any particle  $j$ . Also, let  $\alpha$  be the nearest road particle, so that the condition  $\text{road}(\alpha) \wedge \min_{k=62}^{2000} \{\|\mathbf{p}_k - \mathbf{p}_i\|\} = \|\mathbf{p}_\alpha - \mathbf{p}_i\|$  holds. Let  $\beta$  indicate when the leg particle  $i$  is supposed to

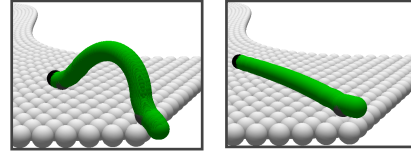


Figure 1: Inchworm edging its way; the state after the first contraction on the left, and the state after the first relaxation on the right.

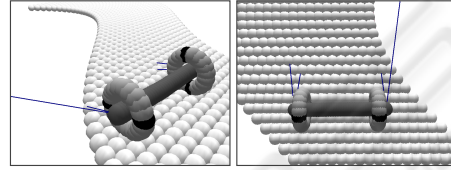


Figure 2: Roller from the side and from the top. In the left image, the left side beam misses the road. In the right image, the right side beam misses the road.

move, that is,  $\beta = \langle i = 60 \rangle \langle \mathbf{m}_{(0,1)} < 0 \rangle + \langle i = 61 \rangle \langle 0 \leq \mathbf{m}_{(0,1)} \rangle$ . Lastly, let  $\gamma$  be the value  $1 - \beta$ . Then, the model of the legs is:

$$\begin{aligned} \mathbf{p}_{i0}^0 &= (15.3, 3.8, -15.3(i - 60) + 4.1) \\ \mathbf{m}_i^0 &= \mathbf{p}_i^0 \\ \mathbf{d}_i &= 0.95 \\ \mathbf{b}_{ij} &= \text{body}(j) \cdot \langle 0 < \|\mathbf{p}_j^0 - \mathbf{p}_i^0\| < 2.5 \rangle \\ \mathbf{c}_{ij} &= \text{road}(j) \\ R_i(\mathbf{p}, \mathbf{v}, \mathbf{m}) &= 0.5(\mathbf{p}_\alpha - \mathbf{p}_i) \\ &\quad + 0.2\beta \langle i = 60 \rangle (\mathbf{p}_i - \mathbf{p}_{61}) + \gamma(\mathbf{m}_i - \mathbf{p}_i) \\ M_i(\mathbf{p}, \mathbf{m}) &= \gamma\mathbf{m}_i + \beta\mathbf{p}_i \end{aligned}$$

### 4 ROLLER

Roller is a system composed of an axle and two wheels. A simpler version of Roller was presented earlier in (Holopainen and Rönkkö, 2006).

The control technique comprises of an observation technique and a steering technique. The observation technique uses four beams, as shown in Figure 2, to detect if there are road particles ahead. Two front beams observe the ground directly at the front of the wheels. Two side beams observe the ground slightly outwards from the front beams.

Based on the observations, the steering technique steers Roller by rotating one or both of the wheels. When both of the wheels rotate to the same direction, Roller moves to that direction. If only one wheel rotates, Roller turns. Roller never rotates wheels to opposite directions, as it results in less controllable motion.

Roller has five steering decisions, shown in Table 1. The decision is based on the beam status. In the table, the numbers show if the wheels are rotated forwards (1), backwards (-1), or freely (0).

Table 1: Decision table for goal directions.

Beams missing the road	Goal direction	
none	$\omega_l = 1$	$\omega_r = 1$
only left side beam	$\omega_l = 0$	$\omega_r = -1$
only right side beam	$\omega_l = -1$	$\omega_r = 0$
both left and right side beams	$\omega_l = -1$	$\omega_r = -1$
left or right front beam	$\omega_l = -1$	$\omega_r = -1$

For identifying the component of Roller's particle  $i$ , we use the functions:

$$\begin{aligned} \text{wheel}(i) &= \langle 0 \leq i < 40 \rangle \\ \text{axle}(i) &= \langle 40 \leq i < 79 \rangle \\ \text{road}(i) &= \langle 79 \leq i \rangle \end{aligned}$$

As Roller does not use memory, the model for an axle particle  $i$ , with respect to any other particle  $j$ , is:

$$\begin{aligned} \mathbf{p}_i^0 &= ((i - 40)/3 - 7, 6, 1.5) \\ \mathbf{d}_i &= 0.999 \\ \mathbf{b}_{ij} &= (1 - \text{road}(j)) \langle i \neq j \rangle \\ \mathbf{c}_{ij} &= \text{road}(j) \\ R_i(\mathbf{p}, \mathbf{v}, \mathbf{m}) &= (0, -0.32, 0) \end{aligned}$$

Consider a wheel particle  $i$  and any particle  $j$ . Also, for  $i$ , let  $\alpha = i \mathbf{div} 20$ ,  $\beta = j \mathbf{div} 20$ , and  $\gamma = i \mathbf{mod} 20$ . Then, the model is:

$$\begin{aligned} \mathbf{p}_i^0 &= 2.1(5\alpha, \cos(0.2\pi\gamma), \sin(0.2\pi\gamma)) + (-6, 6, 1.5) \\ \mathbf{d}_i &= 0.999 \\ \mathbf{b}_{ij} &= \text{wheel}(j) \langle \alpha = \beta \rangle \langle i \neq j \rangle + \text{axle}(j) \\ \mathbf{c}_{ij} &= \text{road}(j) \end{aligned}$$

As mentioned earlier, the *observation technique* uses four beams. Formally, all the beams are identified by an origin,  $\vec{o}$ , and a directional vector,  $\vec{r}$ . Let  $\text{norm}(v)$  denote the normalized vector of  $v$ , that is  $\text{norm}(v) = \frac{v}{\|v\|}$ . Let  $\vec{a}$  be an axle vector, that is,  $\vec{a} = \mathbf{p}_{78} - \mathbf{p}_{40}$ , and let  $\vec{y}$  be  $\vec{y} = (0, 20, 0)$ . Then, the axle's normal is  $\vec{n} = \text{norm}(\vec{a} \times \vec{y})$ , and a planar normal is  $\vec{\tilde{n}} = (3\vec{n}_1, \vec{n}_2, 3\vec{n}_3)$ . Now, the four beams are defined using cross products:

$$\begin{aligned} \text{left front beam: } & \vec{o} = \mathbf{p}_{40} \\ & \vec{r} = (9\vec{n}_1, 20, 9\vec{n}_3) \times \vec{a} \\ \text{left side beam: } & \vec{o} = \mathbf{p}_{40} \\ & \vec{r} = (7\vec{n}_1, 20, 7\vec{n}_3) \times (\vec{a} + \vec{n}) \\ \text{right front beam: } & \vec{o} = \mathbf{p}_{78} \\ & \vec{r} = (9\vec{n}_1, 20, 9\vec{n}_3) \times \vec{a} \\ \text{right side beam: } & \vec{o} = \mathbf{p}_{78} \\ & \vec{r} = (7\vec{n}_1, 20, 7\vec{n}_3) \times (\vec{a} - \vec{n}) \end{aligned}$$

Consider a beam  $\vec{r}$  originating from  $\vec{o}$ , and a vector  $\vec{p}$  capturing the difference from  $\vec{o}$  to a (road) particle  $i$ , that is,  $\vec{p} = \mathbf{p}_i - \vec{o}$ . Then, the beam hits the particle  $i$ , if  $\|\vec{p} - (\frac{\vec{p} \cdot \vec{r}}{\|\vec{r}\|} \frac{\vec{r}}{\|\vec{r}\|})\| < \sqrt{1.5}$  holds.

The *steering technique* is captured by the reaction function for the wheel particles. The reaction functions are actually composed of three forces: gravity, friction, and steering forces. Formally, the reaction function for the left wheel particle  $i$  is stated using a control force variable  $\theta_i$  as the sum of the three forces:

$$\begin{aligned} R_i(\mathbf{p}, \mathbf{v}, \mathbf{m}) &= (0, -0.32, 0) \\ &\quad - 0.02 \langle \exists k > 79 : \|\mathbf{p}_k - \mathbf{p}_i\| < 2 \rangle \mathbf{v}_i \\ &\quad + \theta_i \text{norm}((\mathbf{p}_i - \mathbf{p}_{40}) \times (\mathbf{p}_{45} - \mathbf{p}_{40})) \end{aligned}$$

Here,  $\theta_i$  captures a bounded control force with respect to a desired momentum  $\phi_i$ :

$$\theta_i = \max(-10, \min(\phi_i, 10))$$

The desired momentum,  $\phi_i$ , is computed with respect to the goal direction,  $\omega_l$ , and the current direction,  $\zeta$ , of the left wheel based on the current momentum as:

$$\phi_i = -20\omega_l + \langle \omega_l \neq 0 \rangle \frac{\zeta}{20} \sum_{k=0}^{19} \|\mathbf{v}_{40} - \mathbf{v}_k\|$$

The goal direction,  $\omega_l$ , is obtained from Table 1. The current direction, in turn, is computed by comparing the velocities of the wheel particles headed forward,  $\sigma(k)$ , to all of the wheel particles:

$$\zeta = \begin{cases} 1, & \sum_{k=0}^{19} (2\sigma(k) - 1) \mathbf{v}_{(i,2)} < 0 \\ -1, & \text{else} \end{cases}$$

Here, the function  $\sigma(k)$  is computed with respect to the topmost wheel particle,  $\tau$ , for which  $\max_{k=0}^{19} (\mathbf{p}_{(k,2)}) = \mathbf{p}_{(\tau,2)}$ , and to the lowest wheel particle,  $\beta$ , for which  $\min_{k=0}^{19} (\mathbf{p}_{(k,2)}) = \mathbf{p}_{(\beta,2)}$ , as:

$$\begin{aligned} \sigma(k) &= (\langle \tau > \beta \wedge k > \tau \rangle + \langle \tau > \beta \wedge k < \beta \rangle) \\ &\quad \cdot \langle \tau < \beta \wedge \tau < k \wedge k < \beta \rangle \end{aligned}$$

The steering for the right wheel is analogous to the steering of the left wheel presented above.

## 5 MOTION ON ROUGH ROAD

We study the motion of Inchworm and Roller on Rough Road. Rough Road is a narrow uneven path, having also variation in altitude. Rough Road consists of 1800 immobile particles. Thus, the shape is exhaustively captured by the initial positions. Technically, Rough Road consists of 60 circles, each having 30 particles, as observable in Figure 3. For Rough Road, we identify the a road particle  $i$  with  $\text{road}(i) = \langle 100 \leq i < 1900 \rangle$ . Let  $i$  be a road particle, that is  $\text{road}(i) = 1$ . Also, let  $\alpha = (i - 100) \mathbf{div} 30$ ,  $\beta = (i - 100) \mathbf{mod} 30$ , and  $\gamma = \frac{1}{30}\pi$ . Then, the initial positions are captured:

$$\begin{aligned} \mathbf{p}_i^0 &= (8, 0.5(\alpha \mathbf{mod} 2), 8) \\ &\quad + (\alpha > 30) (0, \frac{1}{6}\alpha, 0) + (\alpha \leq 30) (0, (11 - \frac{1}{5}\alpha), 0) \\ &\quad + 30(\cos(0.5\alpha\gamma), 0, \sin(\alpha\gamma)) + 8(\cos(2\beta\gamma), 0, \sin(2\beta\gamma)) \end{aligned}$$

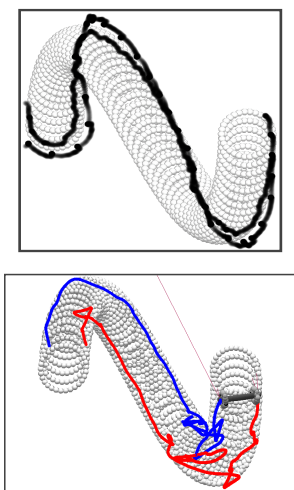


Figure 3: Inchworm (top) and Roller (bottom) on Rough Road. Travelling direction is from left to right.

As the easiest road for both entities is a flat straight road, Rough Road offers many challenges; the shape varies in all dimensions and the surface is bumpy.

The motion of Inchworm is orientation independent, and it makes no predictions of the road ahead. It has a tendency to go straight forward and follow the edges of the road.

Road altitude variations are problematic for Roller. As the observation beams are directed to a constant downward angle, Roller has tendency to understeer and oversteer in downhill and uphill. Roller tries to make predictions of the road ahead and keep both wheels in a safe distance from the edges. In this respect, Rough Road is wide enough. Roller is successful in the first curve, and the problems in the second curve are the result of the rough surface.

Contrary to Inchworm that grips tightly to the road surface and slides slowly along the surface, only the undermost wheel particles of Roller touch the road. Therefore, Roller has problems with bumpy road surface especially in the inner corner of the second curve, where the road elevates sharply.

Figure 3 shows that Roller has a more mechanistic motion than Inchworm. Also, Roller finds its way forward by trial and error, whereas Inchworm edges forward smoothly but stubbornly.

## 6 DISCUSSION

In this paper we studied the use of a specific particle system in motion simulation. The novelty of the particle system is that, with it, everything is formalized systematically and uniformly with matrices.

The results of this study were twofold: First, we demonstrated the applicability of the particle system in simulating plausible motion dynamics. Secondly, we examined the reusability of components that were formalized using the particle system. In this paper, we demonstrated the behavior of Inchworm and Roller. They displayed very different motion dynamics.

The results of this paper complement our earlier results (Holopainen and Rönkkö, 2006; Rönkkö, 2006b). In particular, the reusability results encourage us to find out what kind of interactive tools and techniques could be used for shaping and formalizing particle system components.

We used Atoms (Rönkkö, 2006a) for computing the simulations in this paper. Currently, Atoms is not adequate for running large models. Thus, an important topic for future research is to find out how to improve the performance of Atoms.

## ACKNOWLEDGEMENTS

We wish to thank Vivian Michael Paganuzzi, as well as members of Garry Wong's laboratory.

## REFERENCES

- Eberhardt, B., Weber, A., and Strasser, W. (1996). A fast, flexible, particle-system model for cloth draping. *Computer Graphics and Applications*, 16(5):52–59.
- Holopainen, J. and Rönkkö, M. (2006). Embedding control into a particle system: a case study. In *Proceedings of the Ninth IASTED International Conference on Intelligent Systems and Control*, pages 92–97. Acta Press, Anaheim.
- Miller, G. (1988). Motion dynamics of snakes and worms. *Computer Graphics*, 22:169–178.
- Rönkkö, M. (2006a). Atoms: software development kit. Department of Computer Science, University of Kuopio. <http://www.cs.uku.fi/research/GPL/atoms.tar>.
- Rönkkö, M. (2006b). Hybrid systems: Modeling and analysis using emergent dynamics. *Accepted to Nonlinear Analysis: Theory, Methods & Applications*.
- Wejchert, J. and Haumann, D. (1991). Animation aerodynamics. In *Proceedings of the 18th Annual Conference on Computer Graphics and Interactive Techniques*, pages 19–22. ACM Press, New York.
- Witkin, A. (1997). *Physically Based Modeling: Principles and Practice*. Online Siggraph '97 Course notes.
- Wojtan, C., Mucha, P., and Turk, G. (2006). Keyframe control of complex particle systems using the adjoint method. In *Proceedings of ACM SIGGRAPH/Eurographics Symposium on Computer Animation*, pages 15–23.