Application of the Simplex Method in Tasks for Determining an Optimal Production Program

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Abstract: In this paper is presented a method for solving linear optimization problems. The Simplex method's algorithm is described. The specific practical problem related to finding an optimal solution to an economic task has been formulated and implemented using the Simplex method. The method is versatile, applicable to a wide range of tasks across various fields. It operates as a sequence of finite iterations and allows for identifying model characteristics, such as the existence of alternative optima and unsolvability. The Solver tool from Microsoft Excel is presented for deciding problems in the field of linear and nonlinear programming.

1 INTRODUCTION

The Simplex method, developed by George Dantzig, is a universal approach for solving linear optimization problems. Its main idea is to move from one feasible solution to a better one until the optimal solution is found or it is determined that no solution exists (Ansari, 2019). Many practical problems can be modeled using linear mathematical models (Nabli, 2009). For instance, companies often face challenges in combining available resources to determine which products to manufacture to maximize profits while minimizing costs. The problems associated with the process of maximizing profits are the process of finding optimal solutions in production (Anggoro et al., 2019).

This paper demonstrates the practical application of the Simplex method in a specific economic problem. A mathematical model is created. The model has been adduced to simplex canonical form. The solution has been implemented using the Simplex method and using computer software. The Solver tool from Microsoft Office Excel has been used for the computer implementation. Solver provides an opportunity to solve practical problems that can be mathematically described and represented as a linear or nonlinear optimization model.

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2 MATERIALS AND METHODS

2.1 Steps of Solving Linear Optimization Models with the Simplex Method

The Simplex method analyzes the vertices of a polyhedron using a standard model form and examines feasible basic solutions. The process begins with an initial basic solution, which is checked for optimality. If the solution is optimal, the procedure ends, and the solution is displayed. If not, the plan is improved by transitioning to a neighboring vertex with a better objective function value. If the objective function is unbounded, the model is unsolvable, and the procedure terminates (Avramov & Grozev, 2009).

2.2 Algorithm

Data from the model in simplex canonical form shown on Figure 2 are entered into the Simplex table (Table 1).

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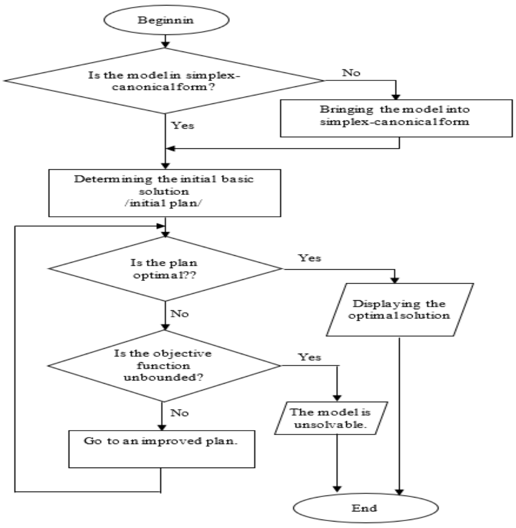

Figure 1: The main stages of solving linear optimization problems using the Simplex method.

$$Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n \rightarrow \min(or \max)$$
$$\alpha_{11}x_1 + \alpha_{12}x_2 + \cdots + \alpha_{1n}x_n = \beta_1$$
$$\alpha_{21}x_1 + \alpha_{22}x_2 + \cdots + \alpha_{2n}x_n = \beta_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$\alpha_{m1}x_1 + \alpha_{m2}x_2 + \cdots + \alpha_{mn}x_n = \beta_m$$

Figure 2: Simplex canonical form.

Table 1: Simplex table.

B	C_B	β	X_1	...	X_{k1}	...	X_{k2}		...	X_{kp}	...	X_q	...	X_{km}	...	X_n
			C_1	...	C_{k1}	...	C_{k2}		...	C_{kp}	...	C_q	...	C_{km}	...	C_n
X_{k1}	C_{k1}	β_1	α_{11}	...	1	...	0		...	0	...	α_{1q}	...	0	...	α_{1n}
X_{k2}	C_{k2}	β_2	α_{21}	...	0	...	1		...	0	...	α_{2q}	...	0	...	α_{2n}
.	.	.	.	.	.	.	.		.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.		.	.	.	.	.	.	.	.
X_{kp}	C_{kp}	β_p	α_{p1}	...	0	...	0		...	1	...	α_{pq}	...	0	...	α_{pn}
.	.	.	.	.	.	.	.		.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.		.	.	.	.	.	.	.	.
X_{km}	C_{km}	β_m	α_{m1}	...	0	...	...		...	0	...	α_{mq}	...	1	...	...
$Z(X)=\Delta_0$			Δ_1	...	0	...	0		...	0	...	Δ_q	...	0	...	Δ_n

The first simplex table is filled in. The value of the objective function and the index estimates are calculated.

- Z(X) is calculated by multiplying the columns C_B and β element by element and adding the resulting products.
- The index estimate Δj of each variable is calculated by multiplying the column CB element by element with the column of the variable, adding the resulting products and subtracting its target coefficient. The index estimates of the basic variables are always 0.

  Check Optimality
- If the index estimates of all variables are non-negative $\Delta_j \geq 0,\ j = \overline{1,n}$, the plan is optimal when searching maximum value of the objective function (Z→max) and the algorithm ends.
- If the index estimates of all variables are less than or equal to zero ($\Delta_j \leq 0,\ j = \overline{1,n}$), the plan is optimal when searching minimum value (Z→min) and the algorithm ends.
- If there is an unfavorable index score (Δq) in whose column there is no positive value ($\alpha_{iq} \leq 0,\ i = \overline{1,m}$) – the objective function is unbounded and the model is unsolvable.
- If for each j for which there is an unfavorable index score Δj there is at least one positive value in the column, then the plan is not optimal and we move on to the next plan.

  Moving to a neighbour plan

  A pivot column is determined – column q, in which the index score (Δ_q) is unfavorable. If there are more than one unfavorable index scores, the more unfavorable one is selected. The variable of this pillar becomes the new basic variable in the new plan.

- A pivot row is determined – If exists $i \in \{1, 2, \dots, m\}$, for $\alpha_{iq} > 0$ which the minimum ratio ($\frac{\beta_p}{\alpha_{pq}} = \min_{\alpha_{iq}>0} \frac{\beta_i}{\alpha_{iq}}$) is selected.
- Pivot number – the intersection of the pivot row with the pivot column determines the pivot number (α_{pq}).
- A new simplex table of a neighbor plan is constructed, in which:
  - In columns B and C_B, only one change is made - X_{kp} is replaced by Xq and C_{kp} is replaced by Cq.
  - The elements of the pivot row are divided by the pivot number.

$$\beta_p = \frac{\beta_p}{\alpha_{pq}} \quad and \quad \alpha_{pj} = \frac{\alpha_{pj}}{\alpha_{pq}}, \quad j = \overline{1,n} \qquad (1)$$

The remaining values are obtained according to the rectangle rule (Avramov & Grozev, 2009).

$$\alpha'_{1n} = \frac{\alpha_{1n}\alpha_{pq} - \alpha_{1q}\alpha_{pn}}{\alpha_{pq}} \qquad (2)$$

From the product of the corresponding element of the old table with the pivot number is subtracted with the product of the element from the same row in the pivot column with the element from the same column in the pivot row and the result is divided by the pivot number (Figure 3).

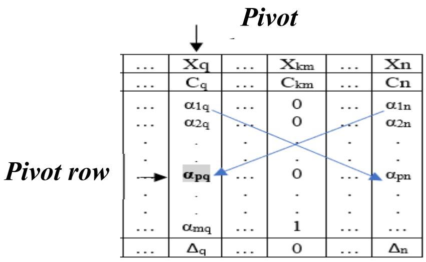

Figure 3: Rectangle rule.

- The value of the objective function and the index estimates are calculated according to the rules from step 1 and are proceed to step 2 of the algorithm.

3 RESULTS AND DISCUSSION

3.1 Problem Statement

A company plans to produce two types of products. Resources required for production are limited, necessitating optimization. The required resources per product type and their availability are given in Table 2.

Table 2: Resource Data

Resource	Item Type		Quantity
	A	B	
Resource 1	2	3	480
Resource 2	4	3	720
Resource 3	3	5	810

How many products of each type should be produced to obtain maximum profit, if it is known that the profit from one product of type A is 4 BGN, and from type B is 5 BGN?

3.2 Solution with Simplex Method

Mathematical description of the problem

- X_1, X_2 - number of products of type A and B.
- $X = (X_1, X_2)$ – production program of the company and $X_1 \geq 0, X_2 \geq 0$.
- Z – total amount of profit for all manufactured products. $Z= 4X_1+5X_2$ -> max
- The consumption of the three types of resources for the desired production program X are:

$$2X_1+3X_2 \leq 480$$
$$4X_1+3X_2 \leq 720$$
$$3X_1+5X_2 \leq 810$$

The task of finding a production program of company is reduced to the following linear optimization mathematical model:

$$Z= 4X_1+5X_2 \rightarrow \max$$
$$2X_1+3X_2 \leq 480$$
$$4X_1+3X_2 \leq 720$$
$$3X_1+5X_2 \leq 810$$
$$X_1 \geq 0,\ X_2 \geq 0$$

The model is abducted into simplex canonical form. The initial basis solution is determined.

$$Z= 4X_1+5X_2 \rightarrow \max$$
$$2X_1+3X_2+X_3=480$$
$$4X_1+3X_2+X_4=720$$
$$3X_1+5X_2+X_5=810$$
$$X_j \geq 0,\ j = \overline{1,5}$$

$X^0(0,0, 480, 720, 810)$ – the initial basis solution

The model is transferred to the first simplex table (Table 3).

Table 3: First simplex table

B	C_B	β	X_1	X_2	X_3	X_4	X_5
			4	5	0	0	0
→ X_3	0	480	2	3	1	0	0
X_4	0	720	4	3	0	1	0
X_5	0	810	3	5	0	0	1
$Z^{(0)}$=0			**-4**	**-5**	**0**	**0**	**0**

$Z^{(0)}=0.480+0.720+0.810=0$

$\Delta_1=0.2+0.4+0.3-4=-4$

$\Delta_1<0$ and $\Delta_2<0$ => The plan is not optimal. The objective function is checked for unboundedness. The pivot column, pivot row and pivot number are determined.

Pivot column – $\min(\Delta_1, \Delta_2)= \Delta_2$

Pivot row – $\min(480/3,720/3,810/5)=480/3$

Pivot number – 3

An improved plan is created. A check for optimality is performed (Table 4).

Table 4: Second simplex table.

B	C_B	β	X_1	X_2	X_3	X_4	X_5
			4	5	0	0	0
→ X_2	5	160	2/3	1	1/3	0	0
X_4	0	240	2	0	-1	1	0
X_5	0	10	-1/3	0	-5/3	0	1
$Z^{(1)}$=800			**-2/3**	**0**	**5/3**	**0**	**0**

One change is made in columns B and C_B, by writing the new basic variable X_2 and its target coefficient 5. The row of the new basic variable is filled in, by dividing the value from the first simplex table on the pivot number. The remaining values are calculated according to the rectangle rule (Figure 4).

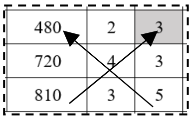

Figure 4: Rectangle rule.

β_3=(810.3-5.480)/3=10

Δ_1<0 => The plan is not optimal. The objective function is checked for unboundedness. The pivot column, pivot row and pivot number are determined.

Pivot column – the column of variable X_1.

Pivot row – the row of variable X_4.

Pivot number – 2.

An improved plan is created. A check for optimality is performed (Table 5).

Table 5: Third simplex table.

B	C_B	β	X_1	X_2	X_3	X_4	X_5
			4	5	0	0	0
X_2	5	80	0	1	2/3	-1/3	0
X_1	4	120	1	0	-1/2	1/2	0
X_5	0	50	0	0	-11/6	1/6	1
$Z^{(2)}$=880			**0**	**0**	**4/3**	**1/3**	**0**

Zmax=880, X_1=120, X_2=80, X_3=0, X_4=0, X_5=50

The company will realize a maximum profit of 880 BGN when it produces 120 products of type A and 80 products of type B. In the optimal solution, the variable X_3 are equal to 0, X_4 is equal to 0, and X_5 is 50. These additional variables that we added to the model reflect the unused quantities of resources. Therefore, resources Resource 1 and Resource 2 are fully used, but there is a rest of 50 units of resource Resource 3.

3.3 Solution with Solver Tool in Microsoft Excel

Solving the problem using the Solver tool in Microsoft Excel requires entering the model into an Excel worksheet (Figure 5).

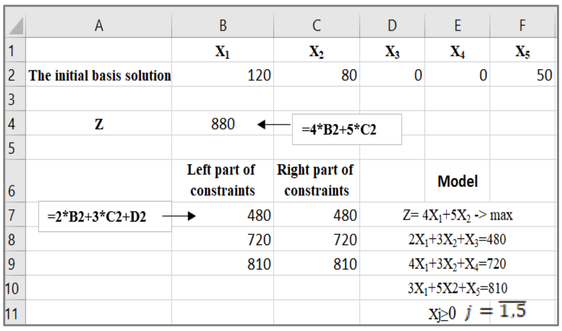

Figure 5: Model entered into Microsoft Excel Worksheet

From the Data menu, the *Solver tool* is selected. In the Solver Parameters window (Figure 6), the model parameters are defined.

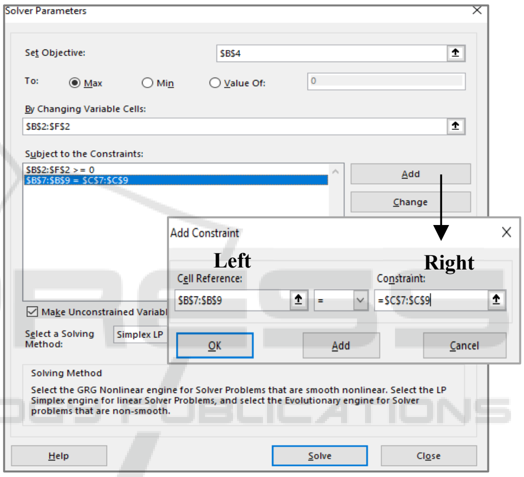

Figure 6: Model entered into window Solver Parameters.

After running the Solver, the optimal solution is displayed, allowing the user to save and generate a report. The optimization algorithm remains hidden from the user (Ivanova, 2014).

4 CONCLUSIONS

This article presents the algorithm of the simplex method for solving linear optimization problems. We explain the theoretical part of the algorithm and illustrated it in one practically task. The method is applied to a specific problem related to the production. This problem we modeled as a linear optimization problem. We used a simplex method to find the maximum of the objective function, respecting all the constraints in the model. The problem was solved using the Solver tool of Microsoft Excel also. The Simplex method is universal, applicable to a wide range of problems in various fields.

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