# DEAD RECKONING FOR MOBILE ROBOTS USING TWO OPTICAL MICE 

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#### Abstract

In this paper, we present a dead reckoning procedure to support reliable odometry on mobile robot. It is based on a pair of optical mice rigidly connected to the robot body. The main advantages are that: 1) the measurement given by the mice is not subject to slipping, since they are independent from the traction wheels, nor to crawling, since they measure displacements in any direction, 2) this localization system is independent from the kinematics of the robot, 3 ) it is a low-cost solution. We present the mathematical model of the sensor, its implementation, and some empirical evaluations.


## 1 INTRODUCTION

Since the very beginning of mobile robotics, dead reckoning was used to estimate the robot pose, i.e., its position and its orientation with respect to a global reference system placed in the environment. Dead reckoning is a navigation method based on measurements of distance traveled from a known point used to incrementally update the robot pose. This leads to a relative positioning method, which is simple, cheap and easy to accomplish in real-time. The main disadvantage of dead reckoning is its unbounded accumulation of errors.

The majority of the mobile robots use dead reckoning based on odometry in order to perform their navigation tasks (alone or combined with other absolute localization systems (Borenstein and Feng, 1996)). Typically, odometry relies on measures of the space covered by the wheels gathered by encoders which can be placed directly on the wheels or on the engineaxis, and then combined in order to compute robot movement along the x and y -axes and its change of orientation. It is well-known that odometry is subject to:

- systematic errors, caused by factors such as unequal wheel-diameters, imprecisely measured wheel diameters and wheel distance, or an imprecisely measured tread (Borenstein and Feng, 1996);
- non-systematic errors, caused by irregularities of the floor, bumps, cracks or by wheel-slippage.

In this paper, we present a new dead reckoning method which is very robust towards non-systematic errors, since the odometric sensors are not coupled with the driving wheels. It is based on the measures taken by two optical mice fixed on the bottom of the robot. We need to estimate three parameteres ( $\Delta x, \Delta y, \Delta \theta$ ), so we cannot use a single mouse, since it gives only two independent measures. By using two mice we have four measures, even if, since the mice have a fixed position, only three of these are independent and the fourth can be computed from them. We have chosen to use optical mice, instead of classical ones, since they can be used without being in contact with the floor, thus avoiding the problems of keeping the mouse always pressed on the ground, the problems due to friction and those related to dust deposited in the mechanisms of the mouse.

In the following section, we will present the motivations for using this method for dead reckoning. In Section 3, we show the geometrical derivation that allows to compute the robot movement on the basis of the readings of the mice. Section 4 describes the main characteristics of the mice and how they affect the accuracy and the applicability of the system. In Section 5, we report the data related to some experiments in order to show the effectiveness of our approach. Finally, we discuss related works in Section 6 and draw conclusions in Section 7.

## 2 MOTIVATIONS

The classical dead reckoning methods, which use the data measured by encoders on the wheels or on the engine-axis, suffer from two main non-systematic problems: slipping, which occurs when the encoders measure a movement which is larger than the actually performed one (e.g., when the wheels lose the grip with the ground), and crawling, which is related to a robot movement that is not measured by the encoders (e.g., when the robot is pushed by an external force, and the encoders cannot measure the displacement).

Our dead reckoning method, based on mice readings, does not suffer from slipping problems, since the sensors are not bound to any driving wheel. Also the crawling problems are solved, since the mice go on reading even when the robot movement is due to a push, and not to the engines. The only problem that this method can have is related to missed readings due to a floor with a bad surface or when the distance between the mouse and the ground becomes too large (see Section 4).

Another advantage of our approach is that it is independent from the kinematics of the robot, and so we can use the same approach on several different robots. For example, if we use classical systems, dead reckoning with omni-directional robots equipped with omnidirectional wheels may be very difficult both for geometrical and for slipping reasons.

Furthermore, this is a very low-cost system which can be easily interfaced with any platform. In fact, it requires only two optical mice which can be placed in any position under the robot, and can be connected using the USB interface. This allows to build an accurate dead reckoning system, which can be employed on and ported to all the mobile robots which operate in an environment with a ground that allows the mice to measure the movements (indoor environments typically meet this requirement).

## 3 HOW TO COMPUTE THE ROBOT POSE

In this section we present the geometrical derivation that allows to compute the pose of a robot using the readings of two mice placed below it in a fixed position. For sake of ease, we place the mice at a certain distance $D$, so that they are parallel between them and orthogonal w.r.t. their joining line (see Figure 1). We consider their mid-point as the position of the robot and their direction (i.e., their longitudinal axis pointing toward their keys) as its orientation.

Each mouse measures its movement along its horizontal and vertical axes. We hypothesize that, during the sampling period (discussed in Section 4), the robot


Figure 1: The relative positioning of the two mice
moves with constant tangential and rotational speeds. This implies that the robot movement can be approximated by an arc of circumference. So, we have to estimate the 3 parameters that describe the arc of circumference (i.e., the ( $\mathrm{x}, \mathrm{y}$ )-coordinates of the center of the circumference and the arc angle), given the 4 readings taken from the two mice. We call $\bar{x}_{r}$ and $\bar{y}_{r}$ the measures taken by the mouse on the right, while $\bar{x}_{l}$ and $\bar{y}_{l}$ are those taken by the mouse on the left. Actually, we have only 3 independent data; in fact, we have the constraint that the respective position of the two mice cannot change. This means that the mice should read always the same displacement along the line that joins the centers of the two sensors. So, if we place the mice as in Figure 1, we have that the x -values measured by the two mice should be always equal: $\bar{x}_{l}=\bar{x}_{r}$. In this way, we can compute how much the robot pose has changed in terms of $\Delta x, \Delta y$, and $\Delta \theta$.

If the robot makes an arc of circumference, it can be shown that also each mouse will make an arc of circumference, which is characterized by the same center and the same arc angle (but with a different radius). During the sampling time, the angle $\alpha$ between the xaxis of the mouse and the tangent to its trajectory does not change. This implies that, when a mouse moves along an arc of length $l$, it measures always the same values independently from the radius of the arc (see Figure 2). So, considering an arc with an infinite radius (i.e., a segment), we can write the following relations:

$$
\begin{align*}
\bar{x} & =l \cos (\alpha)  \tag{1}\\
\bar{y} & =l \sin (\alpha) . \tag{2}
\end{align*}
$$

From Equations 1 and 2, we can compute both the angle between the x -axis of the mouse and the tangent to the arc:


Figure 2: Two different paths in which the mouse readings are the same


Figure 3: The triangle made up of the joining lines and the two radii

$$
\begin{equation*}
\alpha=\arctan \left(\frac{\bar{y}}{\bar{x}}\right) \tag{3}
\end{equation*}
$$

and the length of the covered arc:

$$
l=\left\{\begin{array}{cl}
|\bar{x}|, & \alpha=0, \pi  \tag{4}\\
\frac{\bar{y}}{\sin \alpha}, & \text { otherwise }
\end{array}\right.
$$

In order to compute the orientation variation we apply the theorem of Carnot to the triangle made by the joining line between the two mice and the two radii between the mice and the center of their arcs (see Figure 3):


Figure 4: The angle arc of each mouse is equal to the change in the orientation of the robot

$$
\begin{equation*}
D^{2}=r_{r}^{2}+r_{l}^{2}-2 \cos (\gamma) r_{r} r_{l} \tag{5}
\end{equation*}
$$

where $r_{r}$ and $r_{l}$ are the radii related to the arc of circumferences described respectively by the mouse on the right and the mouse on the left, while $\gamma$ is the angle between $r_{r}$ and $r_{l}$. It is easy to show that $\gamma$ can be computed by the absolute value of the difference between $\alpha_{l}$ and $\alpha_{r}$ (which can be obtained by the mouse measures using Equation 3): $\gamma=\left|\alpha_{l}-\alpha_{r}\right|$.
The radius $r$ of an arc of circumference can be computed by the ratio between the arc length $l$ and the arc angle $\theta$. In our case, the two mice are associated to arcs under the same angle, which corresponds to the change in the orientation made by the robot, i.e. $\Delta \theta$


Figure 5: The movement made by each mouse
(see Figure 4). It follows that:

$$
\begin{align*}
r_{l} & =\frac{l_{l}}{|\Delta \theta|}  \tag{6}\\
r_{r} & =\frac{l_{r}}{|\Delta \theta|} \tag{7}
\end{align*}
$$

If we substitute Equations 6 and 7 into Equation 5, we can obtain the following expression for the orientation variation:

$$
\begin{equation*}
\Delta \theta=\operatorname{sign}\left(y_{r}-y_{l}\right) \frac{\sqrt{l_{l}^{2}+l_{r}^{2}-2 \cos (\gamma) l_{l} l_{r}}}{D} \tag{8}
\end{equation*}
$$

The movement along the x and y -axes can be derived by considering the new positions reached by the mice (w.r.t. the reference system centered in the old robot position) and then computing the coordinates of their mid-point (see Figure 5). The mouse on the left starts from the point of coordinates $\left(-\frac{D}{2} ; 0\right)$, while the mouse on the right starts from ( $\frac{D}{2} ; 0$ ). The formulas for computing their coordinates at the end of the sampling period are the following:

$$
\begin{align*}
x_{r}^{\prime} & =r_{r}\left(\sin \left(\alpha_{r}+\Delta \theta\right)-\sin \left(\alpha_{r}\right)\right) \operatorname{sign}(\Delta \theta)+\frac{D}{2} \\
y_{r}^{\prime} & =r_{r}\left(\cos \left(\alpha_{r}\right)-\cos \left(\alpha_{r}+\Delta \theta\right)\right) \operatorname{sign}(\Delta \theta) \\
x_{l}^{\prime} & =r_{l}\left(\sin \left(\alpha_{l}+\Delta \theta\right)-\sin \left(\alpha_{l}\right)\right) \operatorname{sign}(\Delta \theta)-\frac{D}{2}  \tag{11}\\
y_{l}^{\prime} & =r_{l}\left(\cos \left(\alpha_{l}\right)-\cos \left(\alpha_{l}+\Delta \theta\right)\right) \operatorname{sign}(\Delta \theta) \tag{12}
\end{align*}
$$

From the mice positions, we can compute the movement executed by the robot during the sampling time with respect to the reference system centered in the old pose using the following formulas:

$$
\begin{align*}
\Delta x & =\frac{x_{r}^{\prime}+x_{l}^{\prime}}{2}  \tag{13}\\
\Delta y & =\frac{y_{r}^{\prime}+y_{l}^{\prime}}{2} \tag{14}
\end{align*}
$$

The absolute coordinates of the robot pose at time $t+1\left(X_{t+1}, Y_{t+1}, \Theta_{t+1}\right)$ can be compute by knowing the absolute coordinates at time $t$ and the relative movement carried out during the period $(t ; t+1]$ ( $\Delta x, \Delta y, \Delta \theta$ ) through these equations:

$$
\begin{align*}
X_{t+1} & =X_{t}+\sqrt{\Delta x^{2}+\Delta y^{2}} \cos \left(\Theta_{t}+\arctan \left(\frac{\Delta y}{\Delta x}\right)\right) \\
Y_{t+1} & =Y_{t}+\sqrt{\Delta x^{2}+\Delta y^{2}} \sin \left(\Theta_{t}+\arctan \left(\frac{\Delta y}{\Delta x}\right)\right) \\
\Theta_{t+1} & =\Theta_{t}+\Delta \theta \tag{17}
\end{align*}
$$



Figure 6: The schematic of an optical mouse

## 4 MICE CHARACTERISTICS AND SYSTEM PERFORMANCES

The performance of the odometry system we have described depends on the characteristics of the solidstate optical mouse sensor used to detect the displacement of the mice. Commercial optical mice use the Agilent ADNS-2051 sensor (Agilent Technologies Semiconductor Products Group, 2001) a low cost integrated device which measures changes in position by optically acquiring sequential surface images (frames hereafter) and determining the direction and magnitude of movement.

The main advantage of this sensor with respect to traditional mechanical systems for movement detection is the absence of moving parts that could be damaged by use or dust. Moreover it is not needed a mechanical coupling of the sensor and the floor thus allowing the detection of movement in case of slipping and on many surfaces, including soft, glassy, and curved pavements.

### 4.1 Sensor Characteristics

The ADNS-2051 sensor is essentially a tiny, highspeed video camera coupled with an image processor, and a quadrature output converter (Agilent Technologies Semiconductor Products Group, 2003). As schematically shown in Figure $6^{1}$, a light-emitting diode (LED) illuminates the surface underneath the sensorreflecting off microscopic textural features in the area. A plastic lens collects the reflected light and forms an image on a sensor. If you were to look at the image, it would be a black-and-white picture of a tiny section of the surface as the ones in Figure 7. The sensor continuously takes pictures as the mouse moves at

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Figure 7: Two simple images taken from the mice. Beside the noise you can notice the displacement of the pixels in the images from right to left and top to bottom.

1500 frames per second or more, fast enough so that sequential pictures overlap. The images are then sent to the optical navigation engine for processing.
The optical navigation engine in the ADNS-2051 identifies texture or other features in the pictures and tracks their motion. Figure 7 illustrates how this is done by showing two images sequentially captured as the mouse was panned to the right and upwards; much of the same "visual material" can be recognized in both frames. Through an Agilent proprietary imageprocessing algorithm, the sensor identifies common features between these two frames and determines the distance between them; this information is then translated into $\Delta x$ and $\Delta y$ values to indicate the sensor displacement.

By looking at the sensor characteristics, available through the data sheet, it is possible to estimate the precision of the measurement and the maximum working speed of the device. The Agilent ADNS2051 sensor is programmable to give mouse builders (this is the primary use of the device) 400 or 800 cpi resolution, a motion rate of 14 inches per second, and frame rates up to 2,300 frames per second. At recommended operating conditions this allows a maximum operating speed of $0.355 \mathrm{~m} / \mathrm{s}$ with a maximum acceleration of $1.47 \mathrm{~m} / \mathrm{s}^{2}$.

These values are mostly due to the mouse sensor (i.e., optical vs. mechanical) and the protocol used to
transmit the data to the computer. According to the original PS/2 protocol, still used in mechanical devices featuring a PS/2 connector, the $\Delta x$ and $\Delta y$ displacements are reported using 9 bits (i.e., 1 byte plus a sign bit) espressing values in the range from -255 to +255 . In these kind of mouse resolution can be set to $1,2,4$, or 8 counts per mm and the sample rate can be $10,20,40,60,80,100$, and 200 samples per second. The maximum speed allowed not to have an overflow of the mouse internal counters is obtained by reducing the resolution and increasing the sample rate. However, in modern mice with optical sensor and USB interface these values are quite different, in fact the USB standard for human computer interface (USB Implementer's Forum, 2001) restricts the range for $\Delta x$ and $\Delta y$ displacements to values from -128 to +127 and the sample rate measured for a mouse featuring the ADNS-2051 is 125 samples per second.

### 4.2 Mice Calibration Process

The numbers we have reported in the previous subsection reflect only nominal values for the sensor characteristics since its resolution can vary depending on the surface material and the height of the sensor from the floor as described in Figure 8. This variation in sensor readings calls for an accurate mounting on the robot and a minimal calibration procedure before applying the formulas described in the previous section in order to reduce systematic errors in odometry. In fact systematic errors in odometry are often due to a wrong assumptions on the model parameters and they can be significantly reduced by experimental estimation of right values. Our model systematic errors are mostly due to the distance $D$ between the mice and the exact resolution of the sensors. To estimate the last parameter, we have moved 10 times the mice on a 20 cm straight track and estimated in 17.73 the number of ticks per $m m$ (i.e., 450 cpi ) through the average counts for the mice $\Delta y$ displacements.

The estimation of the distance between the two mice has been calibrated after resolution calibration by using a different procedure. We rotated the two mouse around their middle point for a fixed angle $\eta$ ( $\pi / 2 \mathrm{rad}$ in our experiments) and we measured again their $\Delta y$ displacements. These two measures have to be equal to assure we are rotating around the real middle point and the $\Delta x$ displacement should be equal to zero when the mice are perpendicular to the radius. Provided the last two constraints we can estimate their distance according to the simple formula

$$
\begin{equation*}
D=\Delta y / \eta \tag{18}
\end{equation*}
$$

However, it is not necessary to rotate the mice around their exact middle point, we can still estimate their distance by rotating them around a fixed point


Figure 8: Typical resolution vs. Z (distance from lens reference plane to surface) for different surfaces
on the joining line and measuring the $\Delta y_{1}$ and $\Delta y_{2}$. Given these two measurements we can compute $D$ by using

$$
\begin{equation*}
D=\frac{\Delta y_{1}+\Delta y_{2}}{\eta} \tag{19}
\end{equation*}
$$

## 5 EXPERIMENTAL RESULTS

In order to validate our approach, we take two USB optical mice featuring the Agilent ADNS-2051 sensor, which can be commonly purchased in any commercial store. We fix them as described in Section 3, taking care of making them stay in contact with the ground. We made our experiments on a carpet, like those used in the RoboCup Middle Size League using the recommended operating setting for the sensor (i.e., nominally $400 c p i$ at 1500 frames per second) and calibrating the resolution to 450 cpi and the distance $D$ between the mice to 270 mm with the procedure previously described.

The preliminary test we made is the UMBmark test, which was presented by (Borenstein and Feng, 1994). The UMBmark procedure consists of measuring the absolute actual position of the robot in order to initialize the on-board dead reckoning starting position. Then, we make the robot travel along a $4 x 4 \mathrm{~m}$ square in the following way: the robot stops after each $4 m$ straight leg and then it makes a $90^{\circ}$ turn on the spot. When the robot reaches the starting area, we measure its absolute position and orientation and compare them to the position and orientation calculated by the dead reckoning system. We repeated
this procedure five times in clockwise direction and five times in counter-clockwise. The measure of dead reckoning accuracy for systematic errors that we obtained is $E_{\text {max,syst }}=114 \mathrm{~mm}$, which is comparable with those achieved by other dead reckoning systems (Borenstein and Feng, 1996).

## 6 RELATED WORKS

As we said in Section 1, Mobile Robot Positioning has been one of the first problem in Robotics and odometry is the most widely used navigation method for mobile robot positioning. Classical odometry methods based on dead reckoning are inexpensive and allow very high sample rates providing good short-term accuracy. Despite its limitations, many researcher agree that odometry is an important part of a robot navigation system and that navigation tasks will be simplified if odometric accuracy could be improved.

The fundamental idea of dead reckoning is the integration of incremental motion information over time, which leads inevitably to the unbounded accumulation of errors. Specifically, orientation errors will cause large lateral position errors, which increase proportionally with the distance traveled by the robot. There have been a lot of work in this field especially for differential drive kinematics and for systematic error measurement, comparison and correction (Borenstein and Feng, 1996).

First works in odometry error correction were done by using and external compliant linkage vehicle pulled by the mobile robot. Being pulled this vehicle does not suffer from slipping and the measurement of its displacing can be used to correct pulling robot odometry (Borenstein, 1995). In (Borenstein and Feng, 1996) the authors propose a practical method for reducing, in a typical differential drive mobile robot, incremental odometry errors caused by kinematic imperfections of mobile encoders mounted onto the two drive motors.
Little work has been done for different kinematics like the ones based on omnidirectional wheels. In these cases, slipping is always present during the motion and classical shaft encoder measurement leads to very large errors. In (Amini et al., 2003) the Persia RoboCup team proposes a new odometric system which was employed on their full omni-directional robots. In order to reduce non-systematic errors, like those due to slippage during acceleration, they separate odometry sensors from the driving wheels. In particular, they have used three omni-directional wheels coupled with shaft encoders placed $60^{\circ}$ apart of the main driving wheels. The odometric wheels are connected to the robot body through a flexible struc-
ture in order to minimize the slippage and to obtain a firm contact of the wheels with the ground. Also this approach is independent from the kinematics of the robot, but its realization is quite difficult and, however, it is affected by (small) slippage problems.

An optical mouse was used in the localization system presented in (Santos et al., 2002). In their approach, the robot is equipped with an analogue compass and an optical odometer made out from a commercially available mouse. The position is obtained by combining the linear distance covered by the robot, read from the odometer, with the respective instantaneous orientation, read from the compass. The main drawback of this system is due to the low accuracy of the compass which results in systematic errors.

However, odometry is inevitably affected by the unbounded accumulation of errors. In particular, orientation errors will cause large position errors, which increase proportionally with the distance travelled by the robot. There are several works that propose methods for fusing odometric data with absolute position measurements to obtain more reliable position estimation (Cox, 1991; Chenavier and Crowley, 1992).

## 7 CONCLUSIONS

We have presented a dead reckoning sensor based on a pair of optical mice. The main advantages are good performances w.r.t. the two main problems that affect dead reckoning sensors: slipping and crawling. On the other hand, this odometric system needs that the robot operates on a ground with a surface on which the mice always read with the same resolution.

Due to its characteristics, the proposed sensor can be successfully applied with many different robot architectures, being completely independent from the specific kinematics. In particular, we have developed it for our omnidirectional Robocup robots, which will be presented at Robocup 2004.

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[^0]:    ${ }^{1}$ Images reported in Figure 6, 7 and 8 are taken from (Agilent Technologies Semiconductor Products Group, 2001) and (Agilent Technologies Semiconductor Products Group, 2003) and are copyright of Agilent Technologies Inc.

