

# A Comparative Study on Central Projection and Parallel Projection

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**Keywords:** Parallel Projection, Central Projection, Perspective Transformation, Descriptive Geometry, Computer Graphics.

**Abstract:** A comprehensive comparative study focusing on central projection and parallel projection is conducted in this paper. The two projection methods are systematically analysed from multiple perspectives including their physical definitions, fundamental principles, mathematical theorems, geometric properties, algorithmic implementations, and practical applications. The differences and connections between the two projections are elucidated in detail. Their advantages and disadvantages across various disciplines are also explored, and the study of an application example is given. It is demonstrated that central projection, which simulates human visual perception, provides more realistic scene perception and is suitable for visual arts production field, but non-linear transformations and more complex computation are involved, leading to an inaccurate geometric measurement. In contrast, parallel projection preserves geometric proportions through linear transformations with simpler computations, but lacks spatial depth perception, rendering it ideal for precision-dependent fields such as measurement and manufacturing. Finally, the application of integration on central projection and parallel projection methods and their development in the future are explored. This study provides valuable references and insights for related research fields.


## 1 INTRODUCTION

Projection is the process of mapping a three-dimensional object in space onto a two-dimensional plane. The shadow formed by light rays illuminating an object and casting onto a screen behind it is referred to as a projection. Central projection and parallel projection are two fundamental methods of projection theory, widely applied in fields such as geometry, computer graphics, engineering drafting, photography, painting, and artistic creation (Müller, et al., 2021; Liu et al., 2024; Garcia et al., 2019). Due to their distinct definitions and principles, these two projection methods exhibit different characteristics and are suited to different scenarios. However, they both play important roles in practical applications. This paper conducts a comparative study of these two projection methods by analysing their physical definitions and principles, geometric and mathematical properties, computational approaches, and application fields. Furthermore, their future development prospects are explored.

## 2 DEFINITIONS, PRINCIPLES, AND CHARACTERISTICS

The essence of projection is transmitting the contours of an object onto a designated plane through a set of light rays (projection lines). Therefore, three essential elements are required to form a projection. The first factor is projection center that can be regarded as the light source. For instance, in the case of a shadow cast by sunlight, the sun serves as the projection center. The second is projection object that is being projected. It may consist of geometric elements such as points, lines, or surfaces, or a three-dimensional solid. The third is projection plane, a receiving surface where the image is formed after the light rays pass through the object. It can be a ground surface, a wall, a drawing sheet, etc.

The spatial position of the projection center and the direction of the projection lines influence the shape and size of the projected image on the plane. Based on the mutual relationships between projection lines, projections are classified into central projection

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(perspective projection) and parallel projection (Foley et al., 2018; Coxeter et al., 2003).

## 2.1 Central Projection

Central projection refers to a projection method in which light rays emanate from a single point (the projection center) and diverge radially, passing through an object and intersecting a receiving surface to form a perspective relationship. Geometrically, this process involves extending the lines connecting each point on the object to a fixed projection center until they intersect a plane that does not contain the projection center. The set of intersection points constitutes the central projection of the object onto that plane, as illustrated in figure 1.

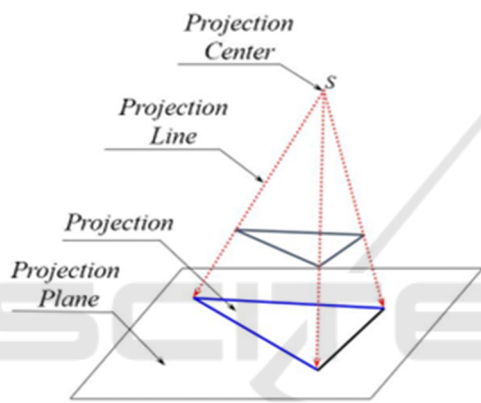


Figure 1: Central Projection (Picture credit: Original).

In central projection, since all projection lines converge at a single point (the projection center), the varying distances between different parts of the object and the projection center result in non-uniform scaling, causing changes in the size and shape of the projected image, whose actual dimensions and angles may undergo nonlinear distortion (Carlson, 2003). Parallel lines in the original object may intersect at vanishing points on the projection plane, leading to significant deviations between the projected image and the original object. These characteristics, to some extent, compromise the metric accuracy of central projection, limiting its widespread application in classical solid geometry. However, the resulting transformations enhance visual intuitiveness and spatial realism, aligning with human visual perception, e.g., near-far size attenuation. Consequently, central projection is extensively employed in artistic domains, such as painting and photography, where it preserves a naturalistic

resemblance to the original object while emphasizing depth and perspective (Peacock, 2001).

## 2.2 Parallel Projection

When the light source at the projection center is relocated to infinity, all projection rays become mutually parallel and intersect the projection plane at a fixed angle, which is termed parallel projection. As illustrated in figure 2, parallel projection can further be categorized into two subtypes:

### ▪ Orthographic Projection

The projection lines are perpendicular to the projection plane. For example, the standard engineering multi-view drawings (including front, top, and side views). This method preserves the true dimensions and shapes of objects, making it indispensable for technical drafting.

### ▪ Oblique Projection

The projection lines intersect the projection plane at an oblique angle ( $\neq 90^\circ$ ). For example, cabinet oblique projection which retains partial depth perception exhibits less metric accuracy compared to orthographic projection.

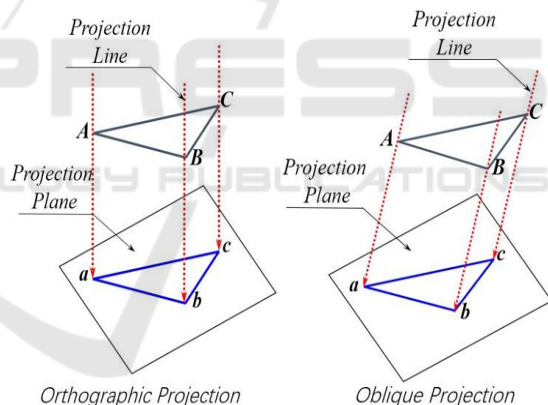


Figure 2: Parallel Projection (Picture credit: Original).

In parallel projection, the shape and size of objects maintain proportional consistency on the projection plane without scaling effects caused by varying distances. Due to its superior metric properties, relatively simple projection rules, and ease of understanding and drafting, parallel projection is broadly applied in fields requiring precise proportions and resistance to perspective distortion. These applications include engineering drawings, mechanical manufacturing, cartography and surveying, architectural design, and computer graphics (Liu, 2022; Zhang et al, 2008; Luo et al., 2009).

### 3 MATHEMATICAL THEOREMS AND RELATED PROPERTIES

#### 3.1 Fundamental Theorems and Properties of Central Projection

- Collinearity Preservation

If three points in space lie on a straight line, their central projections will remain collinear (or all converge to points at infinity).

- Cross-Ratio Invariance

In central projection, the cross ratio of any four collinear points remains invariant. This theorem holds a central position in projective geometry and ensures accurate proportional transformations in perspective projection.

As illustrated in figure 3, four collinear points  $A, B, C, D$  are projected onto another line as  $a, b, c, d$  via central projection. While the lengths of projected segments change and the ratios of individual segments are not preserved, the cross ratio remains invariant, that is:

$$(AB, CD) = \frac{CA}{CB} \cdot \frac{DA}{DB} = \frac{ca}{cb} \cdot \frac{da}{db} \quad (1)$$

Here, all segments are treated as directed lengths with signed magnitudes. Definitions of world coordinates and pixel coordinates are provided in Section 4.

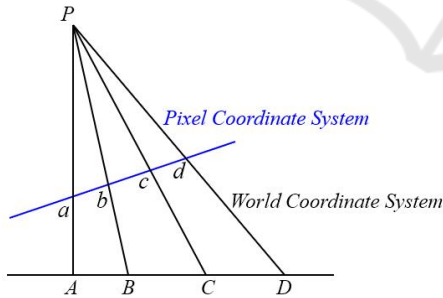


Figure 3: Cross-Ratio in Central Projection (Picture credit: Original).

The cross-ratio invariance has significant applications in computer vision, photogrammetry, and robot navigation. Specific use cases include sun positioning systems (based on shadow measurements) (Zhang et al., 2015), structured-light 3D reconstruction (light-plane calibration) (Chen et al., 2018), Camera self-calibration (using vanishing points) (Cipolla et al., 1999).

- Nonlinearity

Central projection is not a linear transformation (except when the projection center is at infinity, in this case, it degenerates into parallel projection). Notably, As introduced in Desargues' theorem, in homogeneous coordinates, central projection can be represented as a fractional linear transformation.

- Non-Preservation of Distances and Angles

Central projection distorts metric properties (e.g., lengths, angles) of geometric shapes.

- Points at Infinity

In projective geometry, central projection maps lines parallel to the projection plane to vanishing points at infinity, thereby extending Euclidean space into projective space.

- Desargues' Theorem

If the lines connecting corresponding vertices of two triangles meet at a single point (perspector), then the intersections of their corresponding edges lie on a straight line (perspectrix), and vice versa.

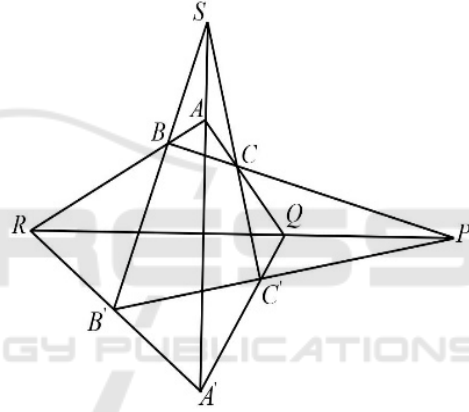


Figure 4: Desargues Theorem (Picture credit: Original).

As illustrated in figure 4, the author considers two triangles  $\triangle ABC$  and  $\triangle A'B'C'$ . If the connecting lines of corresponding vertices  $AA', BB', CC'$  intersect at a common point  $S$  (the perspector), the triangles exhibit point perspective. Conversely, if the intersections of corresponding edges denoted  $RQP$  lie on a straight line (the perspectrix), they exhibit line perspective. A pair of triangles are perspective if either condition is satisfied (Coxeter, 2003; Hartley, 2018).

The conditions and conclusions in Desargues' theorem are mutually inverse and implicative, which demonstrates the fundamental principle of point-line duality in projective geometry. This self-dual characteristic establishes the foundational status of Desargues' theorem in projective geometry and extends its practical utility (Ma, 2011).

Desargues' theorem describes the specific geometric relations satisfied by two triangles under

the condition that central projection preserves collinearity and cross-ratio while potentially altering metric properties such as distances and angles. It reveals profound properties that remain invariant when observing geometric figures under different projection centers. These properties are independent of specific length or angle measurements, relying solely on the relative positional relationships of the figures (Xing et al., 2004).

From an algebraic perspective, Desargues' theorem reflects the preservation of linear dependence. In homogeneous coordinates, collinear points correspond to linearly dependent vectors, while concurrent lines correspond to linearly dependent linear equations. The validity of Desargues' theorem stems from that central projections corresponds to linear transformations in vector space, which preserve this dependency structure (Yu et al., 2011; Sturm et al., 2020).

#### Mathematical Formulation and Transformation Matrices

In computer graphics, central projection is widely used to simulate human visual perception or camera observation of 3D scenes. Its mathematical formulation can be implemented via matrices transformation that generally expressed in homogeneous coordinates to enable perspective division.

Assume that:

The projection center (camera optical center) is located at the origin  $O(0,0,0)$ .

The projection plane is defined as the  $z = f$  plane (where  $f > 0$ ).

A spatial point  $P(X,Y,Z)$  is projected onto the imaging plane as  $p(x,y)$ .

Based on the similar triangle principle (see figure 5, right), the following relationships are obtained:

$$\frac{x}{f} = \frac{X}{Z}, \frac{y}{f} = \frac{Y}{Z} \quad (2)$$

The solving for the projected coordinates is:

$$x = \frac{f \cdot X}{Z}, y = \frac{f \cdot Y}{Z} \quad (3)$$

Here,  $Z$  denotes the depth of point  $P$ , which governs the scaling effect of the projection and serves as the divisor in perspective normalization.

When  $Z = 0$ , the projected point lies behind the projection center, making it geometrically invalid.

When  $Z = \infty$ , the projected point approaches a vanishing point. In Euclidean space, infinite distances cannot be represented with finite coordinates.

To uniformly describe all points while linearizing nonlinear central projections, homogeneous coordinates are introduced (Hartley et al., 2004):

The 3D point  $P(X,Y,Z)$  is represented as  $P(X,Y,Z,1)$  in homogeneous coordinates.

The 2D point  $P(x,y)$  is represented as  $P(x,y,1)$  in homogeneous coordinates.

Using matrix algebra, central projection can be expressed as a linear transformation in homogeneous coordinates. The projected point in homogeneous coordinates is:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix} = \begin{bmatrix} fX \\ fY \\ Z \end{bmatrix} \quad (4)$$

To normalize  $x'$ ,  $y'$  and  $w'$ , 2D coordinates are obtained:

$$x = \frac{x'}{w'} = \frac{f \cdot X}{Z}, y = \frac{y'}{w'} = \frac{f \cdot Y}{Z} \quad (5)$$

### 3.2 Fundamental Theorems and Properties of Central Projection

In contrast to central projection, parallel projection does not exhibit vanishing points but preserves more geometric invariants, with its core parallelism and proportionality preservation. These properties make parallel projection indispensable in engineering drafting, mechanical design, and scientific visualization.

#### Parallelism Preservation

Two parallel lines in space remain parallel in the projection plane unless they are parallel to the projection direction. Lines parallel to the projection direction degenerate to points.

The parallel preservation theorem is one of the most critical and widely applied properties of parallel projection. It guarantees the invariance of parallel relationships in projective transformations, providing the theoretical foundation for operations such as dimension annotation and view correspondence in engineering drawings. For instance, parallel edges of mechanical parts remain parallel in orthographic projections (Shah, 2020).

#### Proportionality Preservation

Parallel projection preserves the proportional lengths of line segments.

The proportionality preservation theorem, rooted in the linearity of affine transformations, guarantees the accurate transfer of geometric relationships. This principle is extensively applied in engineering design, architectural drafting, and computer graphics.

Orthographic projection preserves angles on the projection plane, whereas oblique projection maintains angles only along specific directions (e.g., axial directions), with potential distortion in other orientations.

- **Linearity Theorem**

Parallel projection is a linear transformation expressible as an affine transformation matrix. Its transformation matrix combines a linear matrix and a translation matrix, classified as an affine transformation. This affine structure underpins the validity of the parallelism preservation theorem.

- **Mathematical Formulation and Transformation Matrices**

Orthographic projection is the simplest form of parallel projection, where the mathematical expression of its transformation matrix for front-view projection directly discards the  $Z$  coordinate.

Let the object point be  $P(X, Y, Z)$ , projected onto the plane  $z = 0$  with projection point  $p(x, y)$ . The orthographic projection transformation is given by:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix} \quad (6)$$

Oblique projection may introduce shear distortion while maintaining parallelism, and can be achieved by superimposing orthographic projection with shear transformation (Foley et al., 2018; Shirley et al., 2016). Let the shear parameters be  $-a$  and  $-b$ , then the oblique projection is expressed as:

$$\begin{bmatrix} 1 & 0 & -a \\ 0 & 1 & -b \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}, \quad \begin{cases} a = \cot\theta_x \\ b = \cot\theta_y \end{cases} \quad (7)$$

where  $\theta_x(\theta_y)$  denote the angle between the projection direction and the  $x$  ( $y$ ) axes.

For cabinet Projection,  $\theta = 45^\circ$ ,  $a = b = 1$ .

For cavalier Projection,  $\theta = 63.4^\circ$ ,  $a = b \approx 0.5$ .

Thus, the parameter  $\cot\theta$  governs the degree of distortion in oblique projections.

### 3.3 Comparative Analysis of Central and Parallel Projections

Based on the comprehensive analysis of the definitions, principles, mathematical theorems, and properties of both central and parallel projections, it can be found that these two projection methods exhibit both fundamental differences and intrinsic connections. The detailed comparative analysis is enumerated as follows.

From a definitional perspective: Central projection features a finite distance between the projection center and the projection plane, with non-parallel projection lines converging at a single point, and possesses vanishing points. Parallel projection is characterized by an infinite projection distance between the center and plane, resulting in mutually parallel projection lines with no vanishing points.

From a mathematical perspective: Central projection is a nonlinear transformation in Euclidean coordinate systems but can be linearized through the introduction of homogeneous coordinates, allowing for matrix-based expression. Parallel projection is inherently a linear transformation, and its computation is relatively simple. Whereas central projections require perspective division and normalization, which make its computation becoming complex.

From a geometric perspective: For collinearity preservation, both central (perspective) and parallel projections maintain collinearity. For parallelism preservation, parallel lines of central projection may converge at vanishing points, but parallel lines of parallel projection remain strictly parallel. For length proportionality, central projection exhibits perspective foreshortening with nonlinear scaling in distance, while cross-ratios remain preserved. In contrast, orthographic parallel projection maintains true length for segments parallel to the projection plane, and oblique parallel projection allows adjustable scaling along axial directions. For angular preservation, central projection causes angular distortion. Whereas orthographic parallel projection preserves all angles, and oblique parallel projection only maintains axial angles, with potential distortion in other orientations.

Desargues' theorem and projection applications: Desargues' theorem extends the Euclidean plane to the projective plane by introducing the concepts of points at infinity and the line at infinity, embodying the inclusivity and inherent unification of projective geometry.

Owing to these distinct properties, the two projection methods exhibit different application performances. Central projection is typically employed in scenarios requiring visual effects or artistic expression, whereas parallel projection is primarily utilized in precision-dependent applications such as engineering drawings. Currently, the integration technique of both projection methods which combines geometric accuracy and visual realism has been developed and applied in multiple domains including computer graphics, architectural visualization, medical imaging, augmented reality, as



well as cartography and geographic information systems. Multidimensional representation is achieved by this hybrid approach of applying parallel projection to some objects while using perspective projection for others within the same scene.

## 4 CASE STUDIES OF CENTRAL PROJECTION APPLICATION

Due to space limitations, application case of central projection is exclusively analyzed and discussed in this section.

A canonical application of central projection is the pinhole camera model, whose physical prototype forms inverted images on the projection plane through light rays passing via a small aperture, as illustrated in figure 5.

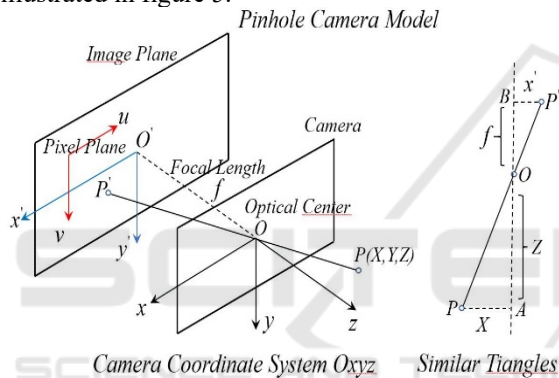


Figure 5: Pinhole Camera Model (Szeliski, 2022).

The process of digital camera image capture is fundamentally an optical imaging procedure, with the pinhole imaging model being the most widely adopted for camera imaging. This model involves four coordinate systems: the world coordinate system, camera coordinate system, image coordinate system, and pixel coordinate system, along with their mutual transformations (Hartley et al., 2004; Szeliski, 2022).

The world coordinate system  $(X, Y, Z)$ , also referred to as the global coordinate system, defines the camera's position in 3D space, while the camera coordinate system originates at the optical center  $O$  with axes  $x, y$ , and  $z$ . The image coordinate system is established on the imaging plane  $O'x'y'$  with coordinates  $(x', y', z)$ , and the pixel coordinate system, essentially a matrix-based system, has its origin at the top-left corner of the image with axes  $u$  and  $v$  parallel to  $x'$  and  $y'$ , where  $(u, v)$  represents the pixel's row and column indices in the matrix.

Here,  $P(X, Y, Z)$  denotes a point in the world coordinate system, and  $P'$  represents its corresponding projected image point.

The transformation of spatial points from the world coordinate system to the camera coordinate system belongs to a rigid-body transformation, involving solely translation  $T$  and rotation  $R$ . The rotation matrix  $R$  between the two coordinate systems can be derived from the axial rotation angles of the three coordinate axes, while the translation  $T$  is determined by the positions of their coordinate origins.

The transformation from pixel coordinates  $(u, v)$  to image coordinates  $(x', y')$  incorporates both scaling and translation due to their distinct origins and scale conventions. The translation parameters and scaling factors can be calculated based on the positional relationship between the origins of the two coordinate systems and the physical dimensions of pixels in the image coordinate system.

After completing the above two transformations, an additional step is required to achieve the full transformation from world coordinates to pixel coordinates: the transformation from camera coordinates  $(x, y)$  to image coordinates  $(x', y')$ . This conversion corresponds precisely to the central projection transformation, as illustrated in figure 5 (right panel).

Based on the principle of similar triangles, the transformation relationship can be derived, that is formula (3) and (5) along with transformation (4) in Section 3.1.

In the process of the complete transformation,  $R$  and  $T$  constitute the extrinsic parameters of the camera, defining its pose in the world coordinate system. The intrinsic parameters, encapsulated in the calibration matrix  $K$ , are derived from the focal length  $f$  and the transformation coefficients between image and pixel coordinates. Both intrinsic and extrinsic parameters can be estimated through established calibration procedures (Zhang, 2000).

Algorithm is implemented and executed according to the above steps:

The extrinsic parameters of the rotation matrix  $R$  and translation vector  $T$  for the world-to-camera coordinate transformation are computed first. Subsequently, the intrinsic parameter calibration of the projection matrix  $K$  encapsulating focal length  $f$  and image-to-pixel coordinate conversions is estimated. Finally, perspective normalization is performed through perspective division to obtain normalized coordinates.

The practical implementation can be achieved through programming in various computer languages.

In this paper, MATLAB programming is adopted, but the source code is not provided here due to space constraints.

## 5 CONCLUSION

In summary, as two fundamental projection methods in projection theory, parallel projection and central projection differ in their definitions, underlying principles, mathematical formulations, exhibited properties, and application domains.

Mathematically, central projection is based on perspective geometry, where all projection lines converge at the viewpoint, forming a conical projection structure. The most distinctive feature that distinguishes it from parallel projection is the convergence of parallel lines at vanishing points on the projection plane. This results in an inverse relationship between object size and the distance from the object point to the projection center (viewpoint), producing the characteristic 'foreshortening' visual effect. By contrast, parallelism in space is preserved in parallel projection, whether orthographic or oblique, with all projection lines remaining parallel, where projected dimensions are independent from the distance (the depth compression ratio of cabinet oblique projection is 0.5). It is this fundamental dichotomy that dictates their divergent applications.

Visually, central projection aligns with human visual perception by generating spatial depth cues, making it suitable for applications requiring realism, such as creating cinematic visual effects and discerning accurate architectural spatial relationships in through strategically placed vanishing points. In contrast, although parallel projection lacks depth perception, its ability to preserve geometric invariants makes it indispensable for technical drawings. Orthographic multi-view projections provide dimensional accuracy for mechanical component designs, while axonometric projections in architectural drafting offer three-dimensional visualization without perspective distortion.

With the development of science and technology, projection technology has also been evolving and innovating. The integration technology combining these two projection methods is being applied in a growing number of fields. It is believed that both projection approaches will further develop toward greater intelligence, automation, and cross-domain integration, enabling broader applications in the future.

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