TRACKING OF A UNICYCLE-TYPE MOBILE ROBOT USING INTEGRAL SLIDING MODE CONTROL

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Abstract: This paper deals with the tracking control for a dynamic model of a wheeled mobile robot in the presence of some perturbations. The control strategy is based on integral sliding mode. Simulations illustrate the results on the studied mobile robot.

1 INTRODUCTION

One of the motivations for tackling the tracking of nonholonomic systems is the large number of applications mobile robots (Laumond, 1998). One difficulty for motion planning and control of a car-like robot arises from the so-called nonholonomic constraints imposed by the rolling wheels.
Obstacles to the tracking of such systems are the uncontrollability of their linear approximation and the fact that the Brockett's necessary condition to the existence of a smooth time-invariant state feedback is not satisfied (Brockett, 1983). To overcome those difficulties, various methods have been investigated: homogeneous and time-varying feedbacks (Pomet, 1992; Samson, 1995), sinusoidal and polynomial controls (Murray and Sastry, 1993), piecewise controls (Hespanha and Morse, 1999; Monaco and Normand-Cyrot, 1992), flatness (Fliess et al., 1995), backstepping approaches (Jiang, 2000) or discontinuous controls (Floquet et al., 2003).

In this paper, it is aimed to design a control law for the unicycle-type mobile robot which:

- solves the disturbance rejection problem for bounded matching perturbations and some unmatched disturbance from the initial time instance,
- is a good compromise between performance and robustness,
- takes into account the actuator dynamics.

This objective will be achieved by using integral sliding mode control law. Sliding mode control (Edwards and Spurgeon, 1998) is a powerful method to control nonlinear dynamic systems operating under uncertainty conditions. A drawback of such a control is that the trajectory of the designed solution is not robust on a time interval preceding the sliding motion. In (Utkin and Shi, 1996; Poznyak et al., 2004; Cao and Xu, 2004), a new sliding mode design concept, namely integral sliding mode (ISM), without any reaching phase was proposed. Thus, the robustness can be guaranteed throughout an entire response of the system starting from the initial time instance.
Here, the proposed controller is based on the integral sliding mode in order to solve the tracking problem in presence of matching and some unmatching perturbations.

The outline of this paper is as follows. Section II formulates the tracking problem. Then, the use of integral sliding mode, in section III, enables to solve the problem of tracking the reference trajectory in spite of perturbations. Finally, in section IV, numerical examples illustrate the effectiveness of the proposed controller.

2 PROBLEM STATEMENT

In this paper, we consider the unicycle-type robot which behavior can be described by the following sys-

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tem (see (de Wit and Sordalen, 1992) for details):

$$\begin{cases} \dot{x} = u\cos(\theta) + p_1(X) \\ \dot{y} = u\sin(\theta) + p_2(X) \\ \dot{\theta} = v + p_3(X) \end{cases}, \tag{1}$$

where $X = [x, y, \theta]$ is the state, x and y are the coordinates of the center gravity of the robot, θ is the orientation of the car with respect to the x-axis. $p_1(X)$, $p_2(X)$ and $p_3(X)$ are some additive perturbations due to parameter variations, unmodeled dynamics or external disturbances assumed to be smooth enough and thus bounded over some compact set. u and v refer respectively to the applied linear and the angular velocities (see Fig. 1).

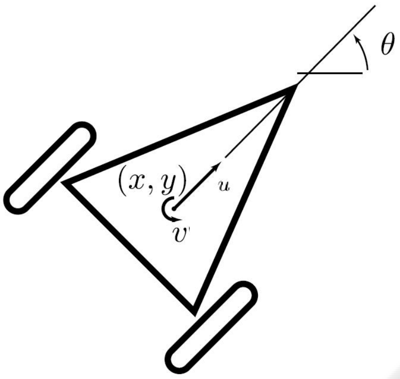

Figure 1: Unicycle robot kinematic

Discontinuous control laws have been developed in the literature in order to stabilize system (1) (Astolfi, 1996; Floquet et al., 2003). The main criticism when applying such strategies to a mobile robot would be the action of a discontinuous control directly on the mechanical part of the system (namely u and v). The purpose of the paper is to define a sliding mode control acting on the electrical parts of the system (which is more realistic since power converters are discontinuous actuators by nature). Taking into account the actuator dynamics remains to include some dynamical extensions (cascaded integrators) in the system (1):

$$\begin{cases} \dot{x} = u\cos\theta + \pi_1 \\ \dot{y} = u\sin\theta + \pi_2 \\ \dot{\theta} = v + \pi_3 \\ \dot{u} = \frac{F}{m} - \alpha_1 u + \pi_4 \\ \dot{v} = \frac{\tau}{J} - \alpha_2 v + \pi_5 \\ \dot{F} = P + \pi_6 \end{cases}, \tag{2}$$

where m is the mass, J is the inertia, α_1, α_2 are positive scalars coming from the actuator dynamics, F is the force, $X = [x, y, \theta, u, v, F]^T$ and $U = [P, \tau]^T$ are respectively the new state and the control input. $\pi = [\pi_1, \ldots, \pi_6]^T$ is an additive perturbation (π_1, π_2, π_3 and π_4 are sufficiently smooth function of time).

Assume that the desired, feasible trajectory $X_r = [x_r, y_r, \theta_r, u_r, v_r, F_r]^T$ satisfies the following dynamics where $U_r = [P_r, \tau_r]^T$ is the reference input.

$$\begin{cases} \dot{x}_r = u_r\cos\theta_r \\ \dot{y}_r = u_r\sin\theta_r \\ \dot{\theta}_r = v_r \\ \dot{u}_r = \frac{F_r}{m} - \alpha_1 u_r \\ \dot{v}_r = \frac{\tau_r}{J} - \alpha_2 v_r \\ \dot{F}_r = P_r \end{cases}. \tag{3}$$

The output tracking error is denoted by:

$$e = [e_1, e_2]^T = [x - x_r, y - y_r]^T. \tag{4}$$

Our control purpose is to design an appropriate controller such that the vehicle (2) is forced to asymptotically track the desired trajectory (3) from some initial tracking errors in spite of the perturbations. In fact, the goal is to asymptotically stabilize the tracking errors e_1 and e_2 and their time derivatives to zero.

3 CONTROLLER DESIGN FOR TRAJECTORY TRACKING

Sliding mode control, which consists in constraining the motion of the system along manifolds of reduced dimensionality in the state space, is quite popular in nonlinear control systems community. One can refer to (Perruquetti and Barbot, 2002) for further details about this theory. Its robustness properties with respect to matching perturbations and its discontinuous character also motivated the authors to consider such an approach for the tracking of the nonholonomic system (2).

Let us differentiate the tracking errors e once with respect to time:

$$\dot{e} = \begin{bmatrix} u\cos\theta - \dot{x}_r \\ u\sin\theta - \dot{y}_r \end{bmatrix} + \begin{bmatrix} \pi_1 \\ \pi_2 \end{bmatrix} \tag{5}$$

Since neither P nor τ appears in (5), one can differentiate further with respect to time until they appear:

$$\ddot{e} = \begin{bmatrix} (\frac{F}{m} - \alpha_1 u)\cos\theta \\ -uv\sin\theta - \ddot{x}_r \\ \\ (\frac{F}{m} - \alpha_1 u)\sin\theta \\ +uv\cos\theta - \ddot{y}_r \end{bmatrix} + \begin{bmatrix} \dot{\pi}_1 + \pi_4\cos\theta \\ -u\pi_3\sin\theta \\ \\ \dot{\pi}_2 + \pi_4\sin\theta \\ +u\pi_3\cos\theta \end{bmatrix} \tag{6}$$

and

$$e^{(3)} = \xi(X)\begin{bmatrix} P \\ \tau \end{bmatrix} + \begin{bmatrix} \phi_1(X) \\ \phi_2(X) \end{bmatrix} + \begin{bmatrix} K_{1,3}(X) \\ K_{2,3}(X) \end{bmatrix} \tag{7}$$

with

$$\xi(X) = \begin{bmatrix} \frac{\cos\theta}{m} & -\frac{u\sin\theta}{J} \\ \frac{\sin\theta}{m} & \frac{u\cos\theta}{m} \end{bmatrix},$$

$$\begin{bmatrix}\phi_1(X)\\ \phi_2(X)\end{bmatrix} = \begin{bmatrix} -x_r^{(3)} - \alpha_1(\frac{F}{m} - \alpha_1 u)\cos\theta \\ -2v(\frac{F}{m} - \alpha_1 u)\sin\theta \\ +u\alpha_2 v\sin\theta - uv^2\cos\theta \\ \\ -y_r^{(3)} - \alpha_1(\frac{F}{m} - \alpha_1 u)\sin\theta \\ +2v(\frac{F}{m} - \alpha_1 u)\cos\theta \\ -u\alpha_2 v\cos\theta - uv^2\sin\theta \end{bmatrix},$$

$$\begin{bmatrix}K_{1,3}(X)\\ K_{2,3}(X)\end{bmatrix} = \begin{bmatrix} (-\alpha_1\pi_4 + \dot{\pi}_4)\cos\theta - 2v\pi_4\sin\theta \\ -2\pi_3(\frac{F}{m} - \alpha_1 u + \pi_4)\sin\theta \\ -u(\pi_5 + \dot{\pi}_3)\sin\theta - 2uv\pi_3\cos\theta \\ (\pi_6 - u{\pi_3}^2)\cos\theta + \ddot{\pi}_1 \\ \\ (-\alpha_1\pi_4 + \dot{\pi}_4)\sin\theta + 2v\pi_4\cos\theta \\ +2\pi_3(\frac{F}{m} - \alpha_1 u + \pi_4)\cos\theta \\ +u(\pi_5 + \dot{\pi}_3)\cos\theta - 2uv\pi_3\sin\theta \\ (\pi_6 - u{\pi_3}^2)\sin\theta + \ddot{\pi}_2 \end{bmatrix}.$$

The decoupling matrix is $\xi(X)$ and

$$det(\xi(X)) = \frac{u}{Jm}.$$

So, a singularity appears at $u = 0$. Therefore, the linear velocity must be ensured to be different from zero.
From (7), let us design

$$\begin{bmatrix}u_1\\ u_2\end{bmatrix} = \xi(X)\begin{bmatrix}P\\ \tau\end{bmatrix} + \begin{bmatrix}\phi_1(X)\\ \phi_2(X)\end{bmatrix} \tag{8}$$

From (5), (6), (7) and (8), two linear sub-systems are obtained, for $i \in \{1, 2\}$:

$$\begin{bmatrix}\dot{e}_i\\ \ddot{e}_i\\ e_i^{(3)}\end{bmatrix} = \begin{bmatrix}0&1&0\\0&0&1\\0&0&0\end{bmatrix}\begin{bmatrix}e_i\\ \dot{e}_i\\ \ddot{e}_i\end{bmatrix} + \begin{bmatrix}0\\0\\1\end{bmatrix}u_i + \begin{bmatrix}K_{i,1}\\ K_{i,2}\\ K_{i,3}\end{bmatrix} \tag{9}$$

where $K_i = [K_{i,1}, K_{i,2}, K_{i,3}]^T$ are external disturbances resulting from (5), (6) and (7):

$$\begin{bmatrix}K_{1,1}\\ K_{2,1}\end{bmatrix} = \begin{bmatrix}\pi_1\\ \pi_2\end{bmatrix},$$

$$\begin{bmatrix}K_{1,2}\\ K_{2,2}\end{bmatrix} = \begin{bmatrix}\dot{\pi}_1 + \pi_4\cos\theta - u\pi_3\sin\theta\\ \dot{\pi}_2 + \pi_4\sin\theta + u\pi_3\cos\theta\end{bmatrix}.$$

Using $E_i = [e_i, \dot{e}_i, \ddot{e}_i]^T$, system (9) can be described in a compact form:

$$\dot{E}_i = AE_i + Bu_i + K_i, \tag{10}$$

The perturbations K_i, $i \in \{1, 2\}$, are split as follows:

$$K_i = h_i + k_i$$

where h_i is smooth uncertainty representing the perturbations which satisfy the so called ”standard matching condition”, that is to say $h_i \in span(B)$, i.e. $h_i = Bq_i$ and k_i represents the unmatched part.

Assumption: q_i and k_i are bounded by known nonlinear functions as follows:

$$\begin{aligned} \parallel q_i \parallel &\leq \varrho_i(E_i), \\ \parallel k_i \parallel &\leq \rho_i(E_i). \end{aligned}$$

For system (10), the control law is defined as follows

$$u_i = u_{i,0} + u_{i,1}. \tag{11}$$

where $u_{i,0}$ is the ideal control and $u_{i,1}$ represents the ISM part which will be designed to be discontinuous in order to reject the perturbation.

The first part of the control design is to find a control law $u_{i,0}$ such that the ideal closed-loop system $\dot{E}_i = AE_i + Bu_{i,0}$ is globally asymptotically stable. The following control enables to stabilize the tracking errors:

$$u_{i,0} = -\beta_i E_i,\ \beta_i = [\beta_{i,1}, \beta_{i,2}, \beta_{i,3}] \tag{12}$$

The constant real coefficients $\{\beta_{i,1}, \beta_{i,2}, \beta_{i,3}\}$ are chosen appropriately such that the ideal system is globally stable.

However, with the control law (12), the system is not robust with respect to the perturbations K_i (see Section 4, Fig. 3). So, to this first controller, a discontinuous term is added based on ISM to ensure the desired performance despite the disturbances.

Define the sliding function σ_i as:

$$\sigma_i = \sigma_{i,0} + z_i \tag{13}$$

The first term $\sigma_{i,0}$ is a linear sliding surface designed as follows:

$$\sigma_{i,0} = D_i E_i$$

where $D_i \in \mathbb{R}^{1\times 3}$ is constant and is selected such that the matrix $D_i B$ is nonsingular (for instance, $D_i = B^+ = [0, 0, 1]$ where B^+ is the pseudo-inverse of B).

The second term z induces the integral term. The main idea of ISM (Utkin and Shi, 1996) is to eliminate the reaching phase by enforcing sliding mode throughout the entire system response. To achieve the stabilization of system (10), the equivalent control of

$u_{i,1}$ (denoted $u_{i,1eq}$), which describes the system trajectory when sliding mode takes place in (13), should fulfill:

$$u_{i,1eq} = -q_i - [D_i B]^{-1} D_i k_i. \qquad (14)$$

Furthermore, in sliding mode, along the system trajectories, one should have:

$$\begin{aligned}\dot{\sigma}_i &= D_i \dot{E}_i + \dot{z}_i \\ &= D_i \left(AE_i + Bu_i + K_i\right) + \dot{z}_i \\ &= D_i (AE_i + Bu_{i,0} + Bq_i + k_i) \\ &\quad + D_i B u_{i,1} + \dot{z}_i \\ &= 0\end{aligned} \qquad (15)$$

Conditions (14) and (15) holds if:

$$\begin{aligned}\dot{z}_i &= -D_i(AE_i + Bu_{i,0}) \\ z_i(0) &= -\sigma_{i,0}(0)\end{aligned} \qquad (16)$$

That is to say:

$$z_i = -D_i E_i(0) - \int_0^t (D_i(AE_i + Bu_{i,0}))ds$$

where the initial condition $z(0)$ is determined from $\sigma_i(0) = 0$. Hence, the sliding mode occurs from the initial time instance.

The control $u_{i,1}$ in (11) is defined to enforce sliding mode along the manifold (13) and is of the following form:

$$u_{i,1} = -M_i(E_i)\,\mathrm{sign}\,(D_i B \sigma_i) \qquad (17)$$

where the switching gain satisfies

$$M_i(E_i) > \varrho_i(E_i) + \|[D_i B]^{-1} D_i\| \rho_i(E_i). \qquad (18)$$

Proposition: The controller defined in (11), (12), (17) solves the tracking problem if the unmatched perturbation satisfy, for $i = 1, 2$:

$$\|I - B[D_i B]^{-1} D_i\| \rho_i(E_i) < \lambda_{i,min} \|E_i\| \qquad (19)$$

where $\lambda_{i,min}$ is the lowest eigenvalue of the matrix $A - B\beta_i$.

Proof: Let us choose the following Lyapunov function

$$V_i = \frac{1}{2}\sigma_i^2$$

From the choice of the switching gain (18), the time derivative of this function can be expressed as:

$$\begin{aligned}\dot{V}_i &= \sigma_i(D_i(AE_i + Bu_i + h_i + k_i) \\ &\quad - D_i(AE_i + Bu_{i,0})) \\ &= \sigma_i D_i (Bu_{i,1} + Bq_i + k_i) \\ &= \sigma_i D_i B(u_{i,1} + q_i + [D_i B]^{-1} D_i k_i) \\ &\leq -\eta_i |D_i B \sigma_i|, \ \eta_i \in \mathbb{R}^{+*} \\ &\leq 0\end{aligned}$$

Thus, the trajectory evolves on the manifold $\sigma_i = 0$ from $t = 0$ and remains there in spite of the perturbations.

Since, in sliding mode, (14) is satisfied, the closed loop dynamics becomes:

$$\dot{E}_i^s = AE_i^s + Bu_{i,0} + \{I - B[D_i B]^{-1} D_i\} k_i$$

where the subscript s denotes the state vector in sliding mode.

Choosing a Lyapunov function as

$$V_s = \sum_{i=1}^{2} \frac{(E_i^s)^T E_i^s}{2},$$

one gets,

$$\begin{aligned}\dot{V}_s &= \sum_{i=1}^{2} (E_i^s)^T (AE_i^s + Bu_{i,0}) \\ &\quad + \sum_{i=1}^{2} (E_i^s)^T (I - B[D_i B]^{-1} D_i) k_i \\ &\leq -\sum_{i=1}^{2} \lambda_{i,min} \|E_i^s\|^2 \\ &\quad + \sum_{i=1}^{2} \|E_i^s\| \|I - B[D_i B]^{-1} D_i\| \rho_i\end{aligned}$$

According to (Qu, 1998), the tracking errors is globally asymptotically stable if the unmatched perturbation satisfy (19).

Remark: As the system (2) without perturbation (π_1 and π_2 are supposed to be vanishing) is flat (Fliess et al., 1995), the tracking errors in orientation will converge to zero. Indeed,

$$\theta - \theta_r = \mathrm{atan}\left(\frac{\dot{y}}{\dot{x}}\right) - \mathrm{atan}\left(\frac{\dot{y}_r}{\dot{x}_r}\right)$$

tends to zero when $\dot{x} \to \dot{x}_r$ and $\dot{y} \to \dot{y}_r$.

4 SIMULATION RESULTS

4.1 Tracking problem

In this simulation, the desired trajectory is circular. The mobile robot is required to track, from an initial point $X(0)$, the circular trajectory:

$$\begin{aligned}x_r(t) &= R\cos(at) \\ y_r(t) &= R\sin(at)\end{aligned}$$

with $R = 20$ and $a = 0.01\pi$ which is represented by the dashed line in the following figures. The reference orientation can be deduced from $\dot{x}_r$ and $\dot{y}_r$, i.e.

$$\dot{\theta}_r = a.$$

The initial state $X(0)$ for the vehicle is:

$$x(0) = 22,\ y(0) = -1,\ \theta(0) = \frac{\pi}{2} + 0.01,$$
$$u(0) = 0.2,\ v(0) = 0,\ F(0) = 0.$$

4.1.1 Simulation without uncertainties

Using the control inputs defined in (12), the tracking errors tend to zero. The tracking problem was simulated with the following design parameters: for $i \in \{1, 2\}$, $\beta_{i,1} = 0.024$, $\beta_{i,2} = 0.26$ and $\beta_{i,3} = 0.9$. $[-0.2, -0.3, -0.4]^T$ are the eigenvalues of the dynamics of the closed-loop system. The performances without any perturbation are depicted in Fig. 2.

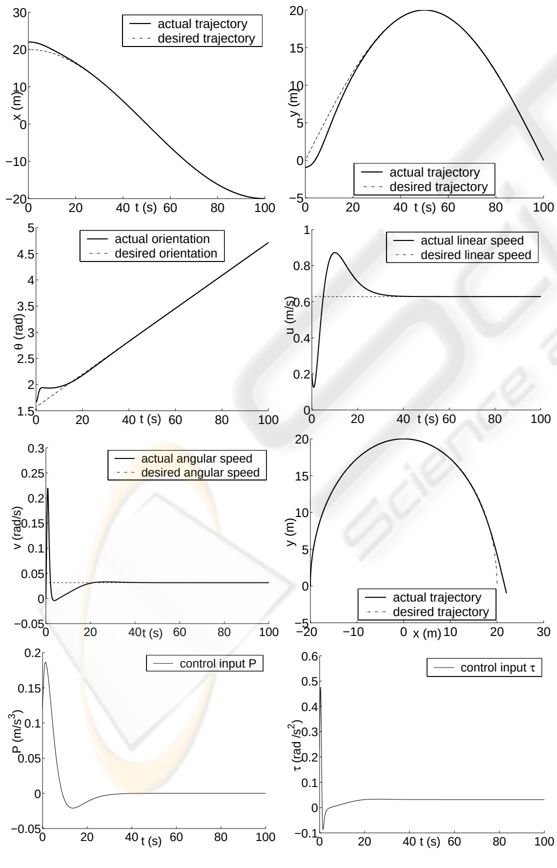

Figure 2: Evolution of variables and inputs without noise

4.1.2 Simulation with uncertainties

Nevertheless, when noise is added, the errors are not stabilized (Fig. 3). Therefore, control (12) is not robust with respect to the perturbations. In order to reject noise, the control law is divided into two parts (11): the ideal control (12) and the discontinuous law described by (17). Perturbations π_5 and π_6 are noise of mean 0.1 and of variance 0.1. The matching noise is bounded and the disturbances $k_i = [K_{i,1}, K_{i,2}, 0]^T$ is the unmatched perturbation which satisfy (19) (i.e. $\rho_i < 0.2\|E_i\|$). In order to avoid the chattering phenomenon, the function signum in (17) is replaced by $\frac{2}{\pi}\,\mathrm{atan}\,(\epsilon\sigma_i)$ with $\epsilon \gg 1$. Using the control law (11) with $D_i = [0, 0, 1]$, the tracking errors tend to zero (Fig. 5) and the system (2) is robust with respect to perturbation from the initial time (ie. sliding function σ_i represented in Fig. 4 is equal to zero from the initial time).

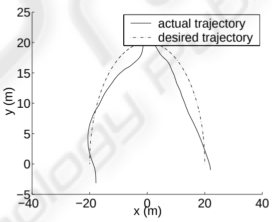

Figure 3: Evolution in the phase plane (x, y) with uncertainties without ISM

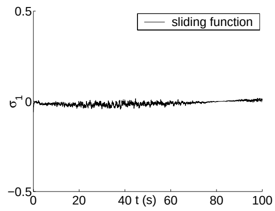

Figure 4: Evolution of the sliding function σ_1

5 CONCLUSION

The problem of robust control design has been considered for the tracking of a unicycle robot system with bounded disturbances and uncertainties. The proposed controller includes terms corresponding to an integral sliding mode component and enables to obtain continuous velocity and acceleration inputs for some practical applications on mechanical systems. The integral sliding mode component compensates

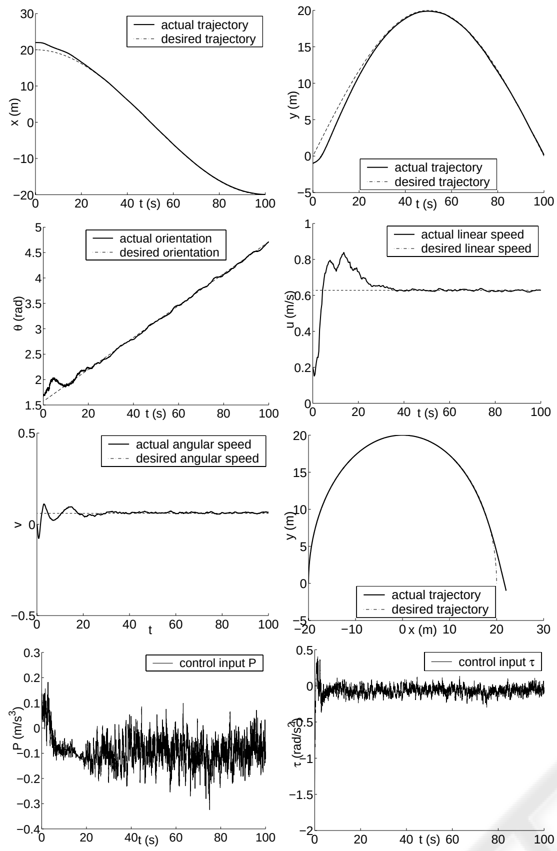

Figure 5: Evolution of variables and inputs without noise using the proposed controller

for the matching perturbation beginning from the initial time and for some unmatched disturbances. Simulations on a unicycle-type mobile robot illustrated the performance of the controller.

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