Adaptive Output Control with a Guarantee of the Specified Control Quality

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Keywords: Dynamic System, Adaptive Control, Tube Method, Coordinate Replacement, Arbitrary Degree.

Abstract: The paper presents a modification of the classical algorithm of adaptive output control in order to guarantee that the signal is found in the set specified by the developer at any moment of time. The paper extends the algorithm to systems with arbitrary relative degree. The aim of current research is to design a control law that will ensure that the error between the output and the reference signal will be in the following set. The effectiveness of the proposed method is illustrated with mathematical modelling.

1 INTRODUCTION

Adaptive control is widely used in control with parametric uncertainty of plant and external bounded disturbances. Often, the goal of adaptive control is to stabilise the output of plant in a limited set for a finite time (Anderson,1985), (Annaswamy, 2021). To date, new adaptive algorithms have been developed to improve the quality of transients and reduce computational costs (Narendra, 2012), (Ioannou, 2012).

Plants with unit relative degree are often studied in the literature and can describe the process of liquid filling in tanks (Arslan, 2001), transmission dynamics in a mechanical gearbox (Farza, 2009), dynamics of oscillating systems (Khalil, 2001), etc. It is important that the same structure of the adaptive control law can be obtained for such objects by different control methods (direct compensation method, velocity gradient method (Chopra, 2008), (Campion, 1989) etc.), (Gnucci, 2021).

Nonlinear control methods (Furtat, 2021) have been proposed earlier with the guarantee of finding the output variables in the given sets. However, these methods are applicable under the conditions of known parameters of the plant, the model of which has unit relative degree.

The paper is organized as follows. Section 2 formulates the problem of adaptive tracking with constraints on the output variable. In Section 3, a control law is first synthesized under the assumption that the derivatives of the plant's output signal are available for measurement. This solution is then generalized to the case when these derivatives are unmeasurable. Section 4 presents a numerical simulation that demonstrates the effectiveness of the proposed solution.

2 PROBLEM STATEMENT

Consider the dynamical system

$$Q(p)y(t) = kR(p)u(t) + f(t), \qquad (1)$$

where $t \geq 0$, $u(t) \in \mathbb{R}$ is the control signal, $y(t) \in \mathbb{R}$ is the measurable output signal, $f(t) \in \mathbb{R}$ is a bounded disturbance, $Q(p)$ and $R(p)$ are linear differential operators with constant coefficients and orders n and m respectively, the coefficients of $Q(p)$ and $R(p)$ are unknown, $k > 0$ is a known high-frequency gain, $p = d/dt$, and the plant (1) is minimum-phase.

Consider the reference model:

$$T(p)\, y_m(t) = k_m g_r(t),$$

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Kolesnik, N.
Adaptive Output Control with a Guarantee of the Specified Control Quality.
DOI: 10.5220/0013823900003982

In *Proceedings of the 22nd International Conference on Informatics in Control, Automation and Robotics (ICINCO 2025) - Volume 1*, pages 502-508
ISBN: 978-989-758-770-2; ISSN: 2184-2809

where $T(p)$ is a known normalized Hurwitz polynomial with real coefficients, $g_r(t)$ is a piecewise continuous, bounded reference signal, $y_m(t)$ is the output of the reference model, $k_m > 0$

The aim of the research is to design a control law that will ensure that the output error signal $e(t) = y(t) - y_m(t)$ is found in the following set of

$$E = \left\{ \underline{g}(t) < e(t) < \overline{g}(t) \right\} \quad \text{for any } \ t \geq 0, \tag{2}$$

where $\underline{g}(t)$ and $\overline{g}(t)$ are bounded functions with their first time derivatives. These functions are chosen by the designer based on the requirements of the system operation.

For example (see Figure 1), one can guarantee transients in a given tube whose boundaries monotonically converge to the neighbourhood of zero in a given time T. The description will be clearly demonstrated in the appendix at the end of the paper.

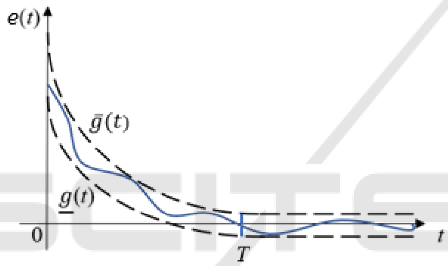

Figure 1: An illustration of output error.

3 SOLUTION

Let us represent the operators in (1) as the following sums:

$$R(p) = R_m + \Delta R(p), \quad Q(p) = Q_m + \Delta Q(p), \tag{3}$$

where $R_m(p)$ and $Q_m(p)$ are known differential operators of orders m and n, respectively, and $R_m(\lambda)$ and $Q_m(\lambda)$ are Hurwitz operators, $\Delta Q(p)$ and $\Delta R(p)$ are polynomials of orders not exceeding $n-1$ and $m-1$, respectively.

For plant (1), we define a reference model of the form

$$Q_m(p) y_m(t) = k_m R_m(p) g_r(t), \tag{4}$$

Let the control law be

$$u(t) = T(p)\upsilon(t), \tag{5}$$

where $T(p)$ is chosen so that the transfer function $\frac{R_m(p)T(p)}{Q_m(p)} = \frac{1}{p+a}$ has unit relative degree. Considering (3), (5), let us rewrite (1) as

$$\begin{aligned} y(t) = {} & \frac{kR_m(p)T(p)}{Q_m(p)T(p)}\upsilon(t) + \frac{kR_m(p)T(p)\Delta R(p)}{Q_m(p)R_m(p)T(p)}\upsilon(t) + \\ & \frac{kR_m(p)T(p)}{Q_m(p)R_m(p)T(p)} f(t) - \frac{kR_m(p)T(p)\Delta Q(p)}{Q_m(p)R_m(p)T(p)} y(t) + \\ & + \frac{kR_m(p)T(p)}{Q_m(p)R_m(p)T(p)} \in(t), \end{aligned} \tag{6}$$

where $\in(t)$ is the exponentially decaying function due to nonzero initial conditions.

Substituting (5) into (6), we obtain

$$\begin{aligned} y(t) = {} & \frac{k}{p+a}\left[\frac{1}{T(p)}\upsilon(t) + \frac{\Delta R(p)}{R_m(p)T(p)}\upsilon(t) - \right. \\ & \left. - \frac{\Delta Q(p)}{R_m(p)T(p)} y(t) + \frac{1}{R_m(p)T(p)} f(t) + \frac{1}{R_m(p)T(p)} \in(t) \right]. \end{aligned} \tag{7}$$

Having isolated the integer part in the summand $\frac{\Delta Q(p)}{R_m(p)T(p)} y(t) = c_{01} y(t) + \frac{\Delta\tilde{Q}(p)}{R_m(p)T(p)} y(t)$, we transform (7) to the form of

$$\begin{aligned} y(t) = {} & \frac{k}{p+a}\left[\upsilon(t) + \frac{\Delta R(p)}{R_m(p)}\upsilon(t) - c_{01} y(t) - \right. \\ & \left. - \frac{\Delta\tilde{Q}(p)}{R_m(p)T(p)} y(t) + \bar{f}(t) + \in(t) \right], \end{aligned} \tag{8}$$

where c_{01} is the integer part remaining when dividing $\Delta Q(p)$ to $R_m(p)T(p)$, c_{02} are coefficients of the polynomial $\Delta\tilde{Q}(p)$, c_{03} are coefficients of the polynomial $\Delta R(p)$ taken with opposite sign. $\bar{f}(t) = \frac{1}{R_m(p)T(p)} f(t)$ is a new bounded disturbance due to the boundedness of the original function $f(t)$ and Hurwitz polynomial $R_m(\lambda)T(\lambda)$. $\zeta_v(t) = \frac{1}{R_m(p)} v(t)$ and $\zeta_y(t) = \frac{1}{R_m(p)T(p)} y(t)$ represent the filtered signals at the output of the respective systems. When dealing with the tracking problem, we additionally consider the filter $\zeta_g(t) = \frac{1}{T(p)} g_r(t)$.

Given (7), let us rewrite (8) as

$$\begin{aligned} y = {} & \frac{k}{p+a}\left[\upsilon(t) - c_{01} y(t) - c_{02}^T \zeta_y(t) - \right. \\ & \left. - c_{03}^T \zeta_v(t) + \bar{f}(t) + \in(t) \right]. \end{aligned} \tag{9}$$

Let us introduce the notations

$$
\begin{aligned}
&c_0^T = -\left[c_{01}, c_{02}^T, c_{03}^T, \frac{k_m}{k}\right],\\
&\omega^T(t) = \left[y(t), \zeta_y(t), \zeta_\upsilon(t), g_r(t)\right],\\
&e(t) = y(t) - y_m(t),
\end{aligned}
\tag{10}
$$

where c_0 is the vector of constant unknown parameters, $\omega(t)$ is the regression vector.

Taking into account (9) and (10), let us write the dynamics of the error $e(t)$ as follows

$$
\dot{e}(t) = -ae(t) + k\left[\upsilon(t) - c_0{}^T\omega(t) + \bar{f}(t) + \epsilon(t)\right]. \tag{11}
$$

According to (Annaswamy, 1998) and (Furtat, 2021) to solve the control problem with given constraints, we introduce a replacement of the output variable y in the form of

$$
e(t) = \Phi(\varepsilon(t), t) = \frac{\bar{g}(t)e^{\varepsilon} + \underline{g}(t)}{e^{\varepsilon} + 1}, \tag{12}
$$

where $\varepsilon(t) \in \mathbb{R}$ is a continuous-differentiable function with respect to $t, \Phi(\varepsilon, t)$ satisfies the following conditions:

(a) $\underline{g}(t) < \Phi\varepsilon(t) < \bar{g}(t)$ for any $t \geq 0$ and $\varepsilon(t) \in \mathbb{R}$;

(b) there exists an inverse mapping $\varepsilon(t) = \Phi^{-1}(e, t)$ for any $e \in E$ and $t \geq 0$;

(c) the function $\Phi(\varepsilon, t)$ is continuous-differentiable with respect to ε and t and $\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon} \neq 0$ for any $e \in E$ and $t \geq 0$;

(d) the function $\frac{\partial\Phi(\varepsilon,t)}{\partial t}$ is bounded at $t \geq 0$ for any $\varepsilon(t) \in \mathbb{R}$. In that case $\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon} = \frac{e^{\varepsilon}(\bar{g}-\underline{g})}{(e^{\varepsilon}+1)^2}$ according to (12).

Now let us determine the dynamics on the variable ε to investigate the stability of the closed-loop system. For this purpose, we find the full time derivative of (12) as

$$
\dot{e}(t) = \frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\dot{\varepsilon} + \frac{\partial\Phi(\varepsilon,t)}{\partial t}.
$$

Since $\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon} \neq 0$, taking into account (12), let us rewrite the last equality as

$$
\begin{aligned}
\dot{\varepsilon} = &\left(\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right)^{-1}\left(-ae(t) + \left[\upsilon(t) - c_0{}^T\omega(t) + \right.\right.\\
&\left.\left. + \bar{f}(t) + \epsilon(t)\right] - \frac{\partial\Phi(\varepsilon,t)}{\partial t}\right).
\end{aligned}
\tag{13}
$$

That is, by using the coordinate transformation (13), the original problem with constraints is reduced to a problem without constraints. Now it is necessary to synthesise a control law u that provides input-state stability of the system (11).

Suppose that the derivatives of $e(t)$ are available for measurement. Let us define an estimation of axillary control signal $\tilde{\upsilon}(t)$. Then consider the control law in the form of

$$
\begin{aligned}
&u(t) = T(p)\tilde{\upsilon}(t),\\
&\upsilon(t) = c^T(t)\omega(t) + \frac{1}{k}\left[ae(t) + \frac{\partial\Phi(\varepsilon,t)}{\partial t} - \alpha\varepsilon(t)\right],
\end{aligned}
\tag{14}
$$

where $c(t)$ is bounded vector of adjustable parameters, $\alpha > 0$.

Substituting (14) into (13), we obtain

$$
\dot{\varepsilon} = \left(\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right)^{-1}\left(-\alpha\varepsilon(t) + \left[(c - c_0)^T\omega(t) + \bar{f}(t) + \epsilon(t)\right]\right). \tag{15}
$$

Let us formulate a theorem, the result of which will be valid with the assumption that the derivatives of $y(t)$ are measurable.

Theorem 1: Let the conditions (a)-(d) be satisfied, $\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}$ for any ε and t, and $sup\left\{\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right\} < \infty$ and the derivatives of $e(t)$ are measurable for the transformation (12) and bounded. Then for any $\alpha > 0, \beta > 0, \gamma > 0$, the control law (14) together with the adaptation algorithm

$$
\dot{c}(t) = -\beta\left(\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right)^{-1}\varepsilon(t)\omega(t) - \gamma c(t) \tag{16}
$$

guarantees that the output error signal $e(t)$ belongs to the set (2).

Let us rewrite the control law (14) taking into account that the derivatives of $e(t)$ are not measurable:

$$
\begin{aligned}
&u(t)=T(p)\tilde{\upsilon}(t),\\
&\tilde{\upsilon}(t)=L\xi(t)\\
&\dot{\xi}(t)=G_0\xi(t)+D_0(\tilde{\upsilon}(t)-\upsilon(t))\\
&\upsilon(t)=c^T(t)\omega(t)+\frac{1}{k}\left[ae(t)+\frac{\partial\Phi(\varepsilon,t)}{\partial t}-\alpha\varepsilon(t)\right].\\
&G_0=\begin{bmatrix}0 & I_{\gamma-2}\\ 0 & 0\end{bmatrix},\\
&D_0^T=\left[-\frac{d_1}{\mu},\ -\frac{d_2}{\mu^2},\ \ldots,\ -\frac{d_{\gamma-1}}{\mu^{\gamma-1}}\right],
\end{aligned} \tag{17}
$$

where the numbers $d_1,\ldots,d_{\gamma-1}$ are chosen so that the matrix $G=G_0-DL$ is Hurwitz, $D^T=[d_1,\ldots,d_{\gamma-1}]$, $\mu>0$ is a sufficiently small number.

Let us introduce vectors $\theta^T(t)=[\upsilon(t),\ldots,\upsilon^{(n)}(t)]$ and $\eta(t)=\Gamma^{-1}(\xi(t)-\theta(t))$, $\Gamma=diag\{\mu^{\gamma-2},\ldots,\mu,1\}$.
Finding the derivative of $\eta(t)$, we obtain

$$
\dot{\eta}(t)=\frac{1}{\mu}G\eta(t)-b\overset{(\gamma)}{\upsilon}(t),\quad \Delta\upsilon(t)=\tilde{\upsilon}(t)-\upsilon(t)=\mu^{\gamma-2}L\eta(t). \tag{18}
$$

Let us rewrite the equation with respect to the output $\Delta\upsilon(t)$:

$$
\dot{\bar{\eta}}(t)=\frac{1}{\mu}G\bar{\eta}(t)-\tilde{b}\dot{\upsilon}(t),\quad \Delta\upsilon(t)=\mu^{\gamma-2}L\eta(t). \tag{19}
$$

Here

$$
\begin{aligned}
&\bar{\eta}_i(t)=\eta_i(t)-\overset{(i-1)}{\upsilon}(t)/\mu^{\gamma-i-1},\quad i=2,\ldots,\gamma-1,\\
&\bar{\eta}_1(t)=\eta_1(t),\quad \tilde{b}^T=[1/\mu^{\gamma-2},0,\ldots,0].
\end{aligned}
$$

Then, based on the control law (17), we reduce the error equation (16) to the form

$$
\begin{aligned}
\dot{e}(t)=&-ae(t)+k\left[\upsilon(t)-c_0{}^T\omega(t)+\right.\\
&\left.+\bar{f}(t)+\epsilon(t)\right]+\mu^{\gamma-2}kL\bar{\eta}(t).
\end{aligned} \tag{20}
$$

Theorem 2: Let the conditions (a)-(d) be satisfied for the transformation (17), $\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}>0$ for any ε and t, and $sup\left\{\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right\}<\infty$ and the derivatives of $e(t)$ are not measurable. Then there exists such $\mu<\mu_0$ that for any $\alpha>0,\beta>0,\gamma>0$, the control law (17) together with the adaptation algorithm (16) guarantees that the output error signal $\mathrm{e}(t)$ belongs to set (2).

4 EXAMPLES

Consider the plant (1) with Q(p) and R(p) given in the form of

$$
Q(p)=(p-1)^2 \ \text{и}\ R(p)=3,
$$

The disturbance is represented as $f(t)=7+5\,sin(3t)+4\,cos(2t)+d(t)$, where $d(t)=sat\{\hat{d}(t)\}$, $sat\{\cdot\}$ is the saturation function, $\hat{d}(t)$ is white noise modelled in Matlab Simulink using the 'Band-Limited White Noise' block with a noise power of 1 and a sampling period of 0.2. The disturbance is passed through a first order aperiodic filter for smoothing. The graph of the disturbance is shown in Figure 2a.

The reference model is given in the form

$$
y_m(t)=\frac{1}{(p+1)^2}g_r(t),\quad g_r(t)=5\cos(1.7t+3)\sin(0.5t).
$$

We choose $T(p)=p+1$. Hence the number a in (17) is 1, and the filters $\zeta(t)$ take the form:

$$
\zeta_y(t)=\frac{1}{p+1},\quad \zeta_g(t)=\frac{1}{p+1}.
$$

One filter is eliminated since deg $R_m(p)=0$.
The regression vector is then equal to

$$
\omega^T(t)=\left[y(t),\zeta_y(t),\zeta_\upsilon(t),g_r(t)\right],
$$

Let's form the control action (17) as

$$
\begin{aligned}
&\dot{\xi}(t)=\xi(t)-\frac{1}{0.01}(\xi(t)-\upsilon(t))\\
&\upsilon(t)=c^T(t)\omega(t)+\frac{1}{k}\left[ae(t)+\frac{\partial\Phi(\varepsilon,t)}{\partial t}-\alpha\varepsilon(t)\right],\\
&u=\xi(t)+1.
\end{aligned}
$$

In (2) we define $\bar{g}(t)=5e^{-0.3t}+0.3$, $\underline{g}(t)=0.2e^{-0.3t}-0.3$. In the control law we set $a=1,\alpha=10$ and $k=1$. In the adaptation algorithm (16), we choose $\beta=10$ and $\gamma=10$. The initial conditions $y(0)=y'(0)=3$. We take all other initial conditions in the closed-loop system as zero.

Figure 2 shows the graphs: disturbances (a), control signal (b), output signal (c), control error $e(t)$ transient with limiting functions (d).

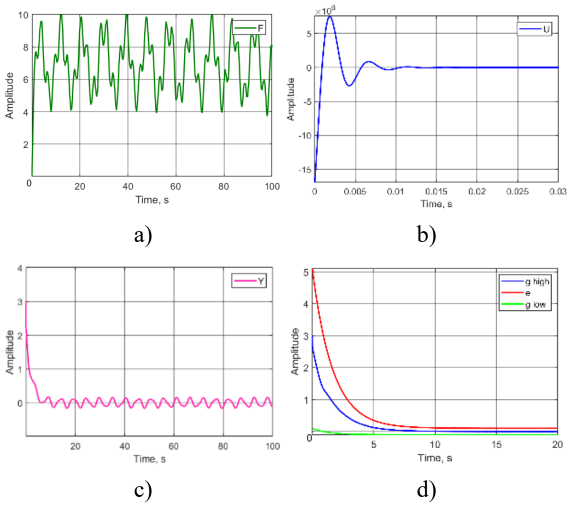

Figure 2: Plant 1 - a) graph of disturbances; b) graph of control signal; c) graph of output signal; d) graph of output signal error with limiting functions $\overline{g}(t)$ and $\underline{g}(t)$, defining the quality of transient.

The advantage of the proposed algorithm, in contrast to (Chopra, 2008), (Campion, 1989), (Gerasimov, 2015) and other classical algorithms is obvious: the transients are always contained in the tube (2), the boundaries of which can define the quality of the transients. Thus, the obtained processes almost exponentially decay to the limit set (-0.1; 0.1) in time 1.5 s., while the algorithms mentioned above are not controllable in terms of transient process and transient process time, as well as it is impossible to determine a priori the quality of the output variable in steady state.

As an example, consider the plant (1) with other parameters:

$$Q(p) = p^3 - 4p^2 + 2p - 1 \text{ and } R(p) = 2,$$
$$Q(p) = p^3 + 6p^2 - 8p + 6 \text{ and } R(p) = 1.5p + 0.5.$$

The error of the systems are shown in Figure 3 (a) and (b) respectively.

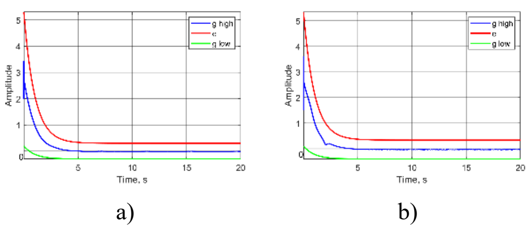

Figure 3: Graphs of the output signal error with limiting functions $\overline{g}(t)$ and $\underline{g}(t)$: a) plant 2; b) plant 3.

5 CONCLUSIONS

In this paper, the methods of classical adaptive control (Fradkov, 1999) and the method of nonlinear control (Annaswamy, 2021) are applied, which allowed us to create a new method of adaptive control that guarantees a given quality of transient throughout the whole process. At first, the new method is used to transform the problem with constraints to a problem without constraints. Then the classical method of adaptive control is applied.

The simulation results confirmed the theoretical conclusions and showed that in classical adaptive control schemes at different parameters of the plant, significantly different uncontrolled transient are observed, while in the new control scheme at the same parameters, the almost given quality of transients is guaranteed.

ACKNOWLEDGEMENTS

The research was carried out with support of the grant of the Russian Science Foundation № 25-19-20075, https://rscf.ru/project/25-19-20075/ in IPME RAS.

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APPENDIX

Proof of Theorem 1. Let us define a Lyapunov function of the form

$$V=\frac{1}{2}\varepsilon^2+\frac{k_m}{2\beta}\left(c-c_0\right)^T\left(c-c_0\right)+ +H\int_t^{\infty}\left(\frac{\partial\Phi(\varepsilon(s),s)}{\partial\varepsilon(s)}\right)^{-1}\in^2(s)ds, \tag{A1}$$

where $H > 0$. Let us find the full time derivative of (A1) using expressions (13) and (15). As a result, we obtain

$$\dot V=\left(\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right)^{-1}\varepsilon\left[-\alpha\varepsilon(t)+k_m\bar f(t)+k_m\in(t)\right]- -k_m\frac{\gamma}{\beta}c^T\left(c-c_0\right)-H\left(\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right)^{-1}\in^2(t). \tag{A2}$$

Let us use the following estimates from above and the relation:

$$\begin{aligned}&\varepsilon(t)\bar f(t)\le 0,5\left(\frac{1}{\nu}\varepsilon^2(t)+\nu\bar f^2(t)\right),\\&\varepsilon(t)\in(t)\le 0,5\left(\frac{1}{\nu}\varepsilon^2(t)+\nu\in^2(t)\right),\\&c^T\left(c-c_0\right)=0.5\left[\left(c-c_0\right)^T\left(c-c_0\right)+c^Tc-c_0^{\,T}c_0\right].\end{aligned} \tag{A3}$$

Given (A3), let us evaluate (A2) in the form

$$\dot V\le\left(\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right)^{-1}\left[-\alpha\varepsilon^2(t)+0,5k_m\left(\frac{1}{\nu}\varepsilon^2(t)+\nu\sup\left\{\bar f\right\}^2\right)+ +k_m0,5\left(\frac{1}{\nu}\varepsilon^2(t)+\nu\in^2(t)\right)-H\in^2(t)\right]- -k_m\frac{\gamma}{2\beta}\left[\left(c-c_0\right)^T\left(c-c_0\right)+c^Tc\right]+\frac{\gamma}{2\beta}c_0^Tc_0.$$

It follows from the previous equation that if the conditions are fulfilled

$$\begin{aligned}&|\varepsilon|>\sqrt{\left(\frac{0,5k_m\nu\sup\left\{\bar f\right\}^2+\in^2\left(0,5k_m\nu-H\right)+\frac{\gamma}{2\beta}c_0^Tc_0\sup\left\{\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right\}}{\frac{k_m}{\nu}-\alpha}\right)},\\&\alpha<\frac{k_m}{\nu},\\&H<0,5k_m\nu\end{aligned}$$

the derivative of the Lyapunov function will be negative. Thus, it is clear from the equation that there always exist α and H that ensure this condition. It follows from condition (b) that the transformation (15) guarantees the fulfilment of condition (2). As a result, function V is bounded and therefore C is bounded. Theorem 1 is proved.

Proof of Theorem 2. Let us rewrite equations (19), (20) in the form:

$$\begin{aligned}&\dot e(t)=-ae(t)+k\left[\bar c^T\omega(t)+\bar f(t)+\in(t)\right]+\mu^{\gamma-2}kL\bar\eta(t),\\&\mu_1\dot{\bar\eta}(t)=G\bar\eta(t)-\mu_2\tilde b\dot\upsilon(t),\\&\dot c(t)=-\beta\left(\frac{\partial\Phi(\varepsilon,t)}{\partial\varepsilon}\right)^{-1}\varepsilon(t)\omega(t)-\gamma c(t),\end{aligned} \tag{A4}$$

where $\mu_1=\mu_2=\mu$. Let us use the lemma (Brusin, 1994).

Lemma (Brusin, 1994). If a system is described by equation $\dot x=f(x,\mu_1,\mu_2), x\in R$, where $f(x,\mu_1,\mu_2)$ is a continuous function that is Lipschitz on x, and at $\mu_2=0$ has a bounded closed dissipativity area $\Omega_1=\{x|F(x)<C\}$, where $F(x)$ is a positively defined, continuous piecewise smooth function, then there exists such $\mu_0>0$ that at $\mu_2<\mu_0$ the original system has the same dissipativity area Ω_1 if for some numbers C_1 and $\bar{\mu}_1$ *with* $\mu_2 = 0$ the following condition is fulfilled

$$\sup_{|\mu_1| \le \bar{\mu}_1} \left(\left(\frac{\partial F(x)}{\partial x} \right)^T f(x, \mu_1, 0) \right) \le -C_1, \qquad \text{(A5)}$$

when $F(x) = C$.

Let us take the Lyapunov function $\mathrm{V}_2 = \bar{\eta}^{\mathrm{T}}(\mathrm{t})\mathrm{H}_2\bar{\eta}(\mathrm{t}), \mathrm{H}_2 = \mathrm{H}_2^{\mathrm{T}} > 0$ is determined from the solution of the equation $\mathrm{H}_2\mathrm{G} + \mathrm{G}^{\mathrm{T}}\mathrm{H}_2 = -\mathrm{Q}_2$, where $\mathrm{Q}_2 = \mathrm{Q}_2^{\mathrm{T}} > 0$, then considering (A.4) we obtain $\dot{V}_2 = -\frac{1}{\mu_2}\bar{\eta}^{\mathrm{T}}(\mathrm{t})\mathrm{Q}_{2\bar{\eta}}(\mathrm{t})$ with $\mu_2 = 0$. It means that at μ_2=0 we obtain the original equations (11), (16) to which we add the independent equation $\mu_1\dot{\bar{\eta}}(t) = G\bar{\eta}(t)$ with asymptotically stable variable $\bar{\eta}(t)$. Hence, for the initial system we have a dissipativity region Ω with an attraction region Ω_1.

Let us take the Lyapunov function as a function $F(x)$

$$\begin{aligned} F = h_1 e^2(t) + \frac{k}{\rho}\bar{c}^T(t)\bar{c}(t) + \zeta_y^T(t)H_3\zeta_y(t) + \\ +\zeta_\upsilon^T(t)H_4\zeta_\upsilon(t) + \bar{\eta}^T(t)H_2\bar{\eta}(t), \end{aligned} \qquad \text{(A6)}$$

where $h_1 > 0$, H_2, H_3, H_4 are positively defined symmetric matrices.

Let us set the number C such that the surface $F(x) = C$, where $x^T(t) = \left[c, \bar{\eta}^T, \zeta_y^t, \zeta_\upsilon^t\right]$ is boundedly closed, is in the area Ω on the variables $x(t)$, and since the set Ω_1 lies in the open area $V(x) < C$ and the system is dissipative, the variables $x(t)$ will tend to attraction area Ω_1, and hence there exists a number C_1 for which (A5) is satisfied. The rate of convergence of the variables $\bar{\eta}(t)$ to zero will depend on the choice of μ_1. Therefore, according to the lemma (Brusin, 1994), there exists $\mu_0 > 0$ such that at $\mu < \mu_0$ the dissipativity area of the system (17), (19), (20) remains the area Ω. Theorem 2 is proved.