

Satellite Navigation Constellation Optimisation Problem Definition for the Application of Genetic Algorithms

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Keywords: Constellation Optimisation, Evolutionary Algorithms, GNSS, LEO-PNT, Multi-Objective Optimisation, Navigation, Problem Characterisation.

Abstract: Global Navigation Satellite Systems (GNSS) are used on a daily basis, providing Positioning, Navigation and Timing (PNT) services for various applications ranging from smartphones over the financial sector up to areas such as aviation and space. Classical GNSS constellations positioned in Medium Earth Orbit (MEO) often experience reduced performance in areas of low visibility like forests and cities. To rectify this, augmentation constellations are deployed, improving the provided positioning accuracy. Recent proposals for augmentation systems have often been based in Low Earth Orbit (LEO), which, for global coverage, require a large number of satellites and are complex to design due to dependencies, coverage requirements and the large search space. This makes the constellation design problem well-suited for applying Genetic Algorithms (GA) to find an optimal solution. However, previous research has only addressed highly constrained versions of the problem. This paper presents an approach for applying GAs to constellation designs with a large search space. In particular, the focus is on the description of the multi-objective fitness function and the simulation necessary for its evaluation, options for the solution encoding, and a discussion of algorithmic features applicable in this scenario.

1 INTRODUCTION

Satellites have come a long way since the launch of the first satellite, Sputnik, in 1957. In today's world satellites have become a backbone of modern civilisation and are used for military as well as civilian applications on a daily basis, spanning a wide range, e.g. telecommunication, television, internet access, reconnaissance, earth observation, deep space telescopes and Position, Navigation and Timing (PNT).

The focus of this work are PNT constellations and mostly augmentative constellations supporting Global Navigation Satellite Systems (GNSS). Currently, there are four GNSS constellations: Global Positioning System (GPS - USA), Global'naya Navigatsionnaya Sputnikovaya Sistema (GLONASS - Russia), Baidou (China) and Galileo (Europa). These systems are mostly positioned in Medium Earth Orbit (MEO) at an altitude between 19000 km and 23000

km, the only exception to this being Baidou, as it integrates augmentation satellites in the system in addition to MEO at Low Earth Orbit (LEO), Geostationary Earth Orbit (GEO) and Geo Synchronous Orbit (GSO). These MEO constellations are stable in their constellation design and improvements are more focused on the onboard technologies. However, in addition to these constellations there are augmentation constellations which are more varying and adaptive in their constellation design and therefore in constant development. These constellations enhance GNSS performance in specific regions, such as the Indian system Navigation with Indian Constellation (NavIC) and the Quasi Zenith Satellite System (QZSS) over Japan. Besides these regional systems, there are also plans for new global constellations in LEO, on governmental side as well as from commercial companies. Constellations in LEO come with certain advantages, for example satellites can use less expensive components due to less radiation, the launch costs are reduced due to the lower orbits (400-2000 km) and the strength of the signal reaching Earth is a lot higher. However, there are drawbacks as well, for global cov-

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erage the number of satellites needs to be higher than for traditional GNSS constellations, also the satellites need to correct their orbits more often due to atmospheric drag at lower altitudes, causing them to have a shorter lifetime than their MEO counterparts. (Li et al., 2023)

These augmentative constellations are much more suited for optimisation than classic GNSS constellations, as either a specific region has to be improved with a specific number of satellites or, on a global scale for LEO, the same performance has to be achieved with a minimal number of satellites. The optimisation of constellations is important for multiple reasons, while the primary objective for the end user of a constellation would be improved performance, for the operator it would be the reduction of costs. Besides performance and costs another topic that has become more important recently is sustainability, one natural point is that a reduced number of rocket launches will reduce the impact on the environment. Furthermore satellites contain rare materials and the long term effects of the residual materials of them burning up in the atmosphere is currently barely researched so until a better solution is found to retire satellites it should be a goal to minimise their number.

In the following sections, the necessary background is provided to enable the understanding of constellation design and performance indicators. Then, an in-depth analysis of the optimisation problem is conducted before showing necessary preparations for the application of a Genetic Algorithm (GA).

2 BACKGROUND

Before describing the considerations necessary for the application of GAs, the requirements and constellation types for the optimisation approach are explained in detail.

2.1 Need for Augmentation

There are some parameters that can be considered for the optimisation of a constellation, but for navigation constellations, the first parameter to look at is the Dilution of Precision (DOP) value of the constellation. The quality of a positioning solution is in direct correlation with the DOP value of the constellation, it accounts for the user aspect with the receiver position, as well as the system design based on the satellite geometry. DOP measures the combined impact of satellite geometry on both 3D position and time accuracy, while Position Dilution of Precision (PDOP) measures only the effect on 3D position accuracy. PDOP

is chosen because it isolates the spatial position quality, providing a clearer assessment of how satellite geometry affects positioning without mixing in timing errors. The PDOP is based on the relationship between the locations of the satellites in Line of Sight (LOS) to the receiver. Within these lines of sight, a reversed squared pyramid can be built between four satellites and the receiver, this is shown in Figure 1. Depending on the volume of the resulting pyramid, the PDOP will change, a larger volume leading to a smaller PDOP and thereby to a better positioning solution. DOP values can be categorised depending on their usability for positioning applications as shown in Table 1.



Figure 1: Visualisation of geometry for the PDOP between receiver and satellites.

Table 1: DOP ratings and classification (Bernhardt et al., 2024).

DOP	Rating	Classification
<1	Ideal	High confidence for high precision
1-2	Excellent	Appropriate for high precision
2-5	Good	Minimum for accurate positioning
5-10	Moderate	Minimum requirement for positioning
10-20	Fair	Rough position estimation
>20	Poor	Measurements should be discarded

The global mean PDOP of Galileo is shown in Figure 2. The data is analysed over one repeat orbit, meaning that the ground tracks of the constellation repeat themselves. For Galileo this happens every ten days. This allows to make viable statements for the behaviour of a constellation with a reduced data set. The data in Figure 2 shows that the average PDOP achieved by Galileo is around two, which is excellent for positioning and, as most common receivers use multi system GNSS, combining compatible systems with each other, these values become even better.

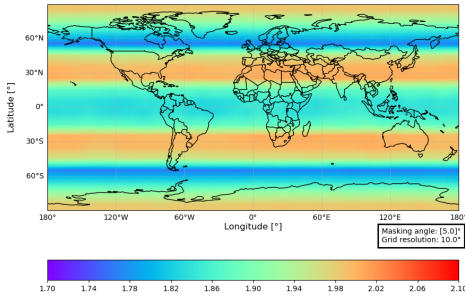


Figure 2: Visualisation of mean PDOP (values represented by the colour bar) for Galileo. The stations were equally distributed every 10° with a masking angle of 5° .

However, this only represents open field conditions with no constraints to visibility and a large part of navigation in daily life happens in cities and, with this, in urban canyons. In urban canyons, the masking angles, depending on building height and positioning of the user, can be much higher, reaching up to 90° and by this greatly reducing the satellites in LOS (Xu et al., 2020). Additionally, as MEO satellites appear to move relatively slowly from an observer's perspective on Earth, the conditions in a given location often remain consistent over a long period. With the reduction of visible satellites this can increase the DOP by a large factor (More et al., 2022) or even prevent positioning when the number of visible satellites falls below four. The introduction of augmentation constellations is meant to increase the number of visible satellites. In the case of GEO satellites, these are in a relatively fixed position mostly at a very high angle over the target region, while for LEO satellites, large numbers combined with faster orbits lead to more frequent changes in the geometry and a greater number of visible satellites. Both approaches increase the performance of GNSS in challenging environments.

Apart from the DOP, other optimisation objectives that could be used for navigation constellations or communication systems are, for example, visibility from a certain region for communication from a ground station to a satellite, or the revisit time it takes the satellite to come back to a certain location.

2.2 Constellation Types

Satellite constellation design is a complex problem, as not only the individual orbits have to be chosen but the different orbits have to be compatible as well. A satellite's orbit and its position on that orbit, is defined through the six Keplerian elements (cf. Figure 3): semi-major axis, half of the major axis of the ellipse, orbit eccentricity, indicating the deviation from a circular orbit, inclination of the orbit with respect to the

equatorial plane, longitude of ascending node, specifying where the orbit passes through the reference plane, argument of periapsis, defining the orientation of the ellipse in the orbital plane and the true anomaly, showing the current position of the satellite on the orbit.

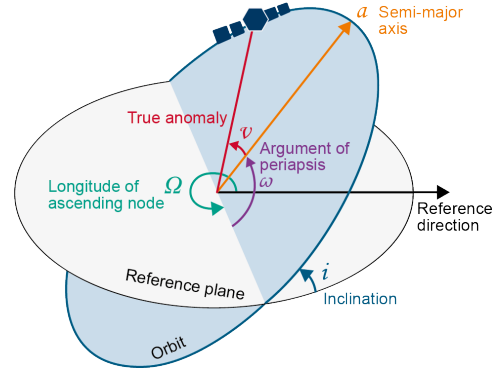


Figure 3: Visualisation of Kepler Elements.

These elements offer an idealised representation of the orbital plane of an object with respect to a reference plane and direction. Figure 3 describes an orbit around an arbitrary body. Since the current satellite constellations are focused on earth some further definitions can be made. The reference plane and direction are defined as the equatorial plane and the vernal equinox respectively. Additionally, for earth centred orbits the terms apogee and perigee are used, as well as Right Ascension of the Ascending Node (RAAN) for the longitude of ascending node.

Since the description of a satellite orbit requires all elements of each individual satellite to be defined, the number of variables quickly becomes impractical. To minimise the number of parameters required to define a constellation, different design frameworks have been developed to define relationships between the satellites within a constellation. Some of the most prominent constellation frameworks include Walker (Delta/Star) constellations (Walker, 1970), Street-of-Coverage (SOC) constellations (Beech and Dutruel-Lecohier, 2013), and the newer Flower constellations (Mortari et al., 2004).

J. G. Walker introduced two circular orbit constellations in 1970, which set all orbital element apart from the RAAN and true anomaly to the same value, and have the orbital planes evenly distributed along the equatorial line. The two constellation types differ in the plane distribution. The Walker Delta (cf. Figure 4) has them distributed along the entire 360° , while the Star pattern spreads them over 180° . In addition to the Keplerian elements of the first satellite, three constellation structure parameters are needed to

define a Walker constellation: the total number of satellites, the number of orbital planes and the phase-shift between adjacent planes. The Walker framework can be used to describe most constellations, such as for example the Galileo constellation (Delta) and the telecommunication constellation Iridium (Star).

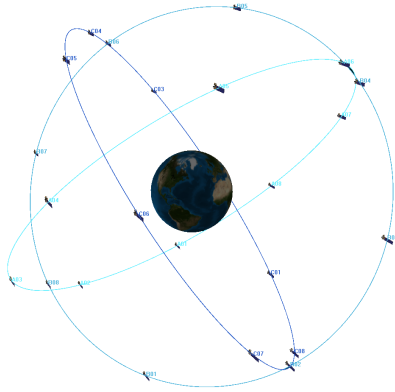


Figure 4: Visualisation of Walker Delta orbits at MEO in an Earth Centered Inertial view.

Streets of Coverage constellations are one of the few asymmetric constellations, distribution the orbits heterogeneously along the equatorial line. Adjacent orbits and satellites in the same orbit are synchronised to offer swaths of coverage. The width of the swath, also called 'street' has to be set, along with the orbital elements of the first satellite. SOC constellations are primarily used for communication.

The newer Flower constellations are designed to have repeating ground tracks, meaning that after a given time all satellites return to their initial positions. This property facilitates designing a constellation for continuous coverage, since a complete constellation analysis can be carried out in the time span of one repetition (Nadoushan and Assadian, 2015). To ensure repeating ground tracks, the framework sets the orbital nodal period, the time needed for a satellite to return to an orbital nodes, equal to the Greenwich nodal period (24 hours). The Flower framework creates very specific constellations and is therefore mostly used for the derivation of more general representations instead of directly in the design of a constellation.

A derivation from the harmonic (symmetric) subclass of Flower constellation results in the Lattice Flower frameworks, which seek to generalise multiple constellation types into one design framework. The 2D Lattice theory combines Walker and Flower constellations and requires the same parameters as the Walker framework (Avendaño et al., 2013). Having set the initial values for the longitude of ascending node and the mean anomaly, the positions of all satel-

lites are defined by a two-dimensional system of equations. Expansion of the theory to higher dimensions such as 3D and 4D is done to increase the degrees of freedom of the design.

The 3D theory includes measures to counter the effects of J_2 perturbations on the constellation orbits. J_2 perturbation are the largest considered perturbation effect caused by earth's oblateness. These disturbance in the gravity field create forces, affecting satellite orbits and distorting them over time, especially non-circular orbits. Instead of planning maintenance manoeuvres to upkeep the repeating ground track, counter measures are included in the constellation design. In the 3D representation, a term is added for varying arguments of perigee, which accounts for the rotation caused by J_2 perturbations (Davis et al., 2013). The argument of perigee rotation changes the relative position of the satellites in one orbital plane to those in another, jeopardising coverage criteria. The constellation can now be described using six integer and six continuous parameters. The 4D Lattice framework introduces an additional variation of the semi-major axis of each orbit (Arnas et al., 2021). Usually, varying satellite altitudes are introduced through different constellation layers, instead of different planes. However, for some regional coverage scenarios, this representation could be of use.

3 RELATED WORK

Due to the relevance of the field, there have been some past research efforts to develop an optimisation method for satellite constellations (Choo et al., 2024). Most literature focuses on the design of communication, navigation and earth observation constellations with different types of coverage requirements: global, regional or discontinuous regional coverage. Different design approaches have been employed to identify optimal constellations, including simulation, analytical analysis and in recent years heuristic optimisation methods, such as GAs. Some of the objectives of the optimisation are to minimise the number of satellites as well as performance metrics such as the revisit time and DOP values. In the following segment, literature designing navigation constellations based on DOP values, as well as its limitations, will be reviewed. Some literature focuses on the minimisation of the Geometric Dilution of Precision (GDOP), however for the reasons stated in Section 2.1 this paper uses the PDOP.

Casanova (Casanova et al., 2014) attempts to reproduce the Galileo constellation, using a basic GA, a Particle Swarm Optimiser (PSO) and a coarse grid

enumeration and compares their results. While all three algorithms produce promising results, they were applied to a search space limited to eccentricity, inclination, and argument of perigee alterations, and still required a day and a half to reach convergence despite the limitations.

Multi-objective constellation design is presented in (Han et al., 2021) utilising a Multi-Objective Particle Swarm Optimiser (MOPSO) to minimise cost, represented by the altitude, minimisation of the PDOP and the maximisation of the number of visible satellites. For the augmentative constellation, to be designed, an altitude interval of 1000km is given and only circular orbits are considered. The paper utilises the 2D Lattice representation, fixing the number of satellites, as well as all orbital parameters except the altitude and inclination. The MOPSO produces a constellation with an improved homogeneous coverage, but neglects the advantages a larger altitude range and eccentric orbits have for global coverage (Ballard, 1980).

An example of a more complete optimisation is presented in (Hitomi and Selva, 2018), which adds the constellation size as a variable. Here, no constellation framework is used, such that each satellite is placed independently. But similarly to other publications, the variable intervals are kept small with the semi-major axis having a range of 600km and the satellite number being limited to twenty. Until now, no optimisation of the complete search space with large constellations has been carried out, warranting further developments.

4 PROPOSED APPROACH

Having discussed basics and previous work in the field of constellation design, the following section seeks to find a suitable formulation for an optimiser to solve the complete design problem.

4.1 Methodology Discussion

In (Casanova et al., 2014), single-objective optimisation is explored using the GDOP as the quality metric, comparing a Simulated Annealing (SA) approach with a classic GA. Here, the GA performs better in regard to the given constellation design problem. Since the design problem does not necessarily have a single optimum (uni-modal), single-solution algorithms such as SA relying on exploitation can get stuck in local optima very easily. The exploration properties provided by population-based approaches, such as GAs, help to avoid this issue, making population-

based algorithms the preferred methodologies for high complexity problems, such as the one described here. In most literature the design problem is run as a single objective optimisation. While some papers focus on the GDOP minimisation (Casanova et al., 2014), in (Paek et al., 2019) a weighted sum of objectives is used as the fitness function, including constellation cost and revisit time in addition to the GDOP. Forming a weighted sum instead of using a multi-objective algorithm simplifies the program structure and reduces computation time. However, due to a significant loss of solution quality the benefits of the simpler formulation are negated. So, while a single-objective formulation can be used for an initial analysis of the problem formulation and constellation representation, an expansion to multiple objectives must be incorporated at a later stage. The topology presented in (Choo et al., 2024) shows that in the context of constellation design the focus lies on optimisation algorithms such as PSO (Han et al., 2021) and GAs (Whittecarr and Ferringer, 2014), with articles such as (Casanova et al., 2014) and (Casanova et al., 2012) comparing their performance. Generally, PSO shows higher fitness results with faster convergence, due to the deterministic nature of the update mechanism compared to a generic GA (Casanova et al., 2014). However, GAs are often preferred for multi-objective optimisation, as extensive research has gone into enabling simplified implementations based on well-established base algorithms and predefined operators (Konak et al., 2006). Additionally, there are enough well documented GA formulations available, such as the Non-dominated Sorting Genetic Algorithm 2 (NSGA-II) (Deb et al., 2002), that allow an easy recreation of the base algorithms, which can then be expanded and adapted for the specific use case. The wide variety of operators available for GAs enables the improvement of the algorithm to match, if not surpass the performance of multi-objective PSO (Konak et al., 2006). Some potential expansions of the algorithm include the introduction of dynamic operator selection, and the reformulation of the problem encoding into a variable-length chromosome representation. This method is based on a theory called building block hypothesis, which suggests that constellations of different sizes will have geometrical similarities like building blocks, meaning constellations with similar orbital parameters will also be similar in quality, even with different amounts of satellites. The use of this representation allows the inclusion of constellation size as a variable to be optimised, avoiding the redundancies caused by running separate optimisations for different constellation sizes. (Hitomi and Selva, 2018) presents an ap-

proach to using this method for constellation design, introducing the necessary constellation representation and operators to solve the problem. While the paper demonstrates a significant reduction of redundancies when optimising for a given range of constellation sizes, and therefore a reduced computational effort and increased convergence rate, the novelty of the method introduces a few concerns. Neither the operators nor the representation presented in (Hitomi and Selva, 2018) can be directly applied to the problem formulation chosen for this setting, making the creation of new operators necessary. While the advantages warrant further research into the use of variable-length chromosomes, particularly for a broad constellation size search space, it will not be a part of the initial algorithm.

4.2 Constellation Representation

As discussed in Section 2.2, using an existing constellation framework simplifies optimisation immensely due to the minimisation of the variables to be optimised. Ideally, the framework is formulated as generic as possible to avoid exclusion of good performing constellations. This is best done using the Lattice Flower constellations, and since elliptical orbits should not be excluded, either the 3D or 4D formulation has to be used. The 3D Lattice constellation includes classical Walker, as well as elliptic Walker, harmonic Flower and Draim constellations, only leaving out non-symmetric constellations such as non-harmonic Flower and SOC constellations. While 4D provides the most generalised option, allowing differing semi-major axes, the complexity of the formulation is not deemed worth it (Arnas et al., 2021). The augmentation constellation to be designed is meant as a single purpose constellation used for GNSS enhancement, and the variation of the semi-major axis within a shell does not seem likely to provide higher performance results for such a constellation, but instead just an increase in launch cost and constellation maintenance. A possible addition of further constellation shells at different altitudes does not require a 4D representation. A representation including non-symmetric constellations is presented in (Davis and Mortari, 2012) but would need to undergo further research to be used, therefore the non-symmetric constellations have to be analysed separately. Since the 3D Lattice, while yielding good results (Xu et al., 2022), is a new approach and requires more parameters, the 2D representation (Casanova et al., 2014; Han et al., 2021) will be used initially to validate the algorithm and fitness function formulation. The later expansion of the algorithm to incorporate additional

variables for the 3D lattice poses no issues, and the constellation creation module simplifies implementing new constellation frameworks. An in-depth explanation of the generation module is given in Section 4.5. For the definition of a 2D Lattice Flower constellation, three integer values and six continuous orbital elements are needed. The integers to be defined are the number of satellites per plane, the number of planes and the phasing parameter. These variables introduce some dependencies that complicate the inclusion into the optimiser loop. Firstly the satellites have to be evenly distributed between planes, so the total number of satellites has to be a multiple of the number of planes. While using the number of satellites per plane as a parameter instead of the total number of satellites does avoid this dependency, new issues pertaining the parameter bounds are introduced. Instead of introducing these dynamic bounds, a correction function is used to round to the closest integer that is a multiple of the number of planes. Additionally the phasing parameter, describing the relative phasing between satellites on different planes, has to be an integer smaller than the number of planes. To ensure this, a second fixing function introduced in Section 4.3 is used. Since the Phasing parameter influences the position of the satellites, it not only affects coverage related metrics, such as the DOP and number of visible satellites, but additionally has an impact on the intra-constellation collision risk, determined by the distance between satellites. To minimise the risk, (Liang et al., 2021) selects the phasing parameter to maximise the minimum distance between satellites during the course of their simulations. Collisions are unlikely to occur during the simulation periods, since they are mostly caused by small deviations of the satellite positions over the long term operation of a constellation. So, for the design of an operable constellation, the collision risk should be taken into account. This can be done by excluding the phasing parameter from the optimisation and selecting it as described in (Liang et al., 2021), or by including the collision risk as an objective to be minimised. To choose the adequate approach, the influence of the phasing parameter on the remaining objectives needs to be quantified, to see whether it is negligible or if the phasing parameter needs to remain as an optimisation variable. The necessary sensitivity study will be carried out during later iterations of the approach presented in this paper.

Excluding the phasing parameter from the optimisation would still leave the number of planes and satellites as integer values. This introduces the additional complexities of mixed integer programming. To consider the desired data types of the different pa-

rameters, they are written in, and read from an input file. Since only continuous intervals are considered, the values marked as integers are rounded after application of the mutation operator. In addition to the integer parameters, the six orbital elements of the first satellite have to be optimised. With these parameters a single layer constellation can be created. However, multi-layer or multiple shell constellations can be advantageous for a lot of applications, since having satellites at different altitudes can combine the higher signal strength of LEO satellites and greater coverage of, for example, MEO constellations. Adding another shell encompasses the full optimisation of an additional constellation, which can be done by extending the parameter list with further sets of constellation parameters. This, however, means each shell is part of the optimisation instead of allowing the number of layers to be an optimisation parameter itself. The variable length chromosome representation can be applied in this scenario. (Hitomi and Selva, 2018) present an approach to optimising a constellation by defining each satellite individually. However, as discussed previously, an optimisation without the use of a framework would cause enormous computational costs as well as complications in the dynamic compatibility of the satellites, meaning this representation cannot be seriously considered for the optimisation of large constellations. Instead the satellite definitions could be replaced by different constellations layers, enabling the creation of multi-layer constellations.

4.3 Update Mechanism and Termination

Choosing not to implement a variable-length chromosome representation simplifies the operator choice considerably, as existing, well researched operators can be used in the algorithm, and it is possible to rely on a program structure that has been tested for its quality and convergence. (Hitomi and Selva, 2018) present two operators specifically designed to deal with the chromosome length, which can be further analysed when considering the expansion to multi-layer constellation design. The implementation of the NSGA-II includes selection, mutation and crossover operators that are written to allow external alteration of their hyperparameters. Tournament selection is carried out to sample the parent individuals to be altered. For this a batch of parents is sampled and the one with the highest rank and lowest crowding distance is chosen to create offspring. The batch size can be adapted to make the algorithm more or less elitist, with the default being set to binary tournament selection. Two-point crossover is used unless

there are fewer than four parameters, in which case NSGA-II falls back to one-point crossover. Since inequality constraints can be set between parameters, a correction function is used in the case that a violation is caused due to crossover results. The correction function chooses a new parameter value from a normal distribution centred at the violated boundary. Mutation of the parameters is carried out based on a truncated normal distribution with the mean set at the parameter of the parent. Sampling from the truncated distribution avoids having to rely on a correction function to fix parameters outside the allowed bounds. The standard deviation can be adjusted to favour exploration or exploitation.

The main focus of (Hitomi and Selva, 2018) lies on the variable-length chromosome representation, but it additionally proposes a tool for Adaptive Operator Selection (AOS). The selection process consists of a credit assignment and operator selection strategy, meaning the performance of each operator is monitored over a given amount of generations and given a score. The operator performance is measured by the quality of solutions produced, as well as the operator's exploration ability, this is important to escape local optima (Wei et al., 2023). Based on the assigned score the algorithm prioritises the use of high-performance operators, while allowing some exploration for possible improvement of lower scored operators (Fialho, 2010). Consistently adapting active operators can lead to an improved convergence and quality of the optimisation. In recent years, approaches to utilise reinforcement learning for the design of a selection strategy have been proposed (Aydin et al., 2023). The implementation of AOS requires further investigation.

Due to the large variety of constellation applications, no universal number of generations can be set for termination. A value for the number of generations can be determined on a case by case basis, based on a target fitness value and the given search space. Adapting these hyperparameters to the problem, can help avoid unnecessary computational effort, while also allowing the discovery of above threshold solutions. Computational cost can be further reduced by not only terminating after a fixed number of generations, but in addition tracking the improvement percentage between generations and setting a minimum for it. When the improvement ends up at a plateau, and sufficient convergence is assumed the optimiser can terminate without having reached the maximum number of generations.

4.4 Fitness Function

While the optimiser formulation presented in this paper can be used for any type of constellation, the focus lies on augmentative constellations for GNSS. Therefore, the fitness function formulation and evaluation will be based on the corresponding requirements. As introduced in Section 2.1, the PDOP is used as the initial objective metric, with the number of satellites as an additional objective. For the design of an augmentation constellation, the constellation to be optimised would be analysed as an overlay to an existing GNSS constellation. The PDOP is a value obtained for defined ground stations on Earth's surface and measured periodically over the simulation period. Therefore, the fitness function needs to be formulated to combine the obtained values in a way, that allows to optimise for the coverage and improvement goals set. In (Casanova et al., 2014), the GDOP is measured on 30 000 globally distributed ground stations over a sixty second propagation period. The fitness function is then chosen so that the maximal GDOP found is minimised. Taking the example of Galileo seen in Figure 2, the above-mentioned fitness formulation could lead to improvements in the lower performance areas, such as the poles, but would only be slightly better in other areas of interest. A more reliable formulation to ensure the wanted results would be to choose the mean PDOP as the fitness. This allows to optimise for a global improvement by evenly distributing the measurements, but also makes reformulation for regional improvements very straightforward; the prioritisation could simply be represented by the ground station distribution without needing additional objectives for the same metric. Once the chosen constellation is propagated, the PDOP is calculated by an analysis module (cf. Figure 5).

Satellite constellations, however, are analysed based on more than one metric, so an expansion to multiple objectives as in (Han et al., 2021) is reasonable. Having decided against a weighted sum of objectives, a different fitness function has to be formulated for each objective. Some of the objectives that will be considered after the initial testing of algorithm and problem formulation include revisit time and cost of the constellation. The minimisation of revisit time and improved visibility from specific locations are vital factors for effective communication with ground stations. Both can be calculated and analysed with the propagated orbits of the constellation.

The calculation of the constellation cost is a more complex problem, as numerous factors have to be considered for a complete representation. The main part of the cost is made up of the individual satellites,

which can be considered by minimising the number of satellites. Additionally, LEO satellites tend to be less expensive when compared to satellites for higher orbits, both due to the launch cost as well as the satellite cost itself. Another expenditure that is highly dependent on the optimisation parameters is the cost of the launch, as it is affected by the specific orbits as well as the number of planes and satellites. (Han et al., 2021) seeks to minimise the orbit altitude to avoid the cost of launching to higher orbits, but a more complete representation is given in (Huang et al., 2021). They include the effects of the orbital inclination with respect to the latitude of the launch site, finding that for high inclinations the advantages gained from the Earth's rotation are negated. This means a minimal difference between orbital inclination and latitude of launch site is preferred. The maintenance costs occurring throughout the life time of the constellation can be taken into account as well. These consist of calculating the manoeuvres needed to counter the orbit degradation caused by atmospheric drag and J_2 perturbations. (Han et al., 2021) proposes a formula for the cost of altitude maintenance for LEO satellites, since for lower orbits the drag effects have to be considered. A formulation for the J_2 effects is not given since only circular constellations are considered in the article and a standalone constellation is analysed for which only the relative position of the satellites within the constellation is relevant, as the J_2 effects are taken as uniform over the whole constellation. However, the algorithm presented in this paper, is meant to be able to optimise non-circular orbits as well, making the perturbations non-negligible. The use of the 3D Lattice representation should allow for minimal maintenance cost, but its effectiveness and the long-term propagation with respect to the main GNSS constellations will have to be further analysed. While the cost factors already require a trade-off, combining them into a single fitness function would avoid the complexities of adding further objectives.

The complete formulation of the constellation design problem is made to allow exploration of augmentation constellations outside of LEO constellations. LEO augmentation constellations have recently gained prominence due to their potential to improve positioning accuracy, particularly in urban canyons. Tall buildings and narrow streets in urban canyons often degrade signals, cause disruptions due to blockage and attenuation, and obstruct satellite visibility (More et al., 2022; Xu et al., 2020), causing positioning inaccuracies and outages. However, due to the disadvantages of LEO constellations detailed in Section 1 an analysis of other augmentation constellations is also deemed necessary.

4.5 Implementation

The optimisation will be implemented inside the System and Service Volume Simulation Environment (S²VSE) (Bernhardt et al., 2024) developed at the Galileo Competence Center. S²VSE is a modular and flexible simulation and performance analysis tool, it enables the reproduction, analysis and study of current and future PNT systems and services. For evaluations different parameter can be adjusted to meet conditions like changes in constellation, impact of new technologies or external factors like space weather, overall this allows the assessment and prediction of Key Performance Indicators (KPIs) like accuracy, availability, continuity and integrity. S²VSE is based on individual modules, which are mainly implemented in Python and are connected via RCE (Remote Component Environment).

The workflow, as depicted in Figure 5, starts with the provision of a baseline constellation for the optimiser, which can either already be a multi-layer system or a single layer constellation. Each layer of the system is described in its own input file, based on the characteristic of the layer like number of planes and satellites per plane, these are then forwarded to the constellation generator. This module will take the parameters from each file and sets up the complete constellation for propagation. Currently, the propagation is done via a third-party software the General Mission Analysis Tool (GMAT) provided by the National Aeronautics and Space Administration (NASA). In order to circumvent some constraints of the software there are ongoing efforts to transition to an alternative solution that is being developed internally. The duration of the propagation is dependent on the created constellation and has to cover a complete repeat orbit. After the propagation finishes, the data will be parsed in a format to be compatible with the subsequent modules. At this point, visualisations can be created, showing the resulting ground tracks or the orbits. The next step is the calculation of the DOP values. For this, in addition to the orbit data, the position of the receiver is necessary. The receiver can be either a real location, an arbitrary number of receivers split over an area or a multitude of receivers purposefully placed to create a certain scenario. After the values have been calculated, the results can be shown either as a plot of a region or as line charts for each single receiver. The DOP values are also used as input data for the optimiser to start the next iteration of the loop with the goal to achieve an optimal solution. Aside from the visualisation mentioned, the data of each module can also be saved as a data file and then used for additional analysis.

5 EXPERIMENT

5.1 Experimental Setup

The validation of the problem definition and workflow set-up was carried out using the NSGA-II (Deb et al., 2002) on a simple scenario. A Walker Delta constellation is to be optimised for minimal PDOP achievable with the minimum number of satellites.

Table 2: Constellation parameters. LB and UB describe the upper and lower bounds, respectively.

Parameter	LB	UB
Number of Planes	4	20
Number of Satellites	32	400
Phasing Parameter	1	-
Semi-major axis	6771 km	-
Argument of periapsis	0°	-
Eccentricity	0	-
Inclination	45°	135°
Right Ascension	0°	-
True Anomaly	0°	-

All but three parameters were initially fixed and a limited search space was given for the parameters to be optimised. The NSGA-II based optimiser was set to have a population size of 30 constellations and to terminate after 20 generations. To cover one repeat orbit, as explained in Section 2.1, the simulations were set up to cover a time span of three days. Since the time to repeat an orbit is dependent upon both altitude and inclination, the maximum time interval was selected to accommodate the chosen parameters. The satellites were propagated with time steps of two minutes to reduce the time needed for simulations, while still having multiple epochs during each overflight per satellite. Even with reduced propagation time and the coarse step size simulation time is around 30 minutes for each constellation. Relying on parallelisation to run all constellations of a generation at the same time it takes 30 minutes per generation and around 10 hours for one optimisation run, this will also change depending on how many receiver are considered during the PDOP analysis. For the initial scenario a single station was simulated without any regard to Non-Line of Sight (NLOS) situations, which will be covered in more detailed follow up scenarios. The fitness of a constellation is calculated by averaging the PDOP values obtained at each time step and penalising the constellations with discontinuous coverage. The penalisation consists in setting the average PDOP to 10000 and the number of satellites to the maximum allowed, essentially disabling the constellation.

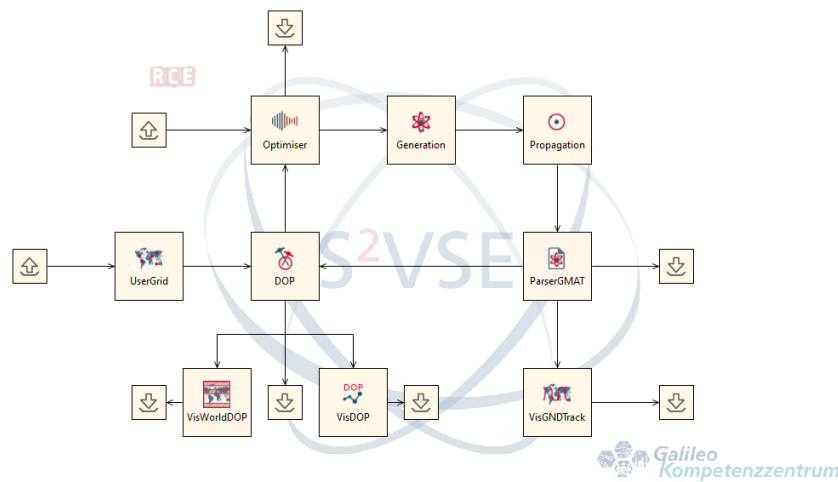


Figure 5: Workflow for the optimisation of constellations in regards to DOP values.

5.2 Simulation Results

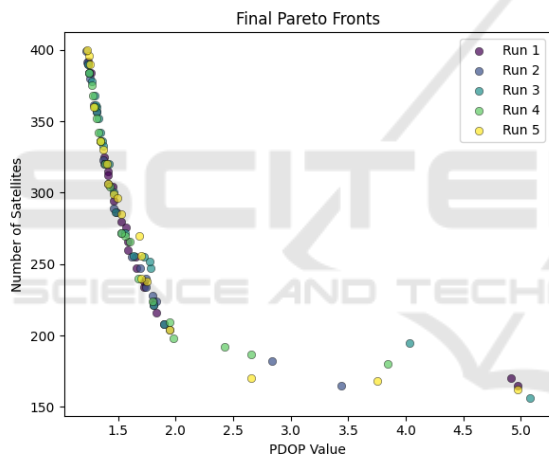


Figure 6: Final Pareto front based on the results of five simulation workflows.

Figure 6 shows the Pareto fronts for the problem described in Section 5.1, generated by running the NSGA-II five times. The algorithm gives consistent and plausible results, as visible in the front created by the different runs and the fact that the PDOP follows the expected outcome for different numbers of satellites. However, an ideal constellation can not be chosen with the given set up, since more application specific information has to be known to pick from the front. This would include for example, more precise data on constellation cost and budget, as well as the required PDOP values.

6 DISCUSSION

The long simulation times, especially when considering global coverage and large receiver grids, required constraints for initial testing, as shown in Section 5, to provide justification for more computationally expensive optimisations. The provided results encourage expanding the simulation scenarios to be able to cover a broader range of use cases. This expansion should also include a switch to a more general constellation framework and the optimisation of more parameters with larger intervals. Additionally, more objectives should be introduced to consider more factors impacting constellation design. Since more than three objectives will be relevant, implementations of additional GAs, such as NSGA-III and Strength Pareto Evolutionary Algorithm II (SPEA-II) will be compared to the current Algorithm. Due to the complexity of the problem, the used algorithms need to be expanded to improve the operators and their use. A static, generation dependent hyperparameter adjustment will be used and later expanded to adaptive operator selection. As mentioned in Section 4.2, an implementation of the 2D Lattice Flower representation and subsequently the 3D representation for perturbation considerations will be used. However, the continuous coefficient representation (Davis and Mortari, 2012) should also be considered, as it includes non-uniform representations, in addition to the uniform constellations included in the 3D Lattice representation. For the optimisation of multi-layer constellations the variable length chromosome representation will be investigated. This paper shows that the utilisation of GAs could give insights into future possibilities in the field of augmentative PNT constellations and aid the complex constellation design process.

ACKNOWLEDGEMENTS

This work was partially funded by the Deutsche Forschungsgemeinschaft DFG (HA 5480/10-1).

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