

Adaptive Trajectory Prediction in Roundabouts Using Moving Horizon Estimation

Selsabil Bougherara¹, Hasni Arezki¹, Chouki Sentouh^{1,2}, Jérôme Floris¹ and Jean-Christophe Popieul^{1,2}

¹LAMIH UMR CNRS 8201, Université Polytechnique Hauts-de-France, Valenciennes, France

²INSA Hauts-de-France, Valenciennes, France

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Abstract: This paper addresses the challenge of trajectory prediction for automated vehicles navigating within roundabouts, where interactions, non-linear motion, and rapid decision-making complicate traditional approaches. We propose a novel prediction framework based on Moving Horizon Estimation (MHE) combined with a non-linear kinematic bicycle model. Unlike conventional methods such as the Extended Kalman Filter (EKF), the proposed MHE-based framework leverages past observations over a sliding time window, enhancing robustness against model uncertainties and noise. The method is validated through simulations using the SHERPA driving simulator in both static and dynamic maneuvering. The results demonstrate that MHE significantly outperforms the EKF in terms of prediction accuracy, particularly during complex vehicle behaviors. This work constitutes a foundational step toward enhancing safety and robustness of decision-making in roundabouts.

1 INTRODUCTION

Autonomous driving has made significant progress in recent years, particularly in relatively simple and well-structured environments such as highways, where vehicle speed remains nearly constant during the decision-making process (Benloucif, 2018; Oudainia, 2023). For instance, authors of (Guo et al., 2018) proposed an automated highway merging strategy that accounts for interactions with surrounding vehicles. However, real-world road environments often involve far more complex structures, such as intersections and roundabouts, where right-of-way rules, multi-agent interactions, and frequent trajectory changes make the decision-making process considerably more challenging. To date, only a limited number of studies have specifically addressed autonomous driving in roundabouts. A notable contribution is that of (Bellingard, 2023), who proposed a decision-making framework for navigation in such environments. However, this work does not rely on explicit trajectory prediction, but rather on sensor data analysis, which can be insufficient when perceived information is uncertain or incomplete.

Navigating through a roundabout consists of three main phases: insertion, circulation, and exit. This

study focuses on the insertion phase, which is considered the most critical, as it requires an immediate decision: the autonomous vehicle must determine whether to merge into the circulating flow or yield to vehicles already in the roundabout. This decision heavily depends on the ability to reliably predict the trajectories of nearby vehicles.

In the literature, trajectory prediction methods are generally categorized into two main groups: parametric and non-parametric approaches. Parametric approaches rely on simplified physical motion models, such as constant velocity or constant acceleration models (Schubert et al., 2008), kinematic bicycle models (Kuwata et al., 2008), or dynamic bicycle models (Brannstrom et al., 2010). While these models can be effective in simple contexts, they fail to capture the complexity of maneuvers typically encountered in roundabouts. They are based on idealized assumptions that do not fully account for vehicle kinematics or the variability of human driving behavior.

Non-parametric approaches, on the other hand, employ machine learning and deep learning techniques (Huang et al., 2022; Wang et al., 2023). Several studies utilize Recurrent Neural Networks (RNNs), particularly Long Short-Term Memory (LSTM) architectures (Altché and de La Fortelle,

2017; Park et al., 2018), to model the temporal dynamics of vehicle trajectories. Although powerful, these methods require large amounts of training data and considerable computational resources, which increase processing time and hinder real-time decision-making. Other works have proposed more elaborate behavioral models. For example, Wiest et al. (Wiest et al., 2012) used Gaussian regression to model the probabilistic distribution of past trajectories, while Firl et al. (Firl et al., 2012) employed a hidden Markov process combined with a reward function to determine the optimal strategy through dynamic programming. However, these methods often assume relatively deterministic motion patterns and struggle to represent the irregular behavior of human drivers in complex environments.

In contrast to the above approaches, several probabilistic methods have been developed to explicitly account for uncertainty in trajectory prediction. The Extended Kalman Filter (EKF) (Guo et al., 2018) has been widely used for state estimation but suffers from linearization errors, especially in nonlinear scenarios such as roundabouts. Monte Carlo-based methods, such as particle filters (Gustafsson, 2010), allow for the representation of non-Gaussian distributions and can model complex interactions between vehicles (Lefèvre et al., 2014). However, they are computationally expensive, sensitive to particle degeneracy, and poorly suited for real-time applications. Another approach involves the use of stochastic reachable sets, which aim to encapsulate all possible trajectories under uncertainty while respecting dynamic and environmental constraints (Du Toit and Burdick, 2011). While this method provides formal safety guarantees, it tends to be overly conservative and difficult to implement online due to its computational complexity. It is important to note that most of these approaches have been applied in relatively simple scenarios, such as highway driving or linear merging situations. In contrast, our study focuses on a more complex environment: predicting the trajectories of vehicles operating within a roundabout.

To overcome the limitations of existing methods, Moving Horizon Estimation (MHE) emerges as a robust and promising alternative for trajectory prediction. Unlike traditional filters that only consider the current state, MHE employs a sliding window of past observations and solves a constrained optimization problem at each step. This approach enables more accurate state estimation and facilitates the handling of physical constraints. Moreover, MHE naturally accommodates nonlinear models without relying on linearization approximations and provides explicit uncertainty quantification. These features make it par-

ticularly well-suited for trajectory prediction in complex urban environments, such as roundabouts.

In this paper, we propose a novel trajectory prediction framework based on Moving Horizon Estimation (MHE), coupled with a nonlinear kinematic bicycle model, for vehicles navigating roundabouts. The approach can be integrated into a multi-level cooperative architecture between human drivers and automated vehicles to handle complex situations such as roundabouts. Our method leverages MHE's robustness in state estimation and employs model propagation for short-term trajectory forecasting.

Simulation results demonstrate significant improvements in prediction accuracy and robustness compared to Extended Kalman Filter (EKF)-based approaches, confirming the effectiveness of MHE in handling the complex dynamics and uncertainties inherent to roundabout scenarios.

The remainder of this paper is organized as follows: Section 2 introduces the kinematic bicycle model used for trajectory prediction. Section 3 presents the proposed MHE-based prediction framework. Section 4 discusses the experimental results and provides a comparison with the baseline EKF-based methods. Finally, Section 5 concludes the paper and outlines directions for future research.

2 VEHICLE MODEL

To represent the motion of surrounding vehicles in a roundabout, we use a nonlinear kinematic bicycle model that captures both longitudinal and lateral vehicle dynamics from (Rajamani et al., 2020). Figure 1 illustrates the structure and geometric parameters of the kinematic bicycle model. The vehicle model is a single-track model, where the two axles are aligned and steering is applied only to the front wheel. We define the system state vector as:

$$x = [X \ Y \ \psi \ v]^T$$

Where X is the longitudinal vehicle position, Y is the lateral vehicle position, ψ is the vehicle yaw angle, and v is the speed of the vehicle. Based on the geometry of the model in Figure 1, the continuous-time nonlinear state dynamics are written as:

$$\dot{x} = \begin{bmatrix} v \cos(\psi + \beta) \\ v \sin(\psi + \beta) \\ \frac{v}{l_r} \sin(\beta) \\ a \end{bmatrix} \quad (1)$$

with

$$\beta = \tan^{-1} \left(\frac{l_r}{l_f + l_r} \tan(\delta_f) \right)$$

with β is the slip angle at the center of mass, accounting for the orientation difference between the velocity vector and the vehicle's longitudinal axis. δ_f is the front steering angle. The constants l_f and l_r represent the distances from the center of mass to the front and rear axles, respectively. The output of the system is given by:

$$y = Cx, \quad \text{with} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Positions X and Y are obtained through vehicle-to-vehicle (V2V) communication. Finally, we assume that the rear wheels are not steerable ($\delta_r = 0$), as is the case in most standard passenger vehicles, and that the acceleration a is either measured or estimated separately.

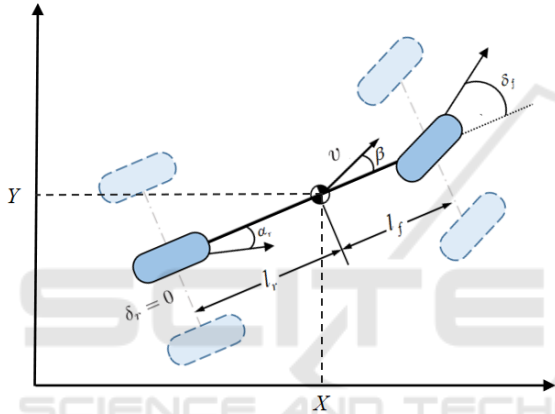


Figure 1: Kinematic bicycle model.

The discrete-time nonlinear kinematic bicycle model is obtained by applying Euler discretization with a fixed time step $\Delta t = 0.1$ seconds to the continuous-time dynamics:

$$x_{k+1} = f(x_k, u_k) = \begin{bmatrix} x_k + \Delta t \cdot v_k \cos(\psi_k + \beta_k) \\ y_k + \Delta t \cdot v_k \sin(\psi_k + \beta_k) \\ \psi_k + \Delta t \cdot \frac{v_k}{l_r} \sin(\beta_k) \\ v_k + \Delta t \cdot a_k \end{bmatrix} \quad (2)$$

with

$$\beta_k = \tan^{-1} \left(\frac{l_r}{l_f + l_r} \tan(\delta_{f,k}) \right)$$

3 PROPOSED METHODOLOGY

As emphasized in the introduction, the insertion phase in roundabouts requires fast and accurate prediction of surrounding vehicle trajectories to enable safe

decision-making. To address the challenge of trajectory prediction in roundabouts, we propose a framework that combines model-based estimation and prediction. A nonlinear kinematic bicycle model is used to represent vehicle dynamics, offering a good trade-off between accuracy and computational efficiency, especially for curved trajectories. For state estimation, we adopt Moving Horizon Estimation (MHE), which outperforms traditional filters like the EKF by optimizing over a sliding window and directly handling nonlinearities and constraints, thus improving robustness to noise, model mismatches, and sudden changes.

MHE is based on the idea of minimizing a quadratic estimation cost function defined on a backward sliding window composed of a finite number of time stages, which will be denoted by the integer $N_e \geq 1$. To this end, we define the standard quadratic objective function:

$$J_t^{N_e}(\hat{x}_{t-N_e}) = \mu |\hat{x}_{t-N_e} - \bar{x}_{t-N_e}|^2 \eta^{N_e} + \nu \sum_{i=t-N_e}^{t-1} \eta^{t-1-i} |y_i - h(\hat{x}_{i|t}, 0)|^2 \quad (3)$$

where $\eta \in (0, 1)$ and $\mu, \nu > 0$ under the constraints:

$$\hat{x}_{i+1|t} = f(\hat{x}_{i|t}, 0), \quad i = t - N_e, \dots, t - 1 \quad (4)$$

and thus $\hat{x}_{t-N_e+1|t}, \dots, \hat{x}_{t|t}$ are generated by $\hat{x}_{t-N_e|t}$. We denote by MHE_{N_e} what results from the minimization of the cost function (3) as follows:

$$\left\{ \begin{array}{l} \hat{x}_{0|t} \in \left\{ \underset{\hat{x}_0 \in \mathcal{X}}{\text{argmin}} J_t^t(\hat{x}_0) \text{ s.t. (4)} \right. \\ \quad \left. \text{holds for } t = 1, \dots, N_e \right\} \\ \hat{x}_{t-N_e|t} \in \left\{ \underset{\hat{x}_{t-N_e} \in \mathcal{X}}{\text{argmin}} J_t^{N_e}(\hat{x}_{t-N_e}) \text{ s.t. (4)} \right. \\ \quad \left. \text{holds for } t = N_e + 1, N_e + 2, \dots \right\} \end{array} \right.$$

together with (4), which provides $\hat{x}_{t|t}$. For further ease of presentation, note that the cost function is given by:

$$J_t^t(\hat{x}_0) = \mu |\hat{x}_0 - \bar{x}_0|^2 \eta^t + \nu \sum_{i=0}^{t-1} \eta^{t-1-i} |y_i - h(\hat{x}_{i|t}, 0)|^2 \quad (5)$$

for all $t \leq N_e$.

The trajectory of a tracked vehicle is typically predicted using model-based propagation. This approach is consistent with the vehicle's dynamics, as defined by the discrete-time nonlinear model (2), and enables accurate state prediction when future control inputs u_{k+i} (acceleration and steering angle) are known or

can be reasonably estimated. Moreover, it inherently respects the physical constraints of the system. An additional key advantage lies in its computational efficiency: the recursive computation of future states using the dynamic model can be performed with minimal computational cost, making it well-suited for real-time applications in embedded systems. Given the discrete-time nonlinear model (2), the predicted trajectory over a prediction horizon of N_p steps is obtained recursively as follows:

$$\begin{aligned}\hat{x}_{k+1|k} &= f(\hat{x}_k, u_k), \\ \hat{x}_{k+2|k} &= f(\hat{x}_{k+1|k}, u_{k+1}), \\ &\vdots \\ \hat{x}_{k+N_p|k} &= f(\hat{x}_{k+N_p-1|k}, u_{k+N_p-1}),\end{aligned}\tag{6}$$

where $\hat{x}_{k+i|k}$ denotes the predicted state at time $k+i$ based on information available at time k , and N_p is the prediction horizon.

4 EXPERIMENTAL VALIDATION

To validate the applicability of our trajectory prediction approach in a realistic environment, we performed simulations using the SHERPA dynamic driving simulator of the LAMIH laboratory. SHERPA is a high-fidelity platform designed to reproduce real-time vehicle dynamics. The experimental scenario consists of a single vehicle navigating a multilane roundabout. The objective is to predict the future trajectory of this vehicle using only its current and past position measurements, while accounting for the curved motion typical of roundabout driving. This setup enables a realistic evaluation of our prediction framework. Figure 2 shows both the SHERPA simulator and a visual representation of the roundabout scenario used during testing.

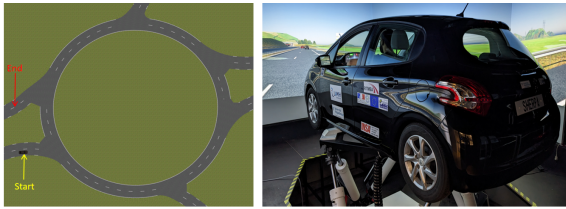


Figure 2: SHERPA dynamic driving simulator on the roundabout scenario.

Utilizing the *MATLAB Yalmip* toolbox, the LMI-based tuning procedure (Arezki et al., 2023) provides the following parameters in (3), which are fixed to $\mu = 0.01$, $\nu = 1$, and $\eta = 0.9$. The estimation horizon is set to $N_e = 10$, meaning the MHE optimization

is performed over a backward sliding window of 10 time steps. Additionally, the prediction horizon is set to $N_p = 10$, enabling the computation of future state trajectories over 10 steps ahead.

Given the fixed sampling time of $\Delta t = 0.1$ seconds, both horizons correspond to a one-second duration. This setup allows the MHE framework to simultaneously benefit from a sufficient estimation memory and a meaningful short-term forecast interval. These parameters were chosen to ensure the robustness and accuracy of the proposed approach. The initial estimated state is set to $[0 \ 0 \ 0 \ 0]^\top$.

Simulation runs are conducted with both system and measurement noise, generated according to zero-mean Gaussian distributions with a covariance of 0.01. For comparison purposes, an Extended Kalman Filter (EKF) was implemented using the same nonlinear kinematic bicycle model. The EKF was initialized with the same initial estimate as the MHE, and the process and measurement noise covariances were selected based on model assumptions and sensor characteristics. The measurement noise covariance matrix R was constructed using the known standard deviations of the position measurements: 1.5 m for the longitudinal position and 0.4 m for the lateral position:

$$R = \begin{bmatrix} (1.5)^2 & 0 \\ 0 & (0.4)^2 \end{bmatrix}$$

The process noise covariance matrix Q was set empirically to reflect reasonable uncertainty in state evolution, particularly during maneuvers. We assumed higher uncertainty in velocity and moderate noise in position and yaw:

$$Q = \text{diag}(0.01, 0.01, 0.001, 0.05)$$

These values were tuned to ensure stable filter behavior and allow for a fair comparison with the MHE-based approach under the same simulation conditions.

4.1 Scenario 1: Without Maneuvering

In this scenario, the vehicle circulates within the roundabout without executing any maneuvers. It follows a trajectory with constant acceleration, indicating the absence of significant braking or acceleration throughout the sequence.

The scenario is illustrated in the following video: *Watch the video on YouTube*. Figure 2 shows, on the left, a thumbnail from the video, and on the right, the SHERPA simulator equipped with a real vehicle used for testing.

Trajectory predictions over a one-second horizon, computed according to the recursive formulation in Equation (6), are presented in Figures 3 and 6. In

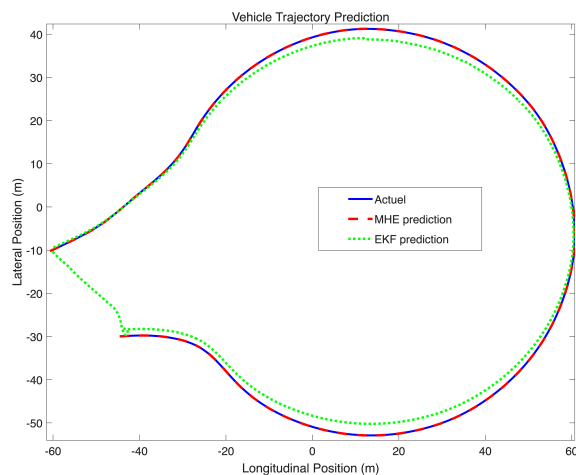


Figure 3: Comparison of state prediction performance between MHE and EKF without manoeuvres.

the absence of maneuvers, the MHE consistently provides predictions that closely match the actual states obtained from the SHERPA simulator, particularly for velocity and yaw angle. The EKF exhibits larger deviations, especially during the initial acceleration phase. These results highlight the superior accuracy and robustness of MHE, even under steady driving conditions.

4.2 Scenario 2: With Maneuvering

In this scenario, the ego vehicle performs two key maneuvers in response to the presence of a red vehicle ahead.

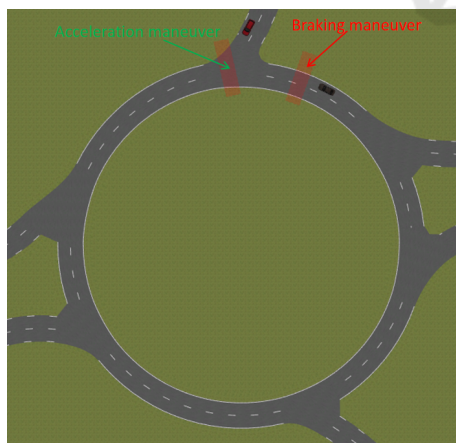


Figure 4: Illustration of the vehicle maneuvers in a roundabout scenario. The ego vehicle performs a braking maneuver (in red) to avoid a slower vehicle ahead, followed by an acceleration maneuver (in green) once the path is clear.

- First, upon detecting the red vehicle, it initiates a braking maneuver, reducing its speed sharply

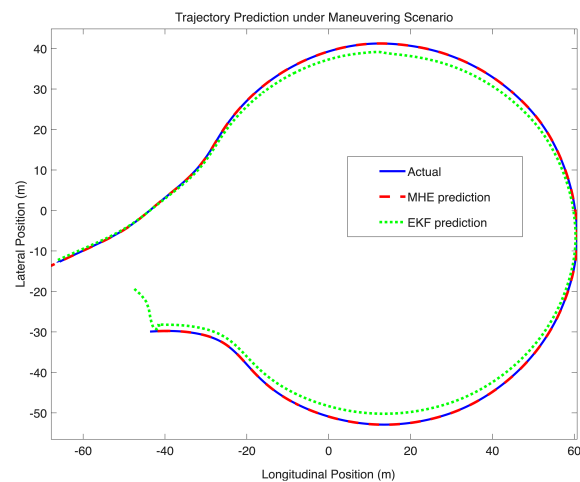


Figure 5: Comparison of state prediction performance between MHE and EKF under manoeuvres

from 30 km/h to 10 km/h in a short time window in order to maintain a safe distance. This deceleration occurs between $t = 20\text{ s}$ and $t = 22\text{ s}$, following a constant cruising speed of 30 km/h from $t = 0\text{ s}$ to $t = 20\text{ s}$.

- Then, once the red vehicle is overtaken or is no longer obstructing the lane, the ego vehicle initiates an acceleration maneuver, increasing its speed from 10 km/h to 40 km/h over the interval $t = 22\text{ s}$ to $t = 30\text{ s}$.

The speed profile, including the abrupt braking and smooth re-acceleration phases, was modeled in the SHERPA simulator using custom event triggers that modify the vehicle's control inputs in real time, as illustrated in Fig. 4. The complete scenario can be viewed in the following video: *Watch the video on YouTube*

Trajectory predictions over a one-second horizon under the maneuvering scenario (Figures 5 and 7), which includes abrupt acceleration and braking events, show that the MHE maintains accurate tracking throughout the entire trajectory. The EKF, however, exhibits notable divergence, particularly in the lateral and longitudinal positions. These observations highlight the superior robustness and adaptability of MHE, even under dynamic and highly nonlinear vehicle behaviors.

To compare the performance of the two trajectory prediction methods, we conducted an analysis based on the Root Mean Square Error (RMSE) computed across multiple state variables. The results, summarized in Table 1, demonstrate the superior accuracy of the MHE approach compared to the EKF, with significant improvements observed across all metrics.

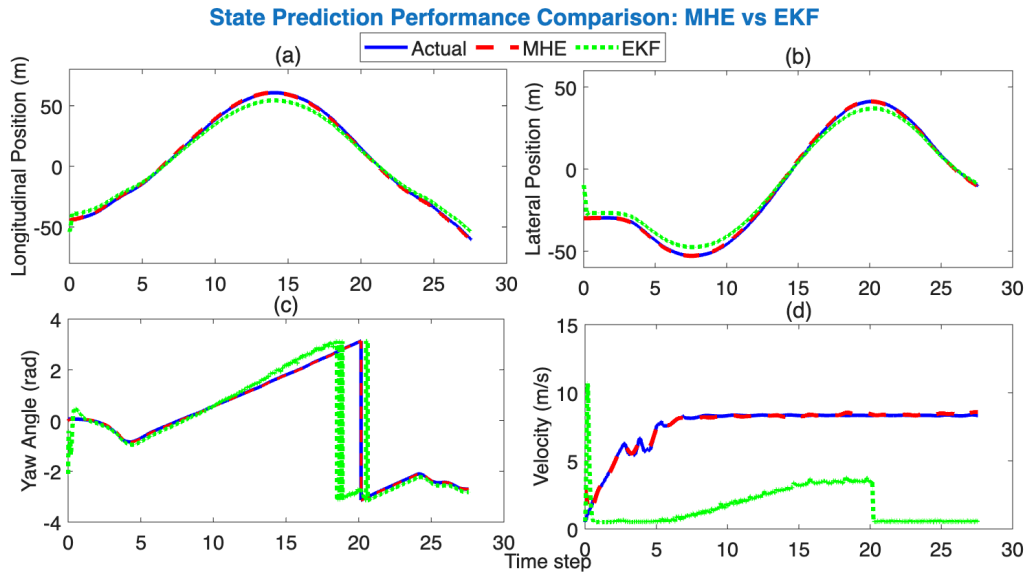


Figure 6: Comparison of state prediction performance between MHE and EKF without maneuvers.

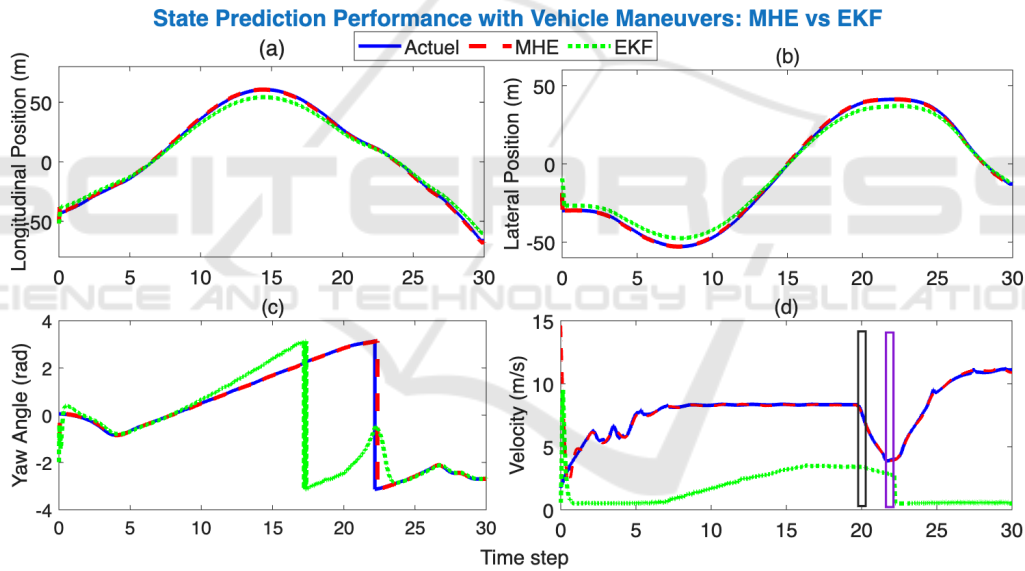
Figure 7: Comparison of state prediction performance under vehicle maneuvers. The ego vehicle performs two maneuvers: (1) a braking phase between $t = 20$ s and $t = 22$ s in response to a slower vehicle ahead (highlighted in black), and (2) an acceleration phase from $t = 22$ s to $t = 30$ s after overtaking (highlighted in violet).

Table 1: RMSE Comparison between MHE and EKF.

Variable	MHE	EKF	Gain (%)
Long. Pos. (m)	0.8790	3.8908	+77.41
Lat. Pos. (m)	0.7133	3.5648	+79.99
Yaw (rad)	0.4845	2.1032	+76.96
Velocity (m/s)	0.5132	6.6501	+92.28

5 CONCLUSION

This work presented a model-based trajectory prediction framework tailored to the complex dynamics of vehicle motion in roundabouts. By integrating a non-linear kinematic bicycle model with Moving Horizon Estimation (MHE), we achieved accurate and robust state predictions, even under maneuvering conditions. The comparative analysis with the EKF demonstrated that MHE provides significant improvements in esti-

mation accuracy and robustness to abrupt behavioral changes, particularly in lateral and longitudinal positions, velocity, and yaw angle.

The proposed trajectory prediction framework lays the groundwork for more advanced decision-making modules. As a future extension, the predicted trajectories will be integrated into a risk-aware decision-making system during the insertion phase into the roundabout. This integration will subsequently be extended to the circulation and exit phases. Such an approach will enable proactive navigation strategies and enhance safety in complex, dynamic environments, where anticipating the behavior of surrounding vehicles is critical.

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