Comparison of the Hubble Tension Measurement from Two Approaches: Distance Ladder and CMB

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Abstract: The Hubble Tension arose when two distinct values of the Hubble constant were calculated, which implied either experimental flaw, or a fundamental misunderstanding of the universe. This study provides an overview of the mathematical meaning of the Hubble constant using Hubble's Law, as well as the Hubble parameter by interpreting the terms in the Friedmann Equation. The distance ladder method will also be elaborated; by explaining the basic mechanism of the method, which is observing the absolute magnitude of the Cepheid variable stars, calibrating them with the Type la Supernovae, eventually establishing a correlation where distance increases linearly with velocity, in which the Hubble constant is the constant of proportionality. The CMB method will be explained particularly with the underlying features of CMB that enable this method: the BAO and the sound horizon, which is integrated into the method of inputting observed values from the angular power spectrum into the ΛCDM to generate Hubble constant. Comparisons, limitations, of the two methods will be addressed, such as the interference to observation by the metallicity of Cepheids dense star-forming regions, the reliance on Cepheids of the Type la Supernovae in the distance ladder method. The sensitive dependence on the ΛCDM model; as well as the potential incompleteness of the model for the CMB method. A further discussion of the scientific meaning of Hubble tension will also be provided.

1 INTRODUCTION

The understanding of the Universe has been greatly expedited during the 20th century. Einstein's creation of General Relativity, followed by his own solutions to the Einstein Field under the assumptions of a spatially homogeneous and isotropic universe by introducing the cosmological constant Λ. Einstein's solution models the Universe as perfectly static; that is neither expanding nor contracting, where Λ counteracts the attraction of gravity.

In 1922 Friedmann proposed a new set of solution to Einstein's Field, one that does not rely on the static nature of the Universe, that the Universe is either expanding or contracting, despite the mathematical solution did not receive any observational proof until 1929. In 1929 Hubble measured the distance of the Milky Way to nearby galaxies using Cepheid Variables, as well as the redshift of light emitted from those galaxies and discovered that the expansion velocity of the galaxies is linearly proportional to the distance, expressed as:

$$v = H_0 d \quad (1)$$

where H_0 is the Hubble Constant (Hubble 1929)

Hubble's discovery reveals that the further a galaxy is, the faster it is receding away from the observer, which means the universe is expanding, as well as the space between neighbouring galaxies. As if all galaxies are mapped on to specific points on a fabric of elastic rubber, as the rubber stretches, the relative distance between two points also increases.

Hubble's discovery lies in the heart of modern cosmology, as it sheds light onto the rudimentary configurations of the Universe, such as age, the past, present and future of the universe. Though the two most popular methods in calculating H_0 – via cosmic distance ladder and cosmic background radiation - show discrepancies in their values, know as the Hubble Tension.

More precise measurement of the Hubble constant in recent years show discrepancies between two fundamentally different methods – the distance ladder and CMB. The major difference between the two methodologies is that the CMB method does not measure the Hubble constant directly, instead a value for Hubble constant is inferred from the ΛCDM simulation from modelling the Universe during the Epoch of Recombination(Yadav 2023). On the other

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hand, the distance ladder offers direct mathematical measurement and computation from the preset-day universe.

The SH0ES collaboration, via the distance ladder method, measures the luminosity of the standard candles (Cepheids and 1a Supernovae) through which the Hubble constant can be calculated through measuring distance and redshift. SH0ES incorporates their research with data from Gaia and Hubble Space Telescope, sets the value of the Hubble constant as

$H_0 = 73.04 \pm 1.04\ \mathrm{kms^{-1}Mpc^{-1}}$ (Riess 2022).

On the other hand, the Planck satellite utilises the CMB method which focuses on measuring the temperature fluctuations thus calculating the Hubble constant by extrapolating distance from the measured angular size of the sound horizon. These fluctuations are then plotted on an angular power spectrum which is modelled in the ΛCDM simulation, through which the Hubble constant is inferred. The value of the Hubble constant is found to be

$H_0 = 64 \pm 0.5\ \mathrm{kms^{-1}Mpc^{-1}}$ (Planck 2018).

The initiative of this paper is to deliver a generic overview of the Hubble tension, starting from the foundations of the Hubble constant as the theoretical implication in determining the configurations of the Universe. Analysis of the experimental approaches of measuring the Hubble constant that give rise to the Hubble tension; particularly from the cosmic distance ladder and CMB methods, by accessing data from pre-existing academic establishments and publicly accessible data such as the Planck satellite. The following parts will be comprised as follows. An explanation to the principles of the Hubble constant and Hubble parameter. A description of the distance ladder method, how Hubble constant is measured using distance ladder, as well as the relevant data analysis such as the Period-Luminosity relation. Subsequently, a description of the CMB method, and relevant approaches in measuring Hubble's constant, and the dependence of the ΛCDM. Afterwards, a comparison of the two values, a discussion of other methods, as well as prospects that would possibly explain the existence Hubble tension, or possible resolution.

2 PRINCIPLES OF THE HUBBLE LAW

The Hubble constant describes the current rate at which the universe is expanding. It displays a linear relationship between the recessional velocity of the galaxy and the distance from the observer. The recessional velocity v unit of the Hubble constant – km/s/Mpc indicates that the recessional velocity of the galaxies increases by a value of H_0 for every megaparsec of separation.

This increase in velocity does not imply the galaxies are 'moving through' space, but the stretching of the space itself, where galaxies consequently separate out from one another. Taking the analogy of the balloon, the galaxies, as dots on the balloon as it is to space; as the balloon expands, the dots move apart from each other, this is analogous to the separation of the galaxies from one another. Therefore, the cosmological redshift is strictly due to the geometric phenomenon of the space expanding, distinctively dissimilar to the Doppler effect in Kinematics (Peacock 1999).

From the equation above, given the Hubble constant at the present time, the age of the universe can be approximated as

$$t \approx [H_0]^{-1} \quad (2)$$

where the age of the universe is approximately 13.8 billion years, this equation is a mere estimation, and further detail will be provided in Part 2. The Hubble constant marks the crucial relationship in converting observed cosmological redshift into physical distances. It is also instrumental in producing large-scale maps of the universe, as well as marking the boundary conditions for computational simulations, such as the ΛCDM (Riess 2022).

The Hubble parameter $H(t)$ defines the expansion rate of the universe as a function of time. It is given by the ratio of the time derivative of the scale factor $a(t)$. Here, $a(t)$ describes the size of the universe at a given time, where the scale of the universe at any time is a ratio to the present time t_0 $a(t_0) = 1$. Therefore, at $a(t_0) > 1$ is the future and $a(t_0) < 1$ the past. The derivative of $a(t)$ Indicates the instantaneous change of the size of the universe with respect to time. Thus, H_0 can also be defined as:

$$H_0 = \frac{1}{a(t_0)} \frac{da(t_0)}{a(t_0)} \quad (3)$$

Following from Part 1, a more refined, accurate computation of the age of the universe, as the Hubble constant does not represent the uniform rate of expansion across the entire time duration, the more accurate age of the universe will be the integral of the Hubble parameter for all values of a. The Hubble parameter is determined by the energy distributions in space, expressed as the Friedmann Equation (Friedmann 1922):

$$H(t)^2 = \frac{8\pi G}{3}\rho(t) - \frac{kc^2}{a(t)^2} + \frac{\Lambda}{3} \quad (4)$$

where the Hubble parameter is dependent upon the energy density at any given time $\rho(t)$, expands to give:

$$\rho(t) = \rho_{r0} a(t)^{-4} + \rho_{m0} a(t)^{-3} + \rho_{\Lambda} \quad (5)$$

In which the radiation and matter density at a given t is inversely proportional to their density at t_0, therefore the overall energy density decreases as $a(t_0) > 1$, increases when $a(t_0) < 1$, relative to the current age of the universe, where as the dark matter density remains constant over time, this is also why the present universe is a dark matter-dominated universe (Weinberg 1972). The relative energy densities of radiation and density are "mass-constant", therefore the total quantity of radiation and matter remain constant, whereas dark energy is "density-constant", therefore the total dark energy existing increases as the universe expands.

The second term $\frac{kc^2}{a(t)^2}$ describes the spatial curvature of the universe, where k takes values of either -1, 0, +1, reflecting the universe being open, flat, or closed, though the numerical value assigned to this mathematical term decreases asymptotically to 0 as $a(t)$ increases. The last term $\Lambda/3$ describes the dark energy content in the universe, acting as though a repulsive force through space. Due to the shifting of significance and numerical scale of each energy density at different epochs of the universe, which implies that the change in the Hubble parameter was not uniform. Therefore, integration is used for a more accurate age of the Universe.

3 MEASURING H_0 VIA DISTANCE LADDER

The distance ladder method is a direct way of determining the Hubble constant. The process involves calibrating stepwise using a series of increasingly distant astronomical objects, where each step of the ladder provides the foundation for the next. The two most relevant standard candles for this method are the cepheid Variable stars, and the Type la Supernovae. These stellar objects are crucial to the "Supernovae and H_0 for the Equation of State" – SH0ES' collaboration work to refine their value for the Hubble constant.

Cepheid variable stars are radially pulsating stars characterised by a periodic variation in their luminosity, as a result of their contracting and expanding outer layer due to the inward gravitation force. The correlation between the periodicity and luminosity is given by the Period-Luminosity relation discovered by Henriette Leavitt. This relation allowed a direct way of determining luminosity, solely with accurately measuring the duration of the varying-luminosity period.

The proportionally between period and luminosity was first discovered by Leavitt via observations of Cepheid variables in the Small Magellanic Cloud. The relation was later calibrated using Cepheid variables within the Milky Way, whose distances were measured in the parallax method, thus establishing the scale between period and absolute magnitude and anchoring the luminosity scale as shown in Figure 1.

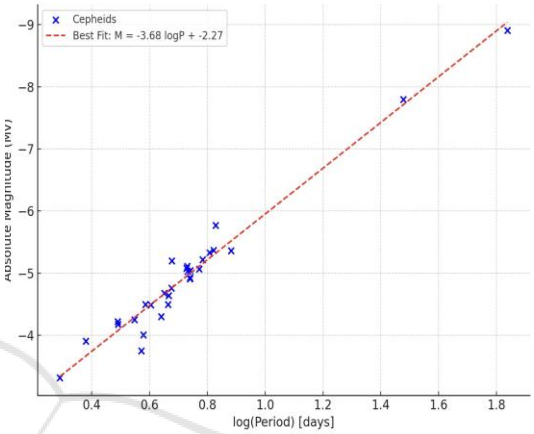

Figure 1: The linear relationship between periodicity and absolute magnitude is displayed by the linearity of the plotted line, which aligns with Leavitt's discovery (Storm J. 2011).

The proportionality is given by:

$$M = a \log P + b \quad (6)$$

where M is the absolute magnitude, P the period in days, a the change in brightness with respect to period, and b the absolute magnitude when the period is 1 day. The Gaia Space Observatory measured parallax for nearby Cepheids with a precision of below 1 milli-arcsecond (Gaia Collaboration 2021). More massive and luminous Cepheids have lower gravity,

$$g = \frac{GM}{r^2} \quad (7)$$

This is because the increase in radius outweighs the increase in mass, yielding a lower gravitational field strength, this consequently increases the period of the expansion and contraction process, thus the period pulsation. By observing the absolute magnitude with the observed magnitude, the distance is given by:

$$m - M = 5\left(\log\left(\frac{\mathrm{d}}{10}\right)\right) \quad (8)$$

where m is the apparent magnitude and M the absolute magnitude, where g is in parsecs. The Cepheids are observed using the Hubble Space Telescope, then calculating the red shift z the recessional velocity of the Cepheid v can be found using $v = cz$, thereby constructing a graph of velocity against measured distance, the gradient is given to be the Hubble constant.

Despite the method via Cepheid variables being straightforward, it is not without its flaws and limitations. Cepheids variable stars are located in dusty star-forming regions where extinction (the reduction in intensity and scattering of light) occurs frequently in the visible light spectrum. Therefore, the F160W band is frequently used nowadays as the subject of the detection, as the infrared radiation emitted from F160W is hardly obscured by the surrounding stars (Riess 2021).

Another limitation of the Cepheid variable stars is their metallicity. Elements heavier than Hydrogen and Helium will absorb radiation emitted by the Cepheids, which changes the frequency of the light received on Earth, this consequently leads to miscalculations regarding the size of the Cepheid, thus the distance, and the Hubble constant. This becomes problematic when two Cepheids that have the same period, mass, would lead to discrepancy and inaccuracy over the calculated Hubble constant value, therefore the inaccuracies must be mitigated using spectroscopic analysis (Riess, 2021; Yuan & Riess, 2023]

Type la Supernovae becomes useful when observing distant galaxies, where the distance is so great that barely any radiation can be detected by the Cepheid Variables. Type la supernovae are the thermonuclear explosions of carbon-oxygen white dwarfs in binary systems. Which occurs when a carbon-white dwarf accretes mass from a companion star mass, where nuclear fusion occurs for carbon and oxygen, overcoming the outward electron degeneracy pressure, which leads to an explosion. The explosion produces a light curve with uniform luminosity and shape, which is used to standardise the luminosity of the Type la supernovae (Riess 2022).

Type la Supernovae are not intrinsically standard candles, as their absolute magnitude cannot be previously known, therefore the external calibration of the luminosity of the la supernovae relies on Cepheid variables acting as anchors. The calibrated luminosity is inferred from a similar equation

$$M_{SN} = m_{SN} - 5\left(\log\left(\frac{d}{10}\right)\right) \quad (9)$$

On this basis, a similar graph of recessional velocity vs. distance can be plotted, provided that the redshift is negligible, in which the gradient is inferred as the Hubble constant.

The most recent determination of the Hubble constant via the distance ladder method is from Riess et al. 2021, by observing over 1000 Cepheids and 42 Type la Supernovae across different host galaxies (Riess 2021), which found the value to be

$$H_0 = 73.2 \pm 1.3\ \mathrm{kms^{-1}Mpc^{-1}} \quad (10)$$

In 2022, the SH0ES collaboration reduced the percentage uncertainty below 2% by using more advanced calibration and a larger dataset demonstrated the consistency of the Hubble constant by measuring using different anchoring galaxies such as NGC 4258, LMC and the Milky Way, the resultant value for the Hubble constant remained consistently above 72km/s/Mpc. (Riess, 2021; Yuan & Riess, 2023)

4 COMPUTING H_0 VIA CMB AND ΛCDM

The CMB method measures the anisotropies in the temperature map imprinted at the Epoch of Recombination (380000 years). Anisotropies are variations of temperature in regions of the CMB map, as red displays higher temperature regions and blue cold. The anisotropies reflect on the distribution of different energy densities across different regions in the primordial universe, as the result of the variations in intensity of Baryon-Acoustic Oscillations (oscillations of photon-baryon plasma), which are imprinted on the angular power spectrum, which is analysed within the ΛCDM model, computing H_0 (Planck 2018). The Angular power spectrum describes the temperature anisotropies specifically from the Epoch of Recombination-when the photons in the CMB had just become free from scattering with electrons-using spherical harmonics. Spherical harmonics break down temperature variations observable in the CMB, correlating the temperature variations to specific angular scales. Multipole moment l corresponds to angular scale, where a small multipole moment means larger angular scale. The peaks in the angular power spectrum indicate Baryon Acoustic Oscillation.

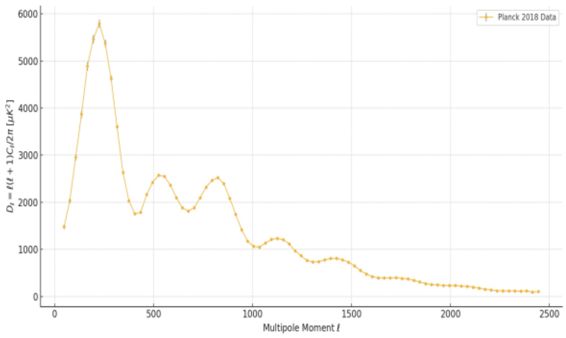

Figure 2: This is a graph displays the relation between angular power spectrum with a range of multipole moments (Planck 2018).

As shown in Figure 2, the peak at $2.2 * 10^2$ indicates the most prominent first harmonic acoustic oscillation in the photon-baryon plasma during the Epoch of Recombination, signifying the largest complete compression mode of the sound wave generated by the oscillation. The oscillations lead to rarefaction and compression in regions of the CMB as sound waves. The sound waves then stop propagating at the Epoch of Recombination, in which the freed photons from the photon-baryon plasmas carry information of these BAO, which then the photons are detected by microwave telescopes (Planck, 2018 & Hu, 2022), that indicate the temperature fluctuations of the CMB around the Epoch of Recombination.

The sound horizon is the distance the sound waves from BAO could travel before the Recombination, where angular scale θ is the ratio between the sound horizon and the angular diameter distance, which measures the distance to the point where the photons became free. The angular power spectrum and multipole moments infer the angular size of the sound horizon determine the angular size of the sound horizon. Where the angular size of the sound horizon is dependent on the expansion rate of the universe, therefore dependent on the Hubble constant. A higher expansion rate leads to smaller angular size, as distance is further, and vice versa.

With the calculated angular size from the observed CMB, the values are inserted into the ΛCMB model, which computes a theoretical value of the Hubble constant, by correlating angular size with the rate of expansion (Planck 2018).

The development of the CMB method is reflected by the three generations of satellites. The cosmic background explorer COBE, 1989-1993 initially detected the anisotropies of the CMB temperature, and that the radiation spectrum of the CMB at 2.725K adheres closely to the black body curve, which implied that the early universe was uniform and in thermal equilibrium, where matter and radiations existed in a very hot and dense condition, the uniformity implied the cooling and expansion of tue universe over time, which aligns with the observed evidence (Smoot 1992). The Wilkinson Microwave Anisotropy Probe 2001-2010 improved the angular resolution from 7 degrees to 0.2 degrees, yielding a refined value of the Hubble constant

$$H_0 = 69.32 \pm 0.80\ \mathrm{kms^{-1}Mpc^{-1}} \quad (11)$$

(Bennet 2013). The Planck mission 2009-2013 provided further observations of temperature anisotropies to 2500 multipole moment, operating closely with the ΛCDM, Planck mission yielded a value of Hubble constant as

$$H_0 = 67.4 \pm 0.5\ \mathrm{kms^{-1}Mpc^{-1}} (12)$$

(Planck 2018).

The major limitation of the CMB method is that it is heavily dependent on the accuracy of ΛCDM model, the given parameters of dark energy, dark matter, and baryonic matter within the model are assumed to be accurate, which means any known, or unknown deviations within this model, will compute a different value for the Hubble constant (Di Valentino 2021).

5 COMPARISON AND PROSPECTS

The two methods both provide a logical, precise path towards the derivation of the Hubble constant, though they do not converge on an unequivocal result, but differ by a difference of 5σ. The two methods fundamentally disparage in a multitude of ways, where the distance ladder observes the modern universe, and the CMB method studies the early universe at the Epoch of Recombination. The distance ladder method being empirical and observation-based, whereas the CMB method relies on a model. The two methods could be subjected to inaccuracies in their measurements, such as the metallicity of the Cepheid variables which would obscure the measurements, or the disproportionality of dark energy, dark and baryonic mater within the configurations the ΛCDM model. Another possible explanation might be the lack of understanding of the 'invisible'; perhaps dark energy is not an invariable, but one that changes according to the scale of the Universe, which might yield a completely different and independent value of H_0.

Another independent method of calculating the Hubble constant is by establishing a direct relationship between the temperature of the CMB with the Hubble constant, a mathematical model, which is subjected to much less inaccuracy in comparison with the distance ladder and CMB method, relying only on the observed value for the temperature of the CMB (Tatum 2024):

$$H_0 = \frac{T_{CMB}^2 k_b^2 32\pi^2}{c\hbar\left(\frac{c^3\hbar}{G}\right)^{\frac{1}{2}}} \quad (13)$$

The Hubble constant calculated to be

$$H_0 = 66.8712 \pm 0.0019\ \mathrm{kms^{-1}Mpc^{-1}}$$

which is very similar to the value computed by the Planck mission. The Hubble tension not only displays a fundamental difference in value via the two most popular methods, but also a reflection of the unknown of the universe. In the future more accurate values of the Hubble constant will be calculated, with finer understanding of the universe, and more advanced methods, though it is most crucial, to lay the foundation for those methods by developing a finer understanding of the ingredients of the Universe.

6 CONCLUSION

To conclude, this study serves as an overview of the two observational methods of the Hubble constant – distance ladder and CMB, and how they fundamentally disagree with each other which results in the Hubble Tension. This paper also delves into the mathematical meaning of the Hubble constant and Hubble parameter, as well as their significance in relation to the age, expansion rate and the dynamics of the universe, as though a blueprint on which all the known and unknown knowledge of the universe is imprinted. The empirical, observational fashion of the distance ladder method, first by establishing a correlation between period and absolute magnitude of Cepheids and Type la Supernovae via parallax and calibration, to find distance and calculate Hubble constant, limited by the metallicity of Cepheids and the dependent nature of the la supernovae. The modelling method of the CMB via ΛCDM, by tracing back to the very beginning of the universe, with data of angular scale, size, multipole moment provided by three generations of satellites, though considered flawed due to the incompleteness of the ΛCDM model. The Hubble constant is a reflection of the rudimentary parameters that govern the universe. In order to establish on a single unfalsifiable value of the Hubble constant, the analysis of the configuration of the universe is the most urgent task for modern Astrophysics. Perhaps only by fully interpreting the known knowledge and unchartered enigmas of the Universe, will the infallible notion of the Hubble constant be ultimately revealed.

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