A STOCHASTIC OFF LINE PLANNER OF OPTIMAL DYNAMIC MOTIONS FOR ROBOTIC MANIPULATORS

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Abstract: We propose a general and simple method that handles free (or point-to-point) motion planning problem for redundant and non-redundant serial robots. The problem consists of linking two points in the operational space, under constraints on joint torques, jerks, accelerations, velocities and positions while minimizing a cost function involving significant physical parameters such as transfer time and joint torque quadratic average. The basic idea is to dissociate the search of optimal transfer time *T* from that of optimal motion parameters. Inherent constraints are then easily translated to bounds on the value of *T*. Furthermore, a stochastic optimization method is used which not only may find a better approximation of the global optimal motion than is usually obtained via traditional techniques but that also handles more complicated problems such as those involving discontinuous friction efforts and obstacle avoidance.

1 INTRODUCTION

Motion planning constitutes a primordial phase in the process of robotic system exploitation. It is a challenging task because the robot behaviour is governed by highly non linear models and is subjected to numerous geometric, kinematic and dynamic constraints (Latombe, 1991) (Angeles, 1997) (Chettibi, 2001). Two categories of motions can be distinguished (Angeles, 1997) (Chettibi, 2000). The first covers motions along prescribed geometric path and correspond, for example, to continuous welding or glowing operations (Bobrow, 1985) (Kang, 1986) (Pfeiffer, 1987) (Chettibi, 2001*b*). The second, which is the focus of this paper, concerns point-to-point (or free) motions involved, for example, in discrete welding or pick-and-place operations (Bessonnet, 1992) (Mitsi, 1995) (Lazrak, 1996) (Danes, 1998) (Chettibi, 2001*a*). In general, many different ways are possible to perform the same task. This freedom of choice can be exploited judiciously to optimize a given performance criterion. Hence, motion generation becomes an optimization problem. It is here referred to as the optimal free motion planning problem (OFMPP).

In the specialized literature, various resolutions methods have been proposed to handle the OFMPP. They can be grouped in two main families; namely: *direct* and *indirect* methods (Hull, 1997) (Betts, 1998). The indirect methods are, in general, applications of optimal control theory and in particular *Pontryagin Maximum Principle* (PMP) (Pontryagin, 1965). Optimality conditions are stated under the form of a boundary value problem that is generaly too difficult to solve (Bessonnet, 1992) (Lazrak, 1996) (Chettibi, 2000). Several techniques, such as the phase plane method (Bobrow, 1985) (Kang, 1986) (Jaques, 1986) (Pfeiffer, 1987), exploit the structure only of the optimal solution given by PMP and get numerical solutions via other means. In general, such techniques are applied to limited cases and have several drawbacks resumed below:

- They require the solution of a N.L multi-point shooting problem (David, 1997) (John, 1998),
- They require analytical computing of gradients (Lazrak, 1996) (Bessonnet, 1992),

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- The region of convergence may be small (Chettibi, 2001) (Lazrak, 1996),
- Path inequality are difficult to handle (Danes, 1998),
- They introduce new variables known as co-state variables that are, in general, difficult to estimate (Lazrak, 1996) (Bessonnet, 1992) (Danes, 1998) (Pontryagin, 1965).
- In minimum time transfer problems, they lead to discontinuous controls (bang-bang) that may create many practical problems (Ola, 1994) (Chettibi, 2001*a*). In fact, the controller must work in saturation for long periods. The optimal control leaves no control authority to compensate for any tracking error caused by either unmodeled dynamics or delays introduced by the on-line feedback controller

To overcome these difficulties, direct methods have been proposed. They are based on discretisation of dynamic variables (states, controls). They seek to solve directly a parameter optimization problem. Then, N.L. programming (Tan, 1988) (Martin, 1997) (Martin, 1999) (Chettibi, 2001*a*) or stochastic optimization techniques (Chettibi, 2002*b*) are applied to compute optimal values of parameters. Other ways of discretisation can be found in (Richard, 1993) (Macfarlane, 2001). These techniques suffer, however, from numerical explosion when treating high dimension problems. Although they have been used successfully to solve a large variety of problems, techniques based on N.L. programming (Fletcher, 1987) (David, 1997) (Danes, 1998) (John, 1998) (Chettibi, 2000) have two essential drawbacks:

- They are easily attracted by local minima ;
- They generally require information on gradiant and hessian that are difficult to get analytically. In addition, continuity of second order must be ensured, while realistic physical models may include some discontinuous terms (frictions).

In parallel to these methods, that take into account both kinematics and dynamics aspects of the problem, numerous pure geometric planners have been proposed to find solutions for the simplified problem that consists of finding only feasible geometric paths (Piano movers problem) (Latombe, 1991) (Overmars, 1992) (Barraquand, 1992) (Kavraki, 1994) (Barraquand, 1996) (Kavraki, 1996) (Latombe, 1999) (Garber, 2002). In spite of this simplification, the problem still remains quite complex with exponential computational time in the degree of freedom (d.o.f.). Of course, any extension (presence of obstacles, for example) adds in computational complexity. Even so, various practical planners have been proposed. Reference (Latombe, 1991) gives an excellent overview of early methods (before 1991) such as: potential field, cell decomposition and roadmap methods, some of which have shown their limits. For instance, a potential field based planner is quickly attracted by local minima (Khatib, 1986) (Latombe, 1991) (Barraquand, 1992). Cell decomposition methods often require difficult and quite expensive geometric computations and data structures tend to be very large (Latombe, 1991) (Overmars, 1992). The key issue for roadmap methods is the construction of the roadmap. Various techniques have been proposed that produce different sorts of roadmaps based on visibility and Voronoi graphs (Latombe, 1991).

During the last decade, interest was given to stochastic techniques to solve various forms of optimal motion planning problems. In particular, powerful algorithms were proposed to solve the basic geometric problem. Probabilistic roadmaps (PRM) or Probabilistic Path Planners (PPP) were introduced in (Overmars, 1992) (Barraquand, 1996) (Kavraki, 1994) (Kavraki, 1996) and applied successfully to complex situations. They are generally executed in two steps: first a roadmap is constructed, according to a stochastic process, then the motion planning query is treated. Due to the power of this kind of schemes, many perspectives are expected as shown in (Latombe, 1999). However, there are few attempts to apply them to solve the complete OFMPP. References (LaValle, 1998) (LaValle, 1999) propose the method of Rapidly exploring Random Trees (RRTs) as an extention of PPP to optimize feasible trajectories for NL systems. Dynamic model and inherent constraints are taken into account.

In (Chettibi, 2002*a*), we introduced a different scheme using a sequential stochastic technique to solve the OFMPP. We present here this simple and versatile method and how it can be used to handle complex situations involving both friction efforts and obstacle avoidance.

2 PROBLEM STATEMENT

Let us consider a serial redundant or non-redundant manipulator with n d.o.f.. Motion equations can be derived using Lagrange's formalism or Newton-Euler formalism (Dombre, 1988) (Angeles, 1997):

$$M(q)\ddot{q} + Q(q,\dot{q}) + G(q) = \tau \qquad (1a),$$

q, $\dot{q}$ and $\ddot{q}$ are respectively joints position, velocity, acceleration vectors. $\mathbf{M}(\boldsymbol{q})$ is the inertia matrix. $\boldsymbol{Q}(q,\dot{q})$ is the vector of centrifugal and *Coriolis* forces in which joints velocities appear under a quadratic form. **G(*q*)** is the vector of potential forces and τ is the vector of actuator efforts.

In order to make the dynamic model more realistic, we may introduce, for the i^{th} joint, friction efforts as follows:

$$\sum_{j=1}^{n} M_{ij}(q(t))\ddot{q}_j(t) + Q_i(q(t),\dot{q}(t)) + G_i(q(t)) + F_i^v \dot{q}(t) + F_i^s \, sign\,(\dot{q}(t)) = \tau_i(t) \quad (1b),$$

F_i^V and F_i^s are, respectively, sec and viscous friction coefficients of the i^{th} joint.

The robot is required to move freely from an initial state $\boldsymbol{P_i}$ to a final state $\boldsymbol{P_f}$, both of which are specified in the operational space. In addition to solving for $\tau(t)$ and transfer time T, we must find the trajectory defined by $\boldsymbol{q}(t)$ such as the initial and the final state are matched, constraints are respected and a cost function is minimized.

The cost function adopted here is a balance between transfer time T and the quadratic average of actuator efforts:

$$F_{obj} = \mu T + \frac{1-\mu}{2}\int_0^T \sum_{i=1}^{n}\left(\frac{\tau_i(t)}{\tau_i^{max}}\right)^2 dt \quad (2).$$

μ is a weighting coefficient chosen from [0,1] and according to the relative importance we would like to give to the minimization of T or to the quadratic average of actuator efforts. The case $\mu = 1$ corresponds to the optimal time free motion planning problem.

Constraints that must be satisfied during the entire transfer ($0 \le t \le T$) are summarized bellow: for $i = 1, \ldots, n$ we have bounds on:

- *Joint torques:*
$$|\tau_i(t)| \le \tau_i^{\max} \quad (3a);$$
- *Joint jerks :*
$$|\dddot{q}_i(t)| \le \dddot{q}_i^{\max} \quad (3b);$$
- *Joint accelerations:*
$$|\ddot{q}_i(t)| \le \ddot{q}_i^{\max} \quad (3c);$$
- *Joint velocities:*
$$|\dot{q}_i(t)| \le \dot{q}_i^{\max} \quad (3d);$$
- *Joint positions:*
$$|q_i(t)| \le q_i^{\max} \quad (3e).$$

Of course, non-symmetrical bounds on the above physical quantities can also be handled without any new difficulty.

Relations (3*a, b, c, d* and *e*) traduce the fact that not all motions are tolerable and that power resources are limited and must be used rationally in order to control correctly the robot dynamic behavior. Also, since joint position tracking errors increase with jerk, constraints (3*b*) are introduced to limit excessive wear and hence to extend the robot life-span (Latombe, 1991) (Piazzi, 1998) (Macfarlane, 2001).

In the case where obstacles are present in the robot workspace, motion must be planned in such a way collision is avoided between links and obstacles. Therefore, the following constraint has to be satisfied :

$$C(\boldsymbol{q}) = False \quad (3f).$$

The Boolean function C indicates whether or not the robot at configuration $\boldsymbol{q}$ is in collision with an obstacle. This function uses a distance function $D(\boldsymbol{q})$ that supplies for any robotic configuration the minimal distance to obstacles.

3 REFORMULATION OF THE PROBLEM

The normalization of the time scale, initially used to make the problem with fixed final time, is exploited to reformulate the problem and to make it propitious for a stochastic optimization strategy. Details are shown bellow.

3.1 Scaling

We introduce a normalized time scale as follows:

$$t = x.T \quad \text{with} \quad x \in [0,1] \quad (4).$$

Hereafter, we will use the prime symbol to indicate derivations with respect to x :

$$q' = \frac{dq(x)}{dx}, \quad q'' = \frac{d^2q(x)}{dx^2}, \quad q''' = \frac{d^3q(x)}{dx^3} \quad (5).$$

Relations (1*a*) and (1*b*) can be written as follows:

$$(1a) \Rightarrow \quad \psi_i(x) = \frac{1}{T^2}H_i + \overline{G}_i \quad (6a)$$

$$(1b) \Rightarrow \psi_i = \frac{1}{T^2}H_i + \overline{G}_i + \frac{1}{T}\overline{F}_i^v q_i' + \overline{F}_i^s sign(q') \quad (6b)$$

where:

$$\psi_i(x) = \frac{\tau_i(x)}{\tau_i^{\max}}, \quad \overline{M}_{ij} = \frac{M_{ij}}{\tau_i^{\max}}, \quad \overline{Q}_i = \frac{Q_i}{\tau_i^{\max}},$$

$$\overline{G}_i = \frac{G_i}{\tau_i^{\max}}, \quad H_i = \sum_{j=1}^{n}\overline{M}_{ij}q''_j + \overline{Q}_i \quad (7)$$

and:

$$\overline{F}_i^v = \frac{F_i^v}{\tau_i^{\max}}, \quad \overline{F}_i^s = \frac{F_i^s}{\tau_i^{\max}} \quad (8)$$

3.2 Cost function

With the previous notations, the cost function (2) becomes without friction efforts:

$$F_{obj} = T\left(S_0 + \frac{S_2}{T^2} + \frac{S_4}{T^4}\right) \qquad (9),$$

Where S_0, S_2 and S_4 are given by:

$$S_0 = \mu + \frac{1-\mu}{2}\int_0^1 \sum_{i=1}^{n} \overline{G}_i^2 dx$$

$$S_2 = (1-\mu)\int_0^1 \sum_{i=1}^{n} H_i \overline{G}_i dx \qquad (10).$$

$$S_4 = \frac{1-\mu}{2}\int_0^1 \sum_{i=1}^{n} H_i^2 dx$$

It must be noted that S_0, S_2 and S_4 are real coefficients that do depend on the joint evolution profile $\boldsymbol{q}(x)$ but that do not depend on T. Also, S_0 and S_4 are always positive. Expression (9) represents a family of curves whose general shape, for any feasible motion, is shown in figure 1*a*. The minimum of each of these curves is reached when T takes on the value $T = T_m$ given by :

$$T_m = \left(\frac{S_2 + \sqrt{S_2^2 + 12 S_0 S_4}}{2S_0}\right)^{1/2} \qquad (11).$$

If friction efforts are taken into account, we introduce the following quantities:

$$K_i = \overline{F}_i^v q', \qquad \overline{\overline{G}}_i = \overline{G}_i + \overline{F}_i^s sign(q') \qquad (12)$$

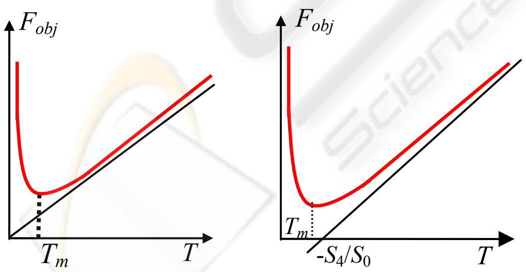

Fig. 1*a* Fig. 1*b*

Figure 1. General shape of the cost function; (a) without friction efforts , (b) with friction efforts

The expression of (2) becomes then :

$$F_{obj} = T\left(S_0 + \frac{S_1}{T} + \frac{S_2}{T^2} + \frac{S_3}{T^3} + \frac{S_4}{T^4}\right) \qquad (13)$$

where:

$$S_0 = \mu + \frac{1-\mu}{2}\int_0^1 \sum_{i=1}^{n} \overline{\overline{G}}_i^2 d\lambda$$

$$S_1 = (1-\mu)\int_0^1 \sum_{i=1}^{n} K_i \overline{\overline{G}}_i d\lambda$$

$$S_2 = \frac{1-\mu}{2}\int_0^1 \sum_{i=1}^{n} \left(K_i^2 + 2 H_i \overline{\overline{G}}_i\right) d\lambda \qquad (14)$$

$$S_3 = (1-\mu)\int_0^1 \sum_{i=1}^{n} H_i K_i d\lambda$$

$$S_4 = \frac{1-\mu}{2}\int_0^1 \sum_{i=1}^{n} H_i^2 d\lambda$$

For a given profile $\boldsymbol{q}(x)$, (13) represents a family of curves whose general shape is shown in figure 1*b*, but now the asymptotic line intersects the time axis at $\boldsymbol{T = -S_1/S_0}$. Furthermore, T_m has to be computed numerically since (11) is no longer applicable.

3.3 Effects of constraints

Constraints imposed on the robot motion will be handled sequentially within the iterative process of minimization described in the next section. Already, we can group constraints into several categories according to the stage of the iterative process at which they will be handled.

3.3.a Constraints of the first category

In the first category, we have constraints that will not add any restriction on the value of T. For example, joint position constraints (3*e*) become:

$$|q_i(x)| \le q_i^{\max} \quad \forall x \in [0,1] \quad i = 1,\ldots,n \qquad (15),$$

and those due to obstacles presence (3*f*) become :

$$C(\boldsymbol{q}(x)) = False \quad \forall x \in [0,1] \qquad (16)$$

In both cases, only the joint position profiles $\boldsymbol{q}(x)$ are determinant.

3.3.b Constraints of second category

In the second category, we have constraints that can be transformed into explicit *lower* bounds on T. For example joint velocity constraints lead to:

$$\frac{1}{T}|q'_i(x)| \le \dot{q}_i^{\max} \Rightarrow T \ge \frac{|q'_i(x)|}{\dot{q}_i^{\max}} \quad i = 1,\ldots, n$$

$$\text{so:} \quad T \ge T_v, \; T_v = \max_{i=1,\ldots,n}\left[\max_{[0,1]} \frac{|q'_i(x)|}{\dot{q}_i^{\max}}\right] \qquad (17).$$

Joint acceleration and jerk constraints are transformed in the same way to give:

For accelerations:

$$T \geq T_a \;,\quad T_a = \max_{i=1,\dots,n}\left[\max_{[0,1]}\left(\frac{\left|q''_i(x)\right|}{\ddot{q}_i^{\max}}\right)^{1/2}\right] \qquad (18),$$

and for jerks:

$$T \geq T_J \;,\quad T_J = \max_{i=1,\dots,n}\left[\max_{[0,1]}\left(\frac{\left|q'''_i(x)\right|}{\dddot{q}_i^{\max}}\right)^{1/3}\right] \qquad (19).$$

Thus, (17), (18) and (19) define three lower bounds on transfer period. In consequence; T must satisfy the following condition:

$$T \geq T^* \,,\qquad T^* = max(T_v,\, T_a,\, T_J) \qquad (20),$$

This type of constraints defines a forbidden region as shown in figure 2. Note that two cases are possible.

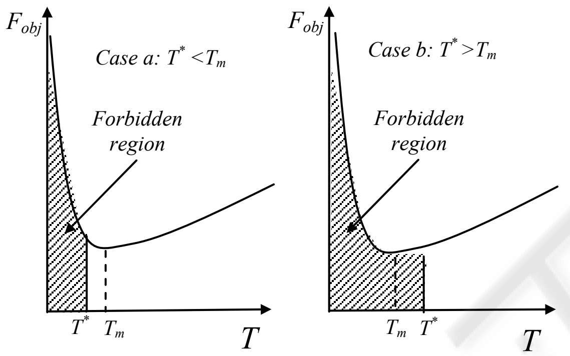

Figure 2: Bounds on transfer time value due to constraints of second category

3.3.c Constraints of third category

In the third category, we have constraints that can be transformed into explicit bilateral bounds on T. For example those imposed on the value of joint torques (3*a*) define, in general, bracketing bounds on T, namely: T_L and T_R. In consequence,

$$T \in [T_L,\, T_R] \qquad (21).$$

A fourth category might be included and would concern any other constraint that *does* add restrictions on T but that cannot be easily translated into simple bounds on T.

4 STRATEGY OF RESOLUTION

The iterative process of minimization proposed here includes the following steps:

Step 1: Generate a random (or guessed) temporal evolution shape $q_i(x)$ for each of the joint variables, taking into account any constraints of the first category (15), (16) as well as any conditions imposed on the initial and the final state.

Step 2: Get the S coefficients from (10) or (14) and T_m from (11) or by numerical means. If $F(T_m)$ is greater than F_{best} obtained so far, then there is no need to continue and hence, return to *Step* 1. Otherwise, a first bracketing interval $[T_1,\, T_2]$ is deduced (Fig. 3) in which F is decreasing from T_1 to T_m and increasing from T_m to T_2.

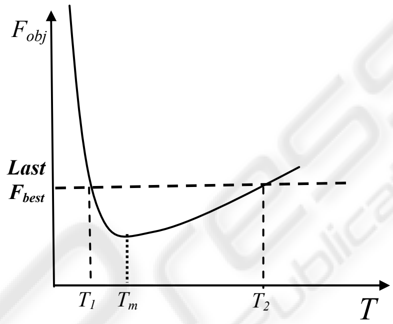

Figure 3: New exploration region defined by a new lower value of F_{best}.

The remaining steps will simply consist of changing T_1, T_m or T_2 while keeping this bracketing.

Step 3: Get T_a, T_v, T_j from (17, 18, 19) and T^* from (20). If $T^* > T_2$ then return to *Step* 1 else modify T_1 and/or T_m according to Fig. 2. That is: in case (*a*) $T_1 \leftarrow T^*$ while in (*b*) $T_1 \leftarrow T^*$ and $T_m \leftarrow T^*$.

Step 4: Get $[T_L,\, T_R]$ from (21). If $T_L > T_2$ or $T_R < T_1$ then return to *Step* 1. Otherwise, we have a new improved F_{best} :

If $T_m \in [T_L,\, T_R]$ then
  $F_{best} \leftarrow F(T_m)$
Else if $T_m < T_L$ then
  $F_{best} \leftarrow F(T_L)$
Else
  $F_{best} \leftarrow F(T_R)$
End if

The above steps can be imbedded in a stochastic optimization strategy to determine better profiles $q_i(x)$, $i = 1, \dots\, n$, leading to lower values of the objective function.

One way to get a guessed temporal evolution shape $q_i(x)$ for the joint variables, at any stage of optimization process, is to use randomly generated clamped cubic spline functions with nodes distributed for $x \in [0,1]$ (Fig. 4).

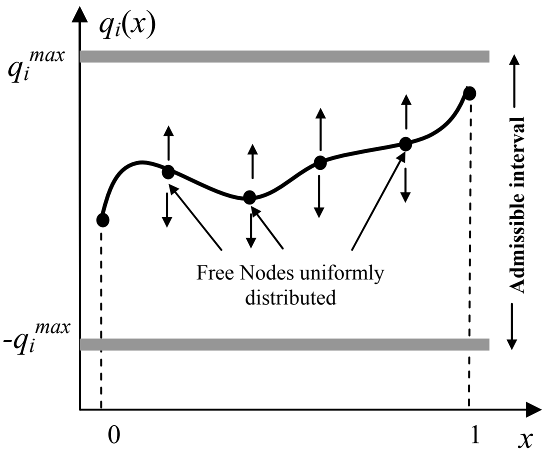

Figure 4: Approximation of joint position temporal evolution.

5 NUMERICAL RESULTS

We consider here a redundant planar robot constituted of four links connected by revolute joints. The corresponding geometric and inertial characteristics are listed in Appendix A. It is asked to move among two static obstacles disposed in it's work space at respectively (2, 1.5) and (-1.5, 1.5) with both unity radius. The robot begin at ($\pi/4$, $-\pi/2$, $\pi/4$, 0) and stops at ($\pi/2$, 0, 0, 0). Boundary velocities are null. The numerical results are obtained with μ=0.5 for both cases: with and without friction efforts. The corresponding optimal motions are depicted in Figures 5*a*, *b*, *c*, *d*, *e* and *f*.

In fact, without introducing friction effort we get : F_{obj} = 2.7745(s) and T_{opt} = 4.9693 (s).In the presence of friction efforts we get a different result: F_{obj} = 3.1119 (s) and T_{opt} = 5.3745 (s). Hence, to achieve the same task, we need more time and more effort in the presence of friction efforts.

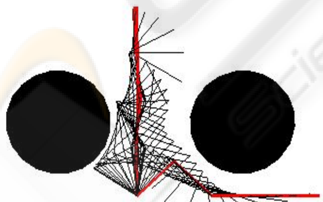

Figure 5*a*: Aspect of motion without friction effect

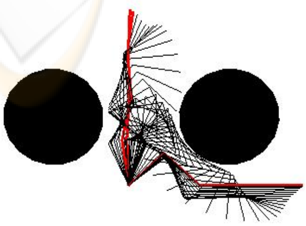

Figure 5*b*: Aspect of motion with friction effect

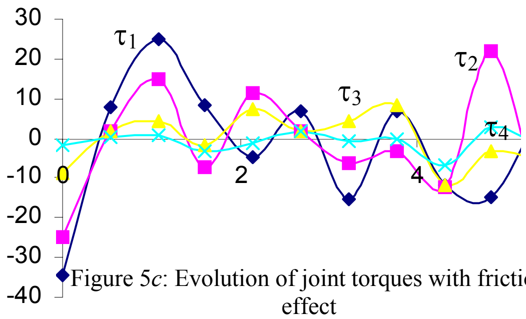

Figure 5*c*: Evolution of joint torques with friction effect

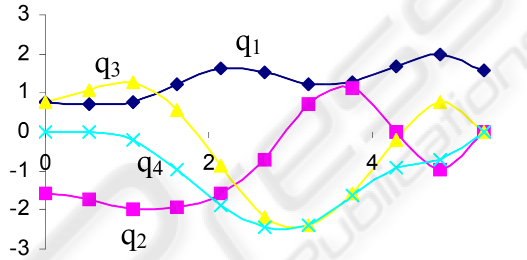

Figure 5*d*. Evolution of joint positions with friction effect

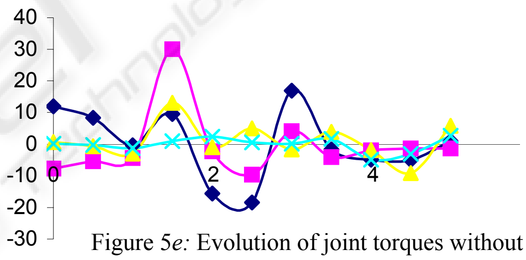

Figure 5*e*: Evolution of joint torques without friction effect

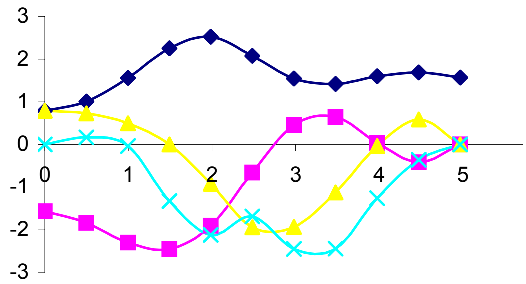

Figure 5*f*: Evolution of joint positions without friction effect

6 CONCLUSION

In this paper we have presented a simple trajectory planner of point-to-point motions for robotic arms. The problem is highly non-linear due first to the complex robot dynamic model that must be verified during the entire transfer, then to the non-linearity of the cost function to be minimized and finally to numerous constraints to be simultaneously respected. The OFMPP is originally an optimal control one and has been transformed into a parametric optimization problem. The optimization parameters are time transfer T and the position of nodes defining the shape of joint variables. The research of T has been separated from that of the others parameters in order to make the computing process efficient and to handle constraints easily by transforming them into explicit bounds on T possible values. In fact, the various possible constraints have been regrouped in four families according to their possible effects on T values and then have been handled sequentially during each optimization step. Nodes, defining $q(x)$ shape, are connected by cubic spline functions and their positions are perturbed inside a stochastic process until the objective function value is sufficiently reduced while all constraints are all satisfied. This ensured smoothness of resulted profiles. The objective function has been written under a weighting form permitting to make balance between reducing T and magnitude of implied torques.

Numerical examples, where a stochastic optimization process, implementing the proposed approach, has been used along with cubic spline approximations, and dealing with complex problems, such as those involving discontinuous friction efforts and obstacle avoidance, have been presented to show the efficiency of this technique. Others successful tests have been made in parallel for complex robotic architectures, like biped robots, will be presented in a future paper.

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Appendix A: *Characteristics of the 4R robot.*

Joint N°	1	2	3	4
α (rad)	0	0	0	0
d(m)	0	1	1	1
r(m)	0	0	0	0
a(m)	0	0	0	0
M(kg)	5	4	3	2
Izz(kg.m^2)	1	0.85	0	0
τ(N.m)	25	20	15	5
F_s (N.m)	0.7	0.2	0.5	0.2
F_v (N.m.s)	1	0.2	0.5	0.2