Application of Linear Programming in Gerrymandering for Optimal Partisan Gain in the United States

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Abstract: This study investigates how linear programming can be applied to engineer optimal electoral districting for partisan advantage, focusing on the 2012 Pennsylvania House of Representatives election as a case study. Although the Republican Party received only 49.9% of the statewide popular vote, they secured 13 out of 18 congressional seats through strategic districting. Using county-level voting and population data, this paper develops a linear programming model in Python to simulate and optimize gerrymandering for the Republican Party. The model, even with relaxed constraints on contiguity and compactness, demonstrates the ability to secure up to 17 out of 18 seats for the party-an outcome more extreme than the real-world result. The model validates core gerrymandering strategies such as "packing" and "cracking," revealing how districts can be drawn to amplify seat share far beyond vote share. This paper also compares the program's output to the actual 2012 district map and discuss the similarities in partisan tactics despite differing structural constraints. The findings underscore the effectiveness of linear programming in modeling partisan manipulation and offer insights into the limits and consequences of gerrymandering under district-based majoritarian systems.

1 INTRODUCTION

The district-based majoritarian system combines two key elements: district-based representation and majoritarian representation.

District-based representation divides a country into districts that each elect one or more representatives. Whoever gets the most votes in a district wins, regardless of national totals. It contrasts with popular voting, which counts all votes equally across the nation.

Majoritarian representation, or "winner takes all," means each district yields a single winner regardless of the winning margin. This model is used in the U.S. House and Electoral College, where winning a district yields total control of its representation. In contrast, proportional representation typically uses multi-member districts, where seats are distributed among parties or candidates in proportion to the votes they receive within that district. While majoritarian systems offer clarity and stable governments, they often distort proportional fairness. A party can win a majority of seats without a majority of votes, leaving large groups of voters effectively unrepresented.

Historically, district-based systems emerged to accommodate geographical identity and civic organization. In 5th century, citizens were grouped into demes that each contributed members to the Boule (Council of 500), balancing urban and rural voices. The Roman Republic similarly divided citizens into 35 voting units. Rome adopted a district-based majoritarian system to preserve elite control-districts were drawn to amplify elite influence, a precursor to modern gerrymandering.

Britain also embraced this system when adopting representative democracy. Today, each of the UK's 650 constituencies elects one through a winner-takes-all vote. This suppresses smaller parties and contributes to a two-party dominance. Meanwhile, in the European Parliament, member states act as districts. Some use winner-takes-all, others use proportional allocation.

In the United States, states retain strong identities and are treated as individual political units. Thus, for both Congress and the presidency, elections rely on district-based majoritarian rules. In the Electoral College, nearly all states give 100% of their electors to the candidate who wins the state, regardless of margin. This model became dominant because it amplified a state's influence and simplified election outcomes-advantages that appealed to politicians despite its undemocratic distortions.

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Controversies arise when the district-based structure combines with majoritarian rules. While it creates clear winners and stable governments, it often undermines political equality. Because only one winner emerges per district, millions of votes can be effectively discarded. Furthermore, while the system appears democratic on the surface, its reliance on district boundaries makes the system vulnerable to manipulation through gerrymandering and can produce outcomes that are deeply unrepresentative and structurally biased.

Gerrymandering-the manipulation of district boundaries for political gain-typically occurs during redistricting cycles (e.g., after the U.S. Census every 10 years). The term originates from 1812, when Massachusetts Governor Elbridge Gerry approved a district shaped like a salamander to benefit his party.

Gerrymandering allows parties to distort the relationship between votes and seats, enabling them to dilute the influence of the opposing party and gain power disproportionate to their support. It relies on the systematical flaw that the winning margin does not matter.

Two key strategies are common: Packing: Concentrating opposition voters into a few districts so they win by overwhelming margins, wasting their votes. Cracking: Spreading opposition voters thinly across many districts so they lack the numbers to win anywhere.

Gerrymandering has happened in the United States numerous times in reality. In 2012, Republicans won 13 of 18 House seats in Pennsylvania with only 49% of the vote due to the gerrymandered map. In North Carolina, Republicans secured 10 of 13 seats despite near-even statewide vote shares. That year, in at least five states, the party that lost the popular vote won the most seats in the House.

Gerrymandering is extremely prevalent in the United States, unlike in many other democracies like Canada, the UK, Australia, and New Zealand, where independent commissions draw districts. In the U.S., redistricting is typically handled by partisan state legislatures, allowing dominant parties to entrench their power.

Some states, including California, Arizona, and Michigan, have established independent redistricting commissions to ensure fairness. Others rely on court challenges and public ballots. For example, Pennsylvania's 2012 map was struck down by a federal court in 2018 for being unconstitutional. North Carolina's congressional map was also invalidated in 2016.

Despite these efforts, enforcement remains weak, and gerrymandering continues to distort representation in American democracy. Legislative reform has been slow, and the issue persists as a deep-rooted challenge.

In this study, the author aims to use the method of linear programming to create a model that can find the optimal districting for a party. This paper will use an approach that is different from past research in a few ways and also try to observe the patterns when people lift some of the restraints. This paper decides to focus on the House of Representatives election in the United States, where gerrymandering is the most prevalent. More specifically, this paper decide to focus this study on the case of Pennsylvania in 2012, one of the most famous and extreme cases of gerrymandering. After creating the model and program, this paper will look for two things: how well the model performs in maximizing the number of seats a party can obtain in a state and how the computer drawn model compares to the real district drawn by the Republican Party in Pennsylvania in 2012.

This study is significant in many ways. First, it can help people better understand how linear programming can be used to model real life scenarios in politics and other fields. Also, creating a linear programming model to find optimal gerrymandering can help people understand the extent gerrymandering could get and the strategies of politicians and state governments. This study could also be useful in court cases by helping with fairness determination and provide guidelines for legislative reforms.

2 LITERATURE REVIEW

Gerrymandering has increasingly been recognized as a problem that can be effectively analyzed using mathematical and computational tools. Scholars have employed geometry, graph theory, and optimization techniques to both detect bias and, conversely, to design maps that maximize partisan gain. Initially, the focus lay on modeling district compactness and contiguity-properties critical for legally acceptable maps-before evolving toward algorithmic detection of bias and, ultimately, toward explicit partisan optimization using integer programming.

Guest et al. pioneered computer-assisted redistricting by posing district design as the minimization of within-district distances. Their weighted k-means clustering approach, tested on real U.S. Census data, produced geometrically "fairer" maps-more compact and contiguous-than many human-drawn counterparts. Their work underscored the capacity of automated methods to expose hidden suboptimalities in conventional maps and established a foundation for later algorithmic fairness benchmarks (Guest et al., 2017).

Building on efficiency metrics, Chatterjee et al. investigated the computational difficulty of minimizing the "efficiency gap"-a measure of wasted votes per side-and demonstrated that although theoretically NP-hard, practical heuristics could yield improved maps in states like Pennsylvania and Wisconsin. Their fast algorithms showed that realistic efficiency gain is tractable, providing tools both for legal challenges and neutral map generation (Chatterjee et al., 2018).

Jacobs and Walch integrated compactness evaluation with partial differential equations to generate large ensembles of alternative maps. Their auction-dynamics and curvature-flow model generated many plausible districtings, enabling statistical outlier detection-an essential method for flagging partisan aberrations (Jacobs and Walch, 2018; Trounstine, 2025).

Turning toward optimization for partisan advantage, Dugošija et al. formalized a graph-based integer linear programming (ILP) framework that enforces population balance, contiguity, and compactness while optimizing either compactness or partisan objective functions. Tested on grid and small-state maps, their ILP models yielded provably optimal plans, illustrating that granular control is feasible with academic-grade solvers (Dugošija et al., 2020; Webb et al., 2025).

Okamoto formulated partisan gerrymandering as a binary optimization problem akin to ILP-using the Ising model and simulated annealing. By applying cell-based redistricting grids to maximize seats for one party under contiguity constraints, he demonstrated near-optimal partisan tilting in synthetic models (Okamoto, 2021).

Most recently, Faure et al. extended linear programming approaches to optimize political or minority representation via mixed-integer programming. Using county-level testbeds, they approximated probit-based objectives under contiguity and population constraints, achieving tight computational bounds, showing that district-scale partisan optimization is now practical for real-world scenarios (Faure et al., 2024; Zhu et al., 2021).

Computational geometry and ensemble methods have become central to the detection of gerrymandering, offering a means to evaluate enacted maps against a vast space of algorithmically generated alternatives. Through techniques such as Markov Chain Monte Carlo sampling and curvature-flow modeling, researchers have generated thousands-sometimes millions-of legally valid districting plans per state, establishing rigorous statistical baselines. These methods have been applied with considerable success in states such as North Carolina, Wisconsin, and Pennsylvania, where enacted maps were shown to be extreme outliers compared to neutral ensembles. On the other hand, linear and mixed-integer programming approaches have demonstrated the feasibility of constructing districting plans optimized for partisan advantage, under realistic legal and geographic constraints. These models have been scaled to handle entire states-such as Indiana, Arizona, and even Pennsylvania-comprising hundreds to thousands of precincts or census blocks. In these applications, solvers have produced maps that outperform existing gerrymanders in terms of seat maximization for a target party, while still satisfying population equality, contiguity, and compactness requirements. In some cases, the optimized maps yielded partisan advantages greater than those seen in enacted maps, underscoring both the potential and the ethical peril of such mathematical precision (Palomares, 2020).

In this study, the author uses an approach that is different from past research in a few ways and lift some of the restraints that are usually required. This paper aims to see if people can observe any meaningful or different patterns.

3 METHODOLOGY

3.1 Data Introduction

In this study, the author decides to specifically focus on the House of Representatives election in Pennsylvania in 2012-one of the most famous and controversial occasions where a party used gerrymandering for its own benefits.

3.2 Method Overview

In this study, the author focuses on the state of Pennsylvania in 2012, and aims to find the most optimal way of drawing electoral districts for the Republican Party using a linear programming model in python-the author aims to maximize the number of seats the Republican Party wins in the state. The author will then compare it to the actual districts in 2012 drawn by a Republican led government which triggered a lot of controversies as well as a lawsuit

(Table 1). The author is looking for two things in the results: How effective is this approach in maximizing the number of seats for the Republican party? In other words, how extreme can gerrymandering be when optimized? Is the computer optimized result similar to the districting that happened in reality in 2012 or is it different? In what ways are they different or similar?

Table 1: This caption has one line so it is centered.

District	Counties Included	Republican Victory?
1	/	/
2	19	Yes
3	21, 39, 42	Yes
4	2, 6, 11, 23, 30, 33, 54, 57	Yes
5	25, 41, 43	Yes
6	47	Yes
7	9, 49	Yes
8	13, 24, 51	/
9	/	Yes
10	5, 60	Yes
11	4, 28, 37, 46, 55, 61, 65	No
12	12, 17, 52, 59	Yes
13	26, 32, 35, 56	Yes
14	36, 58, 63	Yes
15	16, 48	Yes
16	29, 44	Yes
17	14, 66	Yes
18	53	Yes

In the United States, when state governments draw electoral districts, the laws usually require the following rules: integrity, contiguity, no enclaves, compactness, and population equality. In addition, electoral districts should aim to preserve existing boundaries, such as the boundaries of counties.

In this study, to test something different, the author makes the following assumptions. For this study, the author does not take into account the contiguity, no enclaves, and compactness requirements. There are only 2 parties on the ballot, in this study, the Democratic Party and the Republican Party. The author aims to maximize the advantage of the Republican party in this study. The author assumes that everyone in each county votes. Each county must be assigned to 1 and only 1 district; it cannot be split into different districts. The total population for each district must be approximately equal (further explained later). Last, the districts cannot be empty.

Since the author do not allow counties to be split into different districts, it might not be possible to create districts with approximately similar populations. Therefore, the author will start by ignoring this constraint and then gradually making it stricter. For example, the author will first ask the model to try to ensure each district has a population between 70%~130% of the average population; then, the author will tighten the range to 90%~110% of the average population, and so on.

As previously stated, the author aims to model the gerrymandering process with a linear programming model. The author first defines several variables to represent different factors in this model. Then, the author uses these variables to define the function the author aims to maximize and the constraints. Last, the author programs the model in Python and use the "LpProblem" function in the "PuPL" library to solve for the optimal solution.

3.3 Variable Introduction

Each county has a label "i" and each district has a label "j". $x_{ij} = 1$ if county i is in district j; $x_{ij} = 0$ if county i is NOT in district j. $v_i =$ net votes for the Republican party in county i. $y_j = 1$ if the Republican party wins district j, i.e. $\Sigma_i(v_i) \cdot (x_{ij}) > 0$. $p_i =$ the population of county i. $p_{avg} =$ total population / number of districts.

3.4 Optimization Function

The author aims to maximize $\Sigma_j y_j$, the total number of districts the Republican party wins. Only these constraints need to be expressed as equations. Each county is in exactly 1 electoral district: $\Sigma_j x_{ij} = 1$. Every district should have approximately the same population: $k_1 \cdot p_{avg} \le \Sigma_i(p_i) \cdot (x_{ij}) \le k_2 \cdot p_{avg}$. The author will try different sets of k_1 and k_2 to limit the range. The author will start by ignoring this constraint and then do, for example, if people want each district to have a population greater than 70% of the average population and less than 130% of the average, $0.7 \cdot p_{avg} \le \Sigma_i(p_i) \cdot (x_{ij}) \le 1.3 \cdot p_{avg}$. The districts cannot be empty $SUM(x_{ij}) \ge 1$ for each j.

3.5 Program Structure and Explanations

First, the author imports the data to the program. The author creates an empty array to represent the 67 counties and a different one to represent 18 empty districts. There is another array that holds the population of each county. Lastly, the author creates an array that holds the net votes of the Republican party in each county and assign the average population of each district 74248 to the variable pop_ideal.

Then, the author establishes the linear programming model by calling the LpProblem function from the pulp library in Python and create an object prob. After that, the author defines the variables x and y, add the constraints to the object prob, and loop through each district to assign the corresponding 1 or 0 value to each y_j.

For constraint (every district should have approximately the same population), the author starts by excluding it and then gradually tightens the restrictions. The author starts by using the numbers 0.7 and 1.3 for k_1 and k_2; then, the author tries to increase the first number and decrease the second number until the constraint becomes infeasible.

Finally, the author uses the linear programming solver in the library to solve this optimization problem and print the results.

4 RESULTS AND DISCUSSION

4.1 Outputs and Explanations

Table 2 presents the program output when the author excludes lines 28~31 in the code we displayed earlier, therefore ignoring the population constraint (constraint 4). The Republican party wins 17 of the 18 total congressional districts despite only receiving 49.9% of statewide votes. The output indicates that the Republican party wins every district except district 10. The output also displays the exact way of drawing the districts to achieve this optimal solution. For example, district 0 consists only of county 46, Montgomery. Counties 5, 10, 17, and 59 are drawn into district 1; these counties are Bedford, Butler, Clearfield, and Tioga, respectively.

Table 2 presents the program output when this paper includes lines 28~31 and requires the population of each district to be between 70% and 130% of the average population. This turns out to be infeasible, and the assumption that counties cannot be split into different districts is very likely the cause of this issue. Despite being practically insignificant, the program provides a heuristic solution, in which the Republican party wins 15 out of the 16 districts and the other two districts are left empty. The author attempted to apply more lenient restrictions, but the model fails to yield meaningful results until the author loosens the requirement to be greater than 20% of the average population and less than 180% of the average-at that point, the requirement does not have much practical meaning anymore.

Table 2: Program output, without constraint 4

District	Counties Included	Republican Victory?
1	46	/
2	5, 10, 17, 59	Yes
3	7, 13, 39, 42	Yes
4	11, 23, 27, 30, 31, 34, 54, 64	Yes
5	6, 15, 25, 41, 43, 48, 66	Yes
6	47, 62	Yes
7	9, 53	Yes
8	24, 57	/
9	18, 63	Yes
10	49	Yes
11	4, 28, 37, 38, 50, 51, 55, 61, 65	No
12	12, 52	Yes
13	0, 1, 8, 20, 21, 26, 33, 35, 36, 56, 58, 60	Yes
14	29	Yes
15	16	Yes
16	19, 40, 44	Yes
17	2, 3, 14	Yes
18	22, 32, 45	Yes

4.2 Effectiveness of the Approach

The model and approach turned out to be extremely effective. In fact, the author obtained a significantly better outcome for the Republican party compared to the districts the Republican led government drew for the 2012 congressional election. The Republican party received 13 out of 18 seats (72% of the seats) in the actual election, while the model let them have 17 out of the 18 seats (94% of the seats). Note that they only obtained 49.9% of the popular votes-usually, in the United States, for a party to secure 90% or more of the House of Representatives seats, they need to win at least 70% to 75% of the popular votes.

The Republican party loses the popular vote in the 2012 election in Pennsylvania but through gerrymandering, they are able to dominate the House of Representatives. With the model and approach, they are able to obtain an even better result for the Republican Party compared to the real 2012 election. The linear programming model seems to be promising.

4.3 Main Findings

Figure 1 displays the actual congressional districts in Pennsylvania in 2012. Figure 2 displays the districts drawn by the linear programming model without the population equality requirement. In both pictures, each color represents a different district. As presented in these pictures, it is not hard to tell that the districts are drawn in completely different ways. The shapes of the regions seem to resemble each other in no way at all. This is likely caused by the simplifications the author made to the model.

However, despite these districts being drawn in completely different ways-one was drawn by government officials and the other drawn by a computer linear programming model, they share some similarities in the fundamental principles. Similar ideas and strategies seem to appear when both the government officials and the computer program try to draw the districts to maximize the number of seats a certain party wins.

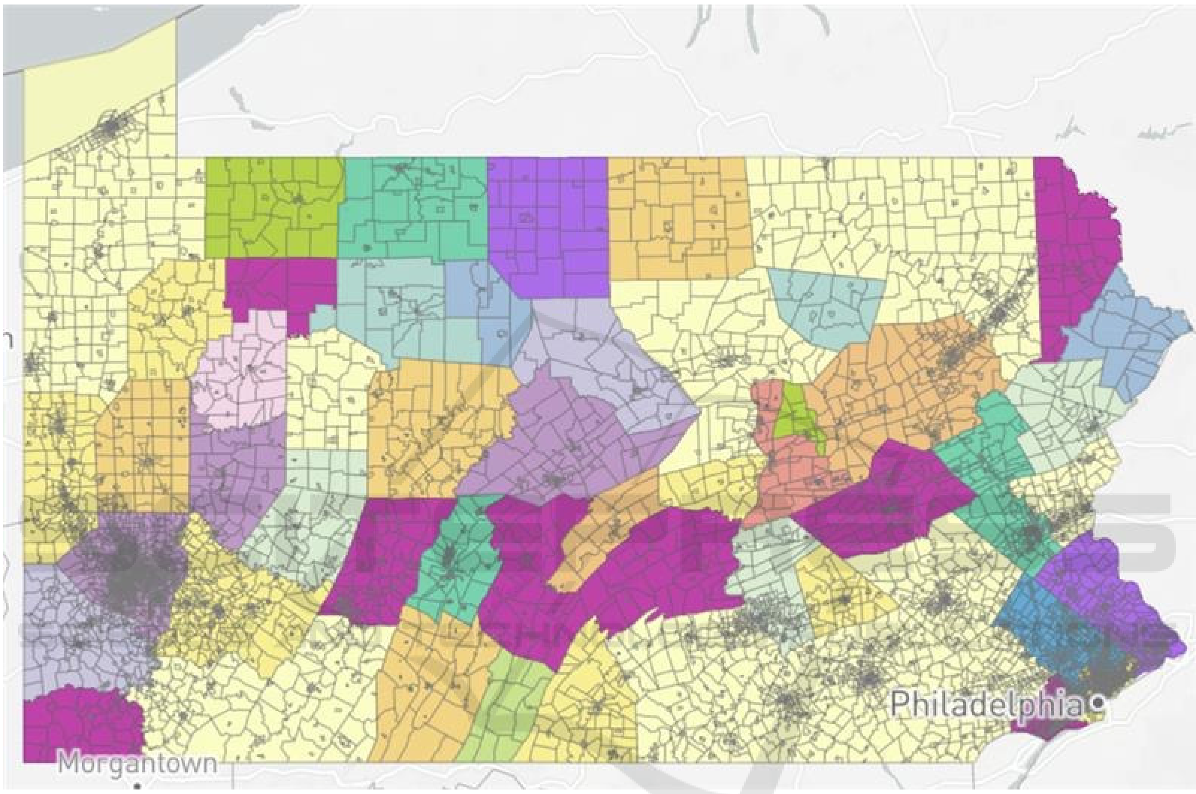

Figure 1: Visualization of the Results (Picture credit: Original).

When elections are done by districts and in a majoritarian manner, winning a district by 1 vote and winning a district by 1000 votes have essentially the exact same effect. Therefore, when a party aims to maximize the seats they get, they try to win many districts with a very small margin and let the opponent party win districts with a large margin. In the 1 district the Democratic Party won in the computer drawn districts and 3 of the 5 districts the Democratic Party won in the actual 2012 districts, the Democrats won by getting over 80% of the votes in the district. In contrary, most of the districts the Republican Party won, in both the computer drawn districts and the actual districts, they got just above 50% of the votes. In addition, when dealing with places where the opponent leads by a little, both the government officials and the linear programming model try to group the region with other ones where the Republicans are leading to achieve an overall Republican winning district.

4.4 Other Observations

In this study, the author only took into account two of the five common requirements of redistricting: integrity and population equality. As a result, the results the author got seem extreme and unrealistic. As shown in figure 2, the districts created by the program do not have any reasonable pattern. A district may involve counties that are separate and far apart from each other. It is impossible for this plan to be implemented in reality.

Meanwhile, the population equality constraint did not function as expected, as evidently shown in figure 2. This is mainly due to the fact that the author did not allow counties to be split into different districts, hoping to satisfy the requirement that existing county boundaries should be preserved as much as possible. Without the population equality constraint, the population of each district differs drastically from each other. When the population equality condition is added, the linear programming model fails to yield meaningful results until the author loosen the requirement to be greater than 20% of the average population and less than 180% of the average-at that point, the requirement does not have much practical meaning anymore. Despite these drawbacks, the results mentioned in previous sections are still significant and valuable to future research in this area.

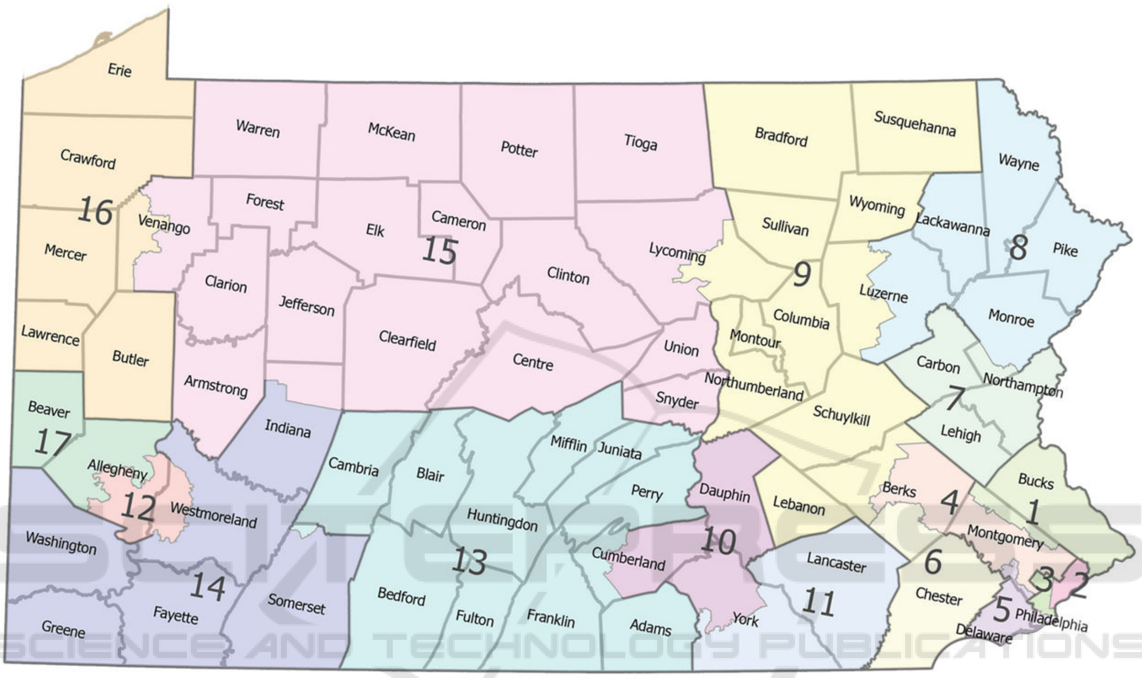

Figure 2: Population Balance of the Districts Drawn by the Program (Picture credit: Original).

5 CONCLUSION

Using the linear programming model, out program successfully provided an optimal districting for the Republican Party in the 2012 Pennsylvania election. With this approach, the author found a solution that can secure 17 out of the 18 seats for the Republican Party despite they only won 49.9% of statewide votes. The computer calculated districts differ significantly from the actual districting in 2012, but they share some similar principles in strategies. For example, when aiming for maximum partisan interest, a party should try to let itself win districts by a small margin and let the opposing party win districts by a large margin.

When the author looks at the results and reflects on this study, the author realize that it may have deeper implications regarding the fairness and reasonableness of doing elections by districts and the majoritarian system. Since the Roman Republic, societies have tried to reflect their citizens' opinions by electing representatives from each area with the hope that the unique opinions of every geographical area, big or small, can be represented.

However, from the House of Commons in Britain, to the Electoral College in the United States, to the House of Representatives election in each state, the district-based majoritarian system has caused troubles because such elections cannot reflect the true popular opinion overall. Indeed, when leaders are chosen based on district elections and under a majoritarian system, the party that is more popular overall in a state or a nation does not always get the dominant influence they deserve. With gerrymandering strategies, a party can obtain influence in a state or a nation that is more than the support they truly have among the people.

These issues force the author to confront with a critical issue: how to balance geographic representation with the accurate reflection of popular will. While the district-based majoritarian system aims to give voice to local communities, they often distort outcomes-especially when manipulated through gerrymandering. To uphold democratic integrity, the author must confront these structural flaws and pursue reforms-this is an urgent, important, yet challenging task for all of people.

First of all, the author did not apply all of the constraints for gerrymandering in this study. As a result, the model and result does not fully reflect the real life scenario. Although the author lifts the restraints on purpose, in the future, researchers should endeavor to create more complex and robust models that also addresses the other three constraints-contiguity, no enclaves, and compactness-since the model did not function well without all of the restraints being included. Researchers can try to model the counties with more complex data structures such as graphs or use vectors to record a county's location relative to other counties.

In addition, the author did not allow counties to be split into different districts, and the population equality constraint was affected by that. In the future, researchers can try to break down a state to the city level for a more realistic and accurate model.

Furthermore, in future studies, researchers can try to replicate this study in different states and in different years and look for patterns on a broader scope. They may also try to aim for the maximum benefit of the Democratic Party instead of the Republican Party or apply a linear programming model to a situation with more than two parties. They may also try to aim for optimal fairness instead of optimal partisan gain when drawing the districts with a linear programming model.

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