


Analysis of the Principle and the State-of-Art Results for Searching Gravitational Waves

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Abstract: As a direct mathematical derivation of Einstein's general theory of relativity, the detection of gravitational waves is direct evidence of the correctness of Einstein's theory of gravity and provides a new perspective for studying the universe. This study focuses on the basic mathematical derivation, detection principles, and latest research achievements of gravitational waves, especially the application of laser interferometry. Through laser interferometry, detectors (e.g., LIGO and Virgo) have successfully detected multiple gravitational wave events, such as the binary neutron star merger event GW170817, verifying the predictions of general relativity and predicting various properties of neutron stars. Gravitational wave detection provides a key tool for studying galaxy evolution, black hole growth mechanisms, and the origin of the universe. At the end of this study, prospects are given for the new generation of detectors and space exploration programs, which will further enhance the sensitivity of gravitational wave detection and reveal more mysteries of the universe.

1 INTRODUCTION


Albert Einstein's general relativity, established in 1915, provided a new research perspective for physics and astronomy, by which there are many astronomical observation phenomena can be explained. For example, the Mercury precession rate was observed as 133'20" per century and eventually proved by general relativity, where an ineligible error was aroused by Newton's gravitational theory (Yahalom, 2023). Although there are many indirect proofs standing for the correctness of General Relativity, theoretical physicists tend to find the direct proof (Cervantes-Cota, et al., 2016). That is the initial motivation for detecting gravitational waves (GW), as it is direct mathematical deduction from Einstein's theory of gravity.

Since GW manifests the perturbation of spacetime, they change the length of objects though the change is extremely unobvious due to the weakness of relativistic effect. In 1960, Joseph Weber described his ideas of detecting the micro change of the length of a cylinder made of aluminium caused by GW (Cervantes-Cota, et al., 2016; Yu, et al., 2019). However, the experiment ended up with no results. It was not for the crude experimental environment (on

the contrary, the environment he set up eliminated a lot of noise, thunderstorms, cosmic rays showers, power supply fluctuations, etc. (Cervantes-Cota, et al., 2016; Yu, et al., 2019)), but for the failure of basic principles. Because there was still a kind of noise caused by thermal agitation (Cervantes-Cota, et al., 2016; Yu, et al., 2019) that cannot be removed and the gravitational signal he wanted was not much greater than this noise.

Then, in 1974, Joseph Hooten Taylor and Alan Russell Hulse found that the rotating radius of binary pulsars was decreasing (Yu, et al., 2019). This discovery can be explained as the radiation of GW leading to such decreasing. It was a new proof of the existence of GW. The progress also gave physicists hope to continue detecting the gravitational signals. Finally, exactly 100 years after the theory of general relativity and the prediction of GW were born, on 14 September 2015, these wave signals were pronounced to be detected by researchers in LIGO (the abbreviation from Laser Interferometer GW Observatory) and have been confirmed to be indeed GW (Yu, et al., 2019).

Besides LIGO, more and more other detectors were developed such as Virgo, located in Europe, also based on the principle of laser interferometer (Barish,

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& Weiss, 1999). With the coordinative operation between LIGO and Virgo, the localization area of GW sources has been reduced from a large range when using LIGO alone to a relatively small sky region, which is crucial for subsequent follow-up observations in the electromagnetic band using other astronomical equipment such as optical telescopes, radio telescopes, etc. For example, GW170817, found in 2017, is a united detecting campaign across many fields, which will be introduced in Sec 4.

In recent years, with the development of space technology, people plan to build GW detectors on a larger scale in space, known as Space GW Detection Program (Ni, 2024). These are cutting-edge concepts for GW detection. For example, LISA, a space GW detection program jointly launched by the European Space Agency (ESA) and the National Aeronautics and Space Administration of the United States (NASA), consisted with an equilateral triangular array composed of three identical spacecraft, located approximately 2.5 million kilometres apart from each other. This greatly increases the spatial scale changes caused by GW, making it easier to detect changes in laser interference fringes. Besides, China's Taiji program (Ni, 2024) also aims to build a GW detection platform in space, using satellite formations to capture GW signals by measuring changes in distance between satellites through laser interferometry. The detection of GW is of great significance for studying the evolution of galaxies, the growth mechanism of black holes, and revealing the origin and early evolution process of the universe.

This research aims to summarize the historical background of theoretical predictions and detection of GW, introduce the basic theoretical formulas of GW and the basic principles and details of some detection methods, as well as their shortcomings. On this basis, this study will discuss some research limitations of GW and some prospects for future research, intending to provide a comprehensive summary and introduction to the study of GW, to encourage inspiration for future research.

2 DESCRIPTIONS OF GW

GW are classified into three types: stochastic, periodic and impulsive (Cervantes-Cota, et al., 2016). Most stochastic waves origin from the very early era of the universe or even in the Big Bang, which acts randomly and are difficult to detect. Because of their irregular fluctuation, it is hard to separate them from the background noises of our instruments and then identify them. For periodic waves, they refer to the

kind of waves that own relatively stable and constant frequency. These waves may generate from binary neutron star (BNS) systems where the two stars rotate with each other (accurately, around the centre of mass). The third type of wave, impulsive, can be analogous to a burst of a signal. These waves correspond to and origin from some instant events, like binary black hole (BBH) merger, the explosion of supernovas or the creation of new black holes. GW are basically the radiation of the energy converted from lost mass of celestial bodies (Cervantes-Cota, et al., 2016).

The following part narrates a short derivation and deduction of the wave functions beginning with Einstein's field equations. Einstein's theory of general relativity is approximated to Newton's theory of gravity when the gravitational field is weak and static, and the particles move much more slowly than the speed of light (Chakrabarty, 1999). One now considers it as a perturbation on the flat Minkowski metric:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1 \quad (1)$$

Indices of any tensor can be raised or lowered using $\eta_{\mu\nu}$ or $\eta^{\mu\nu}$ respectively because of the weakness of the perturbation. Therefore,

$$g^{\mu\nu} = \eta^{\mu\nu} - h^{\mu\nu} \quad (2)$$

The perturbation transforms under Lorentz transformation as a second-rank tensor:

$$h_{\alpha\beta} = \Lambda_{\alpha}^{\mu} \Lambda_{\beta}^{\nu} h_{\mu\nu} \quad (3)$$

By continuously calculating the affine connection and the Riemann curvature tensor, one obtains Ricci scalar as

$$R = \partial_{\lambda} \partial_{\mu} h^{\lambda\mu} - \square h \quad (4)$$

The Einstein tensor, $G_{\mu\nu}$, in weak field is

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} R = \frac{1}{2} (\partial_{\lambda} \partial_{\nu} h_{\mu}^{\lambda} + \partial_{\lambda} \partial_{\mu} h_{\nu}^{\lambda} - \eta_{\mu\nu} \partial_{\mu} \partial_{\nu} h^{\mu\nu} + \eta_{\mu\nu} \square h - \square h_{\mu\nu}) \quad (5)$$

The linearized Einstein field equations are then

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \quad (6)$$

or its equivalent form:

$$\square^2 h_{\mu\nu} - \partial_{\lambda} \partial_{\nu} h_{\mu}^{\lambda} - \partial_{\lambda} \partial_{\mu} h_{\nu}^{\lambda} + \partial_{\mu} \partial_{\nu} h = -16\pi G S_{\mu\nu} \quad (7)$$

where

$$S_{\mu\nu} \equiv T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T \quad (8)$$

The equations have infinitely many solutions because one can always transform the form of one solution to another form by changing the coordinate system. However, one can utilize a specific gauge transformation and work under a selected coordinate system. One such coordinate system is the harmonic coordinate system. The gauge condition is

$$g^{\mu\nu}\Gamma_{\mu\nu}^\lambda = 0 \quad (9)$$

In the weak field limit, this condition reduces to

$$\partial_\lambda h_\mu^\lambda = \frac{1}{2}\partial_\mu h \quad (10)$$

Utilizing it to simplify the linearized Einstein field equations, one obtains

$$\square^2 h_{\mu\nu} = -16\pi G S_{\mu\nu} \quad (11)$$

In vacuum,

$$\square^2 h_{\mu\nu} = 0 \quad (12)$$

It is analogous with the wave function of electromagnetism, and it obviously has the plane-wave solutions

$$h_{\mu\nu} = \epsilon_{\mu\nu} \exp(ik_\alpha x^\alpha) + \epsilon_{\mu\nu}^* \exp(-ik_\alpha x^\alpha) \quad (13)$$

where $k_\alpha = (\omega, \vec{k})$ and $\epsilon_{\mu\nu}$ is the polarization tensor. Plugging in the solution Eq. (13) into the Eq. (12), one obtains

$$k_\alpha k^\alpha = 0 \quad (14)$$

which means $k_\mu = 0$ and GW propagates at the speed of light. Using the harmonic gauge condition, one finds $\epsilon_{\mu\nu}$ is orthogonal to k_μ :

$$k_\mu \epsilon_\nu^\mu = \frac{1}{2} k_\nu \epsilon \quad (15)$$

Therefore, GW is transverse. There are ten independent components in a symmetric rank-2 tensor, so that its degrees of freedom are also ten. One can utilize four equations from Eq. (15) to reduce the degrees of freedom of $\epsilon_{\mu\nu}$ to six. However, under such gauge conditions there are still many coordinate selections. By finalizing the coordinate system used, another four equations can be obtained, which again reduces the number of independent components of $\epsilon_{\mu\nu}$ to 2. Considering a minor coordinate transformation:

$$x'^\mu = x^\mu + \xi^\mu \quad (16)$$

and the metric tensor transforms as:

$$g'_{\mu\nu} = \frac{\partial x'^\alpha}{\partial x^\mu} \frac{\partial x'^\beta}{\partial x^\nu} g_{\alpha\beta} \quad (17)$$

reserving it to the first order:

$$h'_{\mu\nu} = h_{\mu\nu} - \partial_\mu \xi_\nu - \partial_\nu \xi_\mu \quad (18)$$

and the form of ξ^μ one used is

$$\xi^\mu = i e^\mu \exp(ik_\alpha x^\alpha) - i e^{*\mu} \exp(-ik_\alpha x^\alpha) \quad (19)$$

Plugging in it into the Eq. (18), one obtains

$$\epsilon'_{\mu\nu} = \epsilon_{\mu\nu} + k_\mu e_\nu + k_\nu e_\mu \quad (20)$$

Now, one has determined the specific coordinate system with e_μ . It is obvious that there are four more additional conditions, and the final degrees of freedom are 2, which are also the only two physical degrees of freedom. One considers the wave propagates along z-axis, so there are four non-zero components in $\epsilon_{\mu\nu}$:

$$\epsilon_{11} = -\epsilon_{22}, \epsilon_{12} = \epsilon_{21} \quad (21)$$

The trace of $\epsilon_{\mu\nu}$ is obviously zero. Considering the wave is transverse, one names the gauge conditions as transverse traceless (TT) (Flanagan & Hughes, 2005). The matrix form is

$$\epsilon_{\mu\nu}^{TT} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \epsilon_{11} & \epsilon_{12} & 0 \\ 0 & \epsilon_{12} & -\epsilon_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (22)$$

Now, one is going to find the solutions of the Eq. (11).

Using the Green's function, one obtains

$$h_{\mu\nu}(\vec{x}, t) = 4G \int_V d^3\vec{x}' \frac{S_{\mu\nu}(\vec{x}', t - |\vec{x} - \vec{x}'|)}{|\vec{x} - \vec{x}'|} \quad (23)$$

where one uses the natural unit system, and this indicates that the perturbation of the spacetime radiates from the material distribution from the distance of $|\vec{x} - \vec{x}'|$. And the greater the mass, the more intense the radiation (Cervantes-Cota, et al., 2016).

3 PRINCIPLE AND FACILITIES

The Eq. (23) give some inspiration for the detection of GW. To measure the change of scales of objects, one defines the gravitational-wave amplitude h as (Pitkin, et al., 2011):

$$h = \frac{2\Delta L}{L} \quad (24)$$

which is basically the proportion between the change of length and the initial length and one can consider it as perturbation $h_{\mu\nu}$ discussed in Sec 2. As a matter of fact, different astrophysical events radiate waves in different frequencies and with different amplitudes. Most of the amplitudes are very small, only 10^{-21} or even smaller, depending on specific sources (Pitkin, et al., 2011). Therefore, removing various noises is the main technical key point and difficulty for the detection.

Laser interferometry provides the possibility of very high sensitivity over a wide frequency range. It drew inspiration from the design of the Michelson interferometer, which consists of two mirrors and a half mirror (seen from Fig. 1 (Pitkin, et al., 2011)). When the laser travels along the optical path, it is divided into two beams and eventually reassembles to interfere and produce interference fringes. When GW interact with the instrument, the distance between the mirrors is changed, that is, the optical path is changed, so the interference fringes will move. One can use computers to detect GW by converting optical signals into electrical signals. That is the basic principle of laser interferometry.

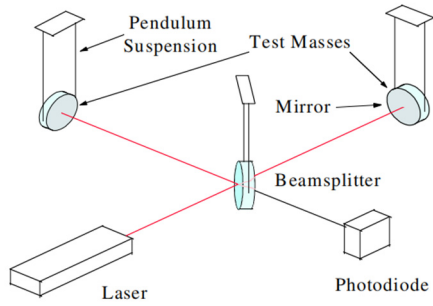


Figure 1: Structure of gravitational-wave detector based on laser interferometry (Pitkin, et al., 2011).

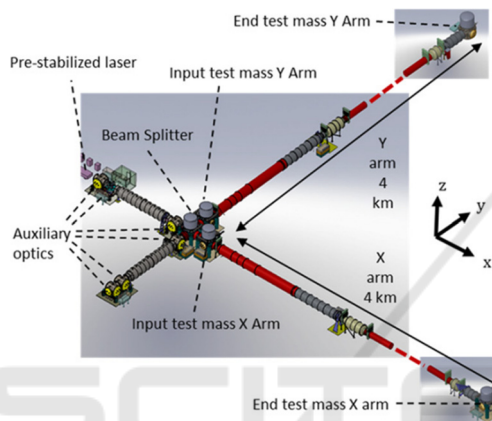


Figure 2: Representation of the LIGO vacuum system (Matichard, et al., 2015).

As the first detector to detect GW signals, LIGO project includes two identical detection devices, located in Hanford (WA) and Livingston (LA). Fig. 2 shows the vacuum enclosure and instrument equipment (Matichard, et al., 2015). Each detector uses 11 vacuum tanks (Matichard, et al., 2015). Five of them are large balanced scorecard chambers (with a diameter of approximately 4.5 meters and 2.5 meters), which contain the core optical system of the interferometer (Matichard, et al., 2015). Six of them are smaller HAM chambers (approximately 2.5 meters high and 2.5 meters wide) that house auxiliary optical equipment for interferometers (Matichard, et al., 2015).

The auxiliary optical system in LIGO is also a major highlight, as the initial laser beam emitted by the laser source may have an uneven intensity distribution or a shape that does not meet the requirements of the interferometer (Matichard, et al., 2015). The beam shaping element in the auxiliary optical system is used to shape the laser beam into a specific shape, such as a Gaussian beam shape. Gaussian beams have the characteristic of high

central intensity and gradually decreasing edge intensity, which is beneficial for propagation in interferometers and reduces aberrations and other issues. By using specially designed optical components such as lens groups or non-spherical mirrors, the intensity distribution of the laser beam can be precisely controlled to ensure that the laser beam has a suitable intensity distribution in various parts of the interferometer, thereby improving the accuracy of interferometric measurements. Some sophisticated structures are shown in Fig. 3 and Fig. 4 (Matichard, et al., 2015).

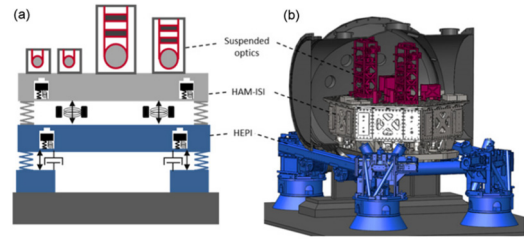


Figure 3: (a) Schematic and (b) CAD model of the isolation systems supporting the auxiliary optics in the HAM chambers (Matichard, et al., 2015).

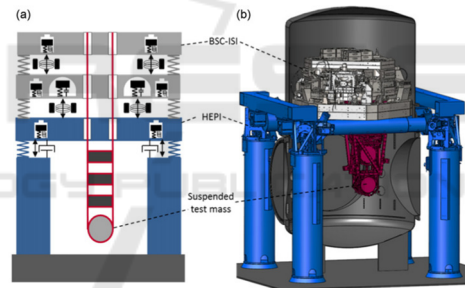


Figure 4: (a) Schematic and (b) CAD model of the isolation systems supporting the core optics in the BSC chambers (Matichard, et al., 2015).

Pulsars are high-speed rotating neutron stars that emit very regular pulse signals. When GWs propagate in the universe, they stretch and compress spacetime, thereby affecting the time interval for pulsar signals to reach Earth. By observing the changes in the arrival time of signals from multiple pulsars and constructing a pulsar timing array, the presence of GW can be detected. If GW pass through the spacetime between Earth and a pulsar, it will cause small regular changes in the arrival time of the pulsar signal on Earth, which can be observed by high-precision radio telescopes (Johnson, et al., 2024). The advantage of pulsar timing array is its high detection sensitivity to low-frequency GW (Johnson, et al., 2024). Unlike laser interferometric GW detectors such as LIGO and Virgo, which mainly target high-frequency GW,

pulsar timing arrays can detect GW in the nanohertz frequency range, which corresponds to some important massive astrophysical processes in the universe, e.g., the merger of supermassive black holes, the formation and evolution of galaxies.

There are many observatories primarily used for observing pulsars to detect GW in the world. For example, the Parkes Radio Telescope Observatory in Australia, the Effelsberg Radio Observatory in Europe, and the Green Bank Observatory in the United States (Johnson, et al., 2024). They have all obtained a large amount of pulsar data, which can be filtered to obtain information about GW. There actually has been some data obtained through the observation by such method, but the analysis of them is still at the fundamental level and it still cannot be sure that if such method is a stable technology and can be generalized to detect GW.

4 OBSERVATION RESULTS AND ANALYSIS

Subsequently, one focuses on the discussion about GW from binary stellar objects like BNS and BBH. One first gives some dynamic and post-Newtonian explanation about the principle of binary system emitting GW and then analyse some data about BNS obtained in recent year.

A binary object system that is constrained by gravity rotates around the centre of mass. In classical Newtonian theory, it is a conservative system and keeps a conservation of energy, that the orbits remain quasi-spherical and will not shrink. However, in post-Newtonian theory, factor v^2/c^2 should be considered (where v is the velocity of object and c is the speed of light). So, the separation between two bodies decreases and the rotating frequency rises, due to the emission of GW causes the decline of orbital angular momentum. The phenomenon of GW rising frequency is called chirping and the waveform is shown as in Fig. 5 (Schmidt, 2021). The orbit is regarded as a plane and the orientation of orbital angular momentum is stable, despite the orbital precession result from any angular perturbation. The negligibility of precession leads to the approximately fixed direction of GW propagation, and the wave can be measured with the component h_{22} . The whole process of binary system evolution falls roughly into three periods: inspiral, merger and ringdown. Figure 5 (right) shows the waveform of late inspiral-period and the merger-period. Through interferometric principle on which detectors based, time-frequency

and time-amplitude data representations can be collected to analyze the properties of GW and further the properties of source by using theoretical results of post-Newtonian theory or, using general relativity for ineligible case when objects are about to merge and the velocities are close to speed of light.

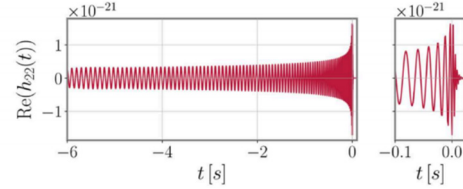


Figure 5: The GW form of a non-spinning $30 + 30M_{sun}$ binary black hole at a distance of 400Mpc (Schmidt, 2021).

For recent data about BNS, asSSS17a/AT 2017gfo was the united campaign organized to precisely observe and analyze captured GW after people in Advanced LIGO realized the GW signal was being received in August 2017. It was the first time for humanity to conduct the united detection across domains of astrophysics, GW and electromagnetism. The signal detected was the strongest GW signal up to that time, with a combined signal-to-noise ratio (SNR) of 32.4 (Abbott, et al., 2017; Abbott, et al., 2019), lasting about 100s (Abbott, et al., 2017), as shown in Fig. 6 (Abbott, et al., 2017). The observation was a joint campaign involving the data derived by two instruments from LIGO (Hanford and Livingston) and the detector from Virgo. From Einstein's general relativity one obtains that as binary system rotating, the rotating frequency, i.e., the frequency of GW emitted from the system, will rise corresponding to the shrink of orbit, which was consistent with discussed earlier. The increasing frequency closely related to a form of combination with stellar masses, which is called the chirp mass (M) (Abbott, et al., 2019), and one has

$$M = (m_1 m_2)^{\frac{3}{5}} (m_1 + m_2)^{-\frac{1}{5}} \quad (25)$$

By analysing the frequencies data from figure 5 collected by the three interferometers, researchers found that the chirp mass of the wave source well matched that of the binary neutron star (BNS) system. Moreover, combining with the mass rate $q = \frac{m_1}{m_2}$, where $m_1 > m_2$, it was verified that this GW was most probably from the process of a BNS merger (despite the possibility of other stellar binary systems (Abbott, et al., 2017) since not much strict and cramped interval of mass, electromagnetic and astrophysical observation afterwards both proved the source was BNS).

During the detection, there was one noise, or glitch, caused by instrumental error of LIGO-Livingston, should be subtracted from the data. The window function (Abbott, et al., 2017) was used to make the effect of the glitch to be the minimum, as the sine-like-shaped brown curve illustrated in Fig. 7 (Abbott, et al., 2017), which was based on sine function mode and mitigated the value from unexpected coordinate band. After that, a rapid binary-coalescence reanalysis (Abbott, et al., 2017) was used to localize the source of BNS with correct data.

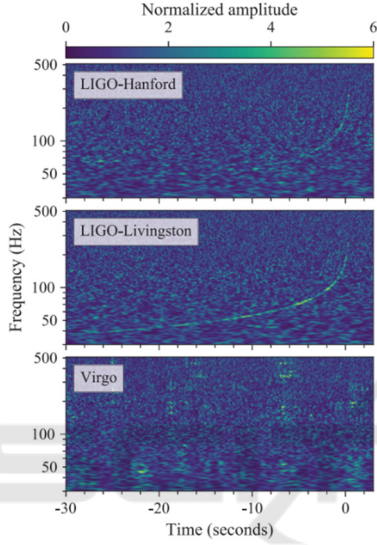


Figure 6: Time-frequency representations of data containing the gravitational-wave event GW170817, observed by the LIGO-Hanford (top), LIGO-Livingston (middle), and Virgo (bottom) detectors (Abbott, et al., 2017).

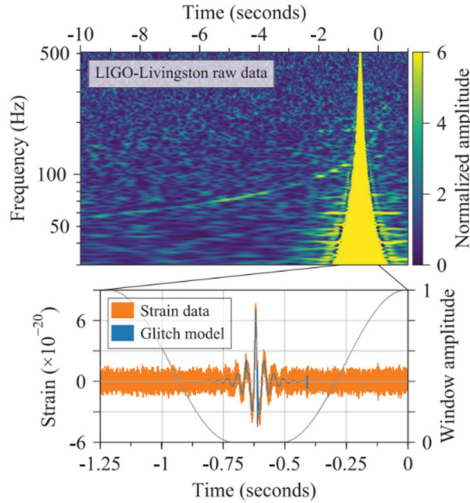


Figure 7: Mitigation of the glitch in LIGO-Livingston data (Abbott, et al., 2017).

Additionally, a γ -ray burst occurred following the merger and was identified by electromagnetic detectors. So, both electromagnetic method and GW method named rapid binary-coalescence reanalysis (Abbott, et al., 2017) were utilized to convincingly localize the source of radiation. Simultaneously, astrophysical observation was also organized, attaining consistent result, and finally determined the location of the BNS source that it was near the galaxy NGC 4993 (Abbott, et al., 2017; Abbott, et al., 2019). The detailed location was presented in Fig. 8 (Abbott, et al., 2019).

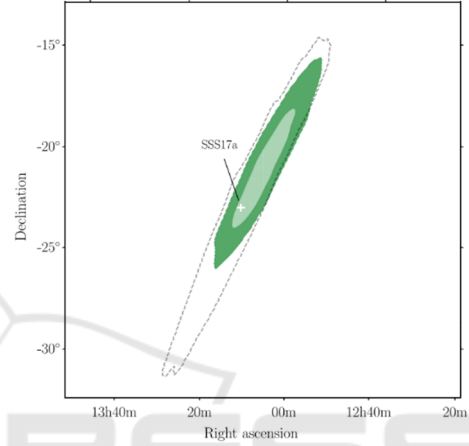


Figure 8: The improved localization of GW170817, with the location of the associated counterpart SSS17a/AT 2017gfo (Abbott, et al., 2019).

5 LIMITATIONS AND PROSPECTS

The current gravitational wave detection technology still has many limitations. The current GW detectors (e.g., LIGO, Virgo, KAGRA) still have limited sensitivity in detecting GW, especially in detecting extreme frequency signals. For low-frequency signals (<10 Hz), ground detectors are difficult to effectively detect because noise on Earth (Pitkin, et al., 2011; Matchard, et al., 2015) (e.g., earthquakes, human activities) can interfere with the signal. For high-frequency signals (>10 kHz), current detectors have low resolution for high-frequency signals and may miss some important physical phenomena.

Meanwhile, GW signals are very weak and easily masked by environmental noise such as earthquakes, waves, temperature changes, etc. Although the impact of noise can be partially reduced through multi detector networks and data analysis techniques, eliminating it remains a challenge. In addition, some

noise, such as quantum noise, cannot be theoretically eliminated because it exists in the form of quantum radiation pressure noise based on photon technology (Abbott, et al., 2017). Among other advances, A+ (the planned advanced LIGO upgrade) will improve LIGO's broadband sensitivity by using the technique of quantum light squeezing to reduce laser phase noise at high frequencies and radiation pressure noise at low frequencies (Coleman Miller, & Yunes, 2022).

In addition, the analysis of gravitational wave signals relies on theoretical models, such as numerical relativistic simulations (Schmidt, 2021) of BBH merger. However, these models may not be entirely accurate, as described in Sec. 4.1, especially under extreme conditions such as extremely high densities, strong gravitational fields, etc. The post Newtonian model is no longer applicable and requires the use of numerical relativity theory to perform numerical calculations and analysis using computers, which can result in numerical errors.

Based on considerations of current limitations, there have been developments and prospects in recent years. For example, the outlook for the construction of a new generation of detectors is that future ground-based gravitational wave detectors such as Einstein Telescope and Cosmic Explorer (Abbott, et al., 2017; Bailes, et al., 2021) (Sensitivity of expected improved Cosmic Explorer is shown in Fig. 9 compared with that of Advanced LIGO and several types of noise) will have higher sensitivity, be able to detect more distant and weaker gravitational wave signals, and cover a wider frequency range (Abbott, et al., 2017). At the same time, in order to obtain a larger observation frequency band, space probes are being planned for construction. Space based gravitational wave detectors (such as LISA, DECIGO, BBO (Bailes, et al., 2021)) will be able to detect low-frequency gravitational waves (in the band of mHz to Hz) and study celestial physical processes such as supermassive black hole mergers and galaxy evolution.

Gravitational waves are also a crucial tool for studying the evolution of the early universe. The Big Bang model shows how the universe inflated from an initial state of extremely high density to the universe we currently inhabit (Ringwald, & Tamarit, 2022). It successfully traces the history of the universe back to a fraction of a second after birth, but direct information about the history of the universe before the Big Bang nuclear fusion can be obtained through observations of gravitational waves (Ringwald, & Tamarit, 2022). Future space probes may be able to directly detect these signals, providing us with new clues about the origin and evolution of the universe.

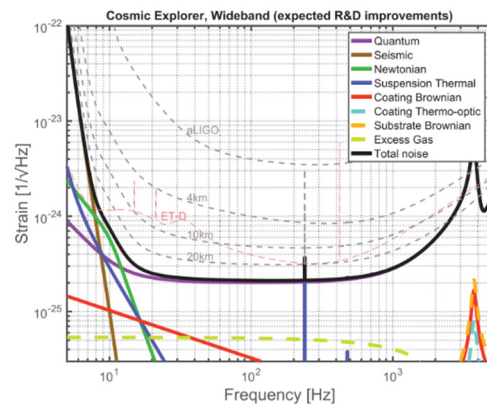


Figure 9. Target sensitivity for a next generation gravitational-wave detector (Abbott, et al., 2017).

6 CONCLUSIONS

To sum up, this study summarizes the historical background, basic theoretical formulas, and detection methods of gravitational wave detection, analyses the limitations of current technology, and discusses future research prospects. By detecting gravitational waves, scientists can study the evolution of galaxies, the growth mechanism of black holes, and the origin and early evolution of the universe. Future space probes and next-generation ground probes will further enhance the sensitivity of gravitational wave detection, enabling us to detect signals that are farther and weaker, thus revealing more mysteries about the universe. This study provides a comprehensive summary and introduction for the in-depth exploration of the field of gravitational waves, inspiring inspiration for future research.

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