Predictors of Freshmen Attrition: A Case Study of Bayesian Methods and Probabilistic Programming

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Abstract: The study explores the use of Bayesian hierarchical linear models to make inferences on predictors of freshmen student attrition using student data from nine academic years and six schools at Marist University. We formulate a hierarchical generalized (Bernoulli) linear model, and implement it in a probabilistic programming platform using Markov chain Monte Carlo (MCMC) techniques. Model fitness, parameter convergence, and the significance of regression estimates are assessed. We compared the Bayesian model to a frequentist generalized linear mixed model (GLMM). We identified college academic performance, financial need, gender, tutoring, and work-study program participation as significant factors affecting the log-odds of freshmen attrition. Additionally, the study revealed fluctuations across time and schools. The variation in attrition rates highlights the need for targeted retention initiatives, as some schools appear more vulnerable to higher attrition. The study provides valuable insights for stakeholders, administrators, and decision-makers, offering applicable findings for other institutions and a detailed guideline on analyzing educational data using Bayesian methods.

1 INTRODUCTION

In higher education, the issue of student dropout rates has always remained a serious concern, and its effects extend well beyond the academic domain. Poor retention of students has negative effects on the prestige and the financial stability of an educational institution. At a time when students and their families often feel that a college education is too expensive and has uncertain outcomes and return of investment, schools must, more than ever, track how many and why students leave their programs. Student attrition rates are especially high during the freshman year, which places the focus on addressing the transition of traditional freshmen students during their first year of college and into their sophomore year. According to the National Center for Educational Statistics (NCES, 2022), for students who entered a 4-year institution in Fall 2019, the overall retention rate was 82%, with a wide spread between the more selective and less selective institutions. At public 4-year institutions the retention rate was 82% overall, 96% at the most selective institutions, and 59% at the least selective institutions. Similarly, at private non-profit institutions, the retention rate was 81% overall, with 92% for the most selective institutions and 64% for the least selective institutions. In private for-profit institutions the overall retention rate was 63% At 2-year degree granting institutions, the overall full-time freshmen retention rate was 61%, with 61% for public institutions, 68% for private nonprofit, and 67% for private for-profit institutions. In comparison, 64% of students who joined 4-year institution in fall 2014 completed the degree within 6 years, which shows that a large large percentage of student attrition happens during or at the end of the freshman year, as also reported in a number of studies (Deberard et al., 2004; NSCRC, 2014). But despite all the research in the academic and learning analytics domain, including student performance and retention analysis (Campbell, 2007; Lauría et al., 2020, for e.g.), and the widespread availability of technology platforms and software systems that can learn from data, very little has been done using the machinery of Bayesian methods and probabilistic programming. Bayesian methods have emerged as a more intuitive and robust alternative to frequentist inference methods, blending prior beliefs with data to compute probability distributions of model parameters (Bertolini et al., 2023). But these methods, techniques and software tools have received considerable less attention by re-

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searchers and practitioners in the educational domain, been mostly relegated to specific niches -Bayesian knowledge tracing, used in many intelligent tutoring systems (Yudelson et al., 2013; Dai et al., 2021), natural language processing and text mining (Chen et al., 2022), and to a lesser extent, Bayesian networks (Delen, 2010). A simple experiment reinforces these assertions: a search on Google Scholar with keywords *student retention* (or *attrition*), *predictors*, and *machine learning* / *statistical analysis* / *data analysis* yields several thousand results. When we replace the latter with a combination of keywords *Bayesian* and *MCMC* / *Markov chain Monte Carlo* / *variational inference* / *NUTS* (all common terms in a Bayesian inference setting), the number of search results drops way below one hundred.

This paper explores the use of Bayesian hierarchical linear regression models using data from multiple academic years to derive insights on freshmen student retention. Hence, the paper makes the following contributions: 1) It frames freshmen retention analysis through the lens of Bayesian inference. 2) It describes the use of Bayesian hierarchical linear models, demonstrating the use of hierarchical models to account for data organized in multiple layers or levels; 3) It highlights the computational power and simplicity of probabilistic programming tools, providing reproducible data-driven workflow practices in this context. The paper is organized in the following manner: we start with a section on Bayesian methods and probabilistic programming, introducing key concepts and their application to modeling freshmen attrition. Next, we describe the study, including research questions, data, methods, technology platform, and results. The paper ends with a summary of our conclusions.

2 BAYESIAN METHODS AND PROBABILISTIC PROGRAMMING

2.1 Overview

Bayesian methods provide a framework to make inferences from data using probability to quantify uncertainty. Bayesian models are grounded in Bayes' theorem, which describes the relationship between the prior or initial beliefs on the probability distributions of the model parameters, the likelihood function, and the posterior distribution, the probability of observing the data given the model parameters, also known as the likelihood function, and the posterior distribution of the model parameters, representing the conditional distribution of the parameters representing the updated beliefs after observing the data. To formulate Bayes theorem, let's consider observable data X, and a vector of unobservable parameters θ. Bayes' theorem is therefore expressed as:

$$P(\theta|X) = \frac{P(X|\theta)P(\theta)}{P(X)} \tag{1}$$

where $P(\theta)$ in Equation 1 is the prior distribution of vector parameter θ; $P(X \mid \theta)$ is the likelihood of the data X given θ; $P(\theta \mid X)$ is the posterior distribution of θ after observing X; and $P(X) = \int P(X \mid \theta)\, p(\theta)\, d\theta$ is the marginal likelihood of X, which serves as a normalization constant.

Why is the Bayesian framework important? Because it provides a common-sense, straightforward interpretation of statistical conclusions. A Bayesian high-density interval (HDI) for an unknown quantity of interest provides a range of values where the quantity of interest is most likely to be, given the observed data and the priors, in contrast to a frequentist confidence interval (CI), which may strictly be interpreted only in relation to a sequence of repeated experiments (Gelman et al., 2013). For example, a 95% HDI is the region of values that contains 95% of the posterior probability mass, whereas the 95% CI means that approximately 95% of the intervals calculated from the repeated experiments would contain the quantity of interest. In frequentist statistics, a p-value summarizes the probability over many trials of observing the effect on the outcome, whereas Bayesian methods estimate the posterior probability of the effect on the outcome after observing the data; the difference which seems subtle, has led to misinterpretations that have spread throughout the scientific literature, with students, practitioners and researchers mistaking p-values as posterior probabilities (Greenland et al., 2016). The p-value is also sensitive to the amount of observations: a large sample size can give way to small p-values that can in turn lead to erroneous assertions about significant effects. Bayesian methods are less affected by the size of the sample, as they provide a full probability distribution of the effect of the model parameters on the outcome after observing the data.

2.2 Approximate Bayesian Computation and Probabilistic Programming

Bayes rule, as depicted in Equation 1, hides the complexity of computing the marginal likelihood of the data $P(X) = \int P(X \mid \theta)\, p(\theta)\, d\theta$, which can be circumvented through the use of conjugate priors -one where the product of the prior times the likelihood yields a statistical distribution of the same form (in the same family) as the prior distribution. The use of conjugate priors can be advantageous but also restrictive, and not a well-grounded hypothesis in many real-world situations. But when conjugate priors are not valid, the marginal likelihood $P(X)$ is an integral that can be difficult or even impossible to compute, especially over multiple dimensions. Bayesian inference has to resort then to approximate numerical techniques such as the Markov Chain Monte Carlo (MCMC) family of algorithms, or variational inference. MCMC methods such as Metropolis-Hastings (Hastings, 1970) and Gibbs sampling (Geman and Geman, 1984), originally derived from statistical mechanics, produce samples approximating the sought posterior distribution using a proposal distribution to accept or reject those samples -in the case of Gibbs sampling, the proposal distribution is the conditional distribution of one parameter given the others, and there is no rejection step. Hamiltonian Monte Carlo or HMC (Duane et al., 1987), another popular variation of Metropolis-Hastings, simulates the Hamiltonian trajectory of a particle in a potential energy field, formulating a proposal distribution that renders more efficient sampling by avoiding random walk behavior and reducing the correlation between successive samples. The NUTS (No-U-Turn Sampler) algorithm (Homan and Gelman, 2014), an extension of HMC, uses a recursive algorithm that dynamically determines the length of the simulated Hamiltonian trajectory, avoiding backtracks and therefore leading to better sampling. MCMC sampling methods are the most common algorithms for Bayesian computation, but other approaches have been developed as well. Variational inference (Blei et al., 2017) is an alternative and generally more scalable method that approximates the posterior distribution of the parameters of interest using a surrogate distribution instead of resorting to sampling, and estimates the parameters of the surrogate distribution using optimization algorithms (e.g. variations of gradient descent) such that the resulting distribution resembles as much as possible the posterior distribution. Variational inference is usually faster than MCMC, but it can be less accurate and it can converge to local minima, which further restricts accuracy. Where do probabilistic programming platforms come into play? They allow for flexible specification of probabilistic models, and provide the tools to perform Bayesian inference, hiding the intricate details of approximate Bayesian inference computation (MCMC and variational inference) so that researchers and practitioners can focus on model design, development, and testing, leaving the probabilistic programming platform to handle the computational details for them. A number of probabilistic programming platforms have been developed over the years, among them BUGS (Gilks et al., 1994), JAGS (Plummer, 2003), Stan (Carpenter et al., 2017) and PyMC (Abril-Pla et al., 2023), Stan and PyMC are popular platforms in active development.

2.3 Using Bayesian Methods to Model Freshmen Attrition

In a Bayesian context, a typical approach to model student attrition is to use a binary logistic regression, a generalized linear model where the response variable y, representing whether the student has dropped out, follows a Bernoulli distribution with parameter θ, which in turn is a nonlinear (logistic) function of a linear combination of m scaled predictors (i.e. $b_0 + \sum_{j=1}^{m} b_j \cdot x_j$), representing student demographics, high school and college student activity, characteristics and academic performance. Numeric data is typically scaled to improve numerical stability and aid the convergence and speed of computation. To obtain stable logistic regression coefficients and circunvent nonidentifiability issues due to separation (when a linear combination of the logistic regression predictors perfectly predicts the outcome), sparsity, collinearity, or the inclusion of numerous binary predictors, the literature prescribes mild regularization through the use of noninformative or weakly informative priors on the regression coefficients, by constraining parameter estimates so that they are not too extreme or unrealistic (Gelman et al., 2008; Westfall, 2017). This formulation is known as a ”flat” or ”pooled” model, in the sense that it does not take into account the structure associated with potential grouping and nesting of the data. Educational data is inherently complex, multilayered, and nested: students join majors and minors, which are part of academic units -departments and schools. Each level within this layered structure has its own characteristics which can have an effect on student academic performance and student attrition. Also, student attrition data has strong temporal components, which give way to repeated measurements of freshmen attrition over multiple academic years, affected by internal and external factors (e.g., attrition rates affected by COVID-19). Of course, the immediate solution that comes to mind when dealing with variability among groups is to fit a separate regression model for each of the groups. This ”unpooled” approach addresses the issue of considering the particular characteristics of each group, but since a model is fitted independently for each group, it is difficult to extract findings that are common to all groups. Moreover, unpooled models may be constrained by the availability of data: there may not be enough data in particular groups to fit the regression model. The multi-layered structure of educational data introduces this additional wrinkle in the analysis: data can be sparse at each level. Specialized majors and minors might have low enrollment figures, and new programs might carry limited historical data. If the analysis examines data using student demographic characteristics, certain groups and minorities might be highly underrepresented, giving way to parameter estimates with high variance, or lead to unstable, very large or potentially infinite regression coefficients, a phenomenon described as complete, or quasi-complete separation. Also, models trained with small amounts of data are prone to overfitting and may not generalize well when applied to new or unseen data. This is especially critical when applying frequentist methods on multi-level data (e.g., frequentist logistic regression), as Bayesian models introduce priors that can help mitigate high variance and bias of regression coefficients as well as separation issues. Hierarchical models strike a balance between effects that are group-specific and those that apply more broadly across group populations (Gelman and Hill, 2006). Although frequentist hierarchical models (e.g., mixed-effect models or GLMMs) address variability among groups, there are key advantages in applying Bayesian hierarchical models compared to frequentist approaches. Bayesian hierarchical models apply partial pooling through hierarchical priors, naturally regularizing group-level estimates towards the overall distribution, and therefore preventing large coefficient estimates in groups with small counts. Frequentist hierarchical models can incorporate regularization, but it is not inherently built in. In small groups, frequentist models may struggle to accurately estimate the variance of random effects (e.g. how much of the variability in attrition is due to differences between schools). On the downside, Bayesian hierarchical models are computationally more expensive, especially for large datasets, than their frequentist counterparts, which rely on optimization methods such as maximum likelihood estimation (MLE).

3 STUDY DESIGN

In this study, we investigate the use of Bayesian generalized linear models to ascertain potential predictors of freshmen attrition drawn from student data. We built hierarchical binary logistic regression models using a probabilistic programming platform to compute the posterior probability distributions of the logistic regression coefficients and derived metrics. The quality of the fitted models and the significance of the regression coefficients are measured using the metrics described in section 2.3.2. We also analyze random effects due to variability in the different academic years under study and across different schools within the academic institution. In doing so, the study addresses the following research questions:

RQ1: How do student demographics, high school and university academic performance, and student activities affect the odds of freshmen attrition?

RQ2: Is there considerable fluctuation in freshmen attrition across different academic years and among different schools?

RQ3: How does the Bayesian hierarchical model compare to its frequentist counterpart?

3.1 Datasets

We considered Freshmen data from nine academic years (2012-2018, and 2021-2022) extracted from the institution's data warehouse. We decided to skip (2019-2020) data as it corresponds to the COVID-19 pandemic period, which could introduce variability in the results (we are aware that 2021 and 2022 data could be also different from the pre-pandemic years, but we performed preliminary testing and did not find significant variation). See Table 1 for details. School codes correspond to the following schools: CC - Computer Science & Mathematics; CO - Communications & the Arts; LA - Liberal Arts; SB - Social & Behavioral Sciences; SI - Science; SM - Management.

Data was imputed using K-nearest neighbors (KNN). Each record corresponds to each accepted and registered freshman student in the Fall of the corresponding academic year, enriched with school data and demographics using the record format depicted in Table 1. The dataset included 10921 records, with 1154 instances of attrition. Dropouts are considered over the full academic year (institutional research data was used to determine if a student who came in in the Fall of a given academic year, did not return in the Fall of the following academic year). The overall attrition ratio in the dataset was 10.57%. Dataset was checked to identify outliers and influential observations. Numeric variables were scaled as z-scores. Correlations among predictors were checked: *EffectiveGPA* correlates with *HSGPA* (0.49), *isDeanList* (0.59), and *NumAPCourses* (0.26), indicating a relationship between past and current academic performance. *EFC* (Expected Family Contribution) is negatively correlated with *UnmetNeed* (-0.33) and *PellAmount* (-0.21), reflecting expected patterns in financial aid allocation. *PellAmount* is correlated with *UnmetNeed* (0.40), *hasLoans* (0.19), and *isCampusWorkStudy* (0.23). We also checked the Variance Inflation Factor (VIF). Predictors had a VIF < 5, meaning multicollinearity is not a major concern. *EffectiveGPA* had a VIF between 2.5 - 3.0, which warrants some consideration.

Table 1: Data Description.

Identifier	Description
Academic Performance	
EffectiveGPA (academic year)	Numeric
isDeansList (made it to Dean's list)	Binary (1/0)
TutoringClassCount (classes tutored in)	Numeric
HSGPA (high school GPA)	Numeric
NumAPCourses (taken during high school)	Numeric
Demographics	
UScitizen	Binary (1/0)
Gender	Binary (F, M)
StudentofColor	Binary (1/0)
isFirstGeneration (college student)	Binary (1/0)
DistanceFromHome (miles)	Numeric
Institutional and Enrollment Factors	
isCampusWorkStudy	Binary (1/0)
isDivisionI (athlete)	Binary (1/0)
WaitListed (before admitted)	Binary (1/0)
Financial Aid and Need	
EFC (Expected Family Contribution, in $)	Numeric
UnmetNeed (after financial aid, in $)	Numeric
HasLoans	Binary (1/0)
PellAmount (federal grant, in $)	Numeric
AcademicYear	Discrete
School ((CC, CO, LA, SB, SI, SM)	Discrete
didNotReturnNextFall (response variable)	Binary (1/0)

3.2 Statistical Modeling

We built a hierarchical binary logistic regression model using the predictors depicted in Table 1. We consider random effects in the model associated with potential variability in the log-odds of freshmen attrition across multiple academic years and different schools. The model, depicted in Equation 2, includes varying intercepts for both academic year and school; $b_{AcademicYear}$ is a group-level effect that captures the effect on the likelihood of the outcome tied to being in a particular year. Similarly, b_{School} captures the effect of attending a particular school on the log-odds of dropping out during or at the end of the freshman year. Regression coefficient priors are denoted by P_β. We refined the prior distributions of the model, using the following criteria:

- The choice of the Intercept's prior as Normal(-2.2, 1.0) is based on the approximate 10% attrition rate. The intercept represents the log-odds when all predictors are at the mean value, as they are scaled as z-scores. Therefore if $P(y=1) = \frac{e^{Intercept}}{1+e^{Intercept}}$, when $P(y=1) = 0.10 \implies \log(0.10/0.90) = -2.2$.
- We used StudentT(ν=3, ν=0, σ=2.5) on correlated predictors or predictors width moderate outliers. ν=3 allows for some large deviations and σ=2.5 keeps the prior weakly informative.
- We used normal weakly informative priors - Normal(0,2.5) for all other predictors.
- For group effects (School and AcademicYear), we used HalfNormal(2.5). A $\sigma = 2.5$ allows for moderate variation while keeping the group-specific intercepts within a rather similar scale as the fixed-effects intercept.

The prior distributions with their calculated values using the criteria described above are depicted in Table 2.

$$
\begin{aligned}
\sigma_{b_{\text{AcademicYear}}} &\sim \text{HalfNormal}(\sigma_{\text{AcademicYear}}) \\
\sigma_{b_{\text{School}}} &\sim \text{HalfNormal}(\sigma_{\text{School}}) \\
b_{\text{AcademicYear}} &\sim \text{Normal}(0, \sigma_{b_{\text{AcademicYear}}}) \\
b_{\text{School}} &\sim \text{Normal}(0, \sigma_{b_{\text{School}}}) \\
b_0 &\sim \text{Normal}(\mu_{b_0} = -2.2, \sigma_{b_0} = 1.0) \\
b_j &\sim P_\beta \quad \text{for} \quad j = 1, 2, \ldots, m \qquad (2) \\
\text{logit} &= b_{\text{AcademicYear}} + b_{\text{School}} + b_0 + \sum_{j=1}^{m} b_j x_j \\
\theta &= \frac{1}{1+\exp(-\text{logit})} \\
y &\sim \text{Bernoulli}(p = \theta)
\end{aligned}
$$

3.3 Probabilistic Programming Platform

We used Bambi (Capretto et al., 2022) -Bayesian Model Building Interface, a Python package for generalized linear models built on top of PyMC (Abril-Pla et al., 2023) and the ArviZ package for exploratory analysis (Kumar et al., 2019) to develop and run the Bayesian logistic regression models and produce visualizations. The Bayesian models developed in Bambi used the No U-Turn (NUTS) algorithm to obtain the posterior distribution samples of the regression parameters. We chose the NumPyro (Phan et al., 2019) implementation of the NUTS sampler. Samples of the posterior distributions for the logistic regression parameters were computed using 4 chains of 5000 samples each, with a warm-up period of 1000 samples for each of the chains (the number of samples initially discarded until the chain converges to the stationary posterior distribution). The chains were tested and no divergences were found in any of the chains of

Table 2: Prior Distributions for Fixed and Group Effects.

Component	Prior Distribution
Fixed Effects	
Intercept	Normal(μ:-2.2, σ:1.0)
EffectiveGPA	StudentT(ν:3.0, μ:0.0, σ:2.5)
isDeansList	StudentT(ν:3.0, μ:0.0, σ:2.5)
TutoringClassCount	Normal(μ:0.0, σ:2.5)
HSGPA	StudentT(ν:3.0, μ:0.0, σ:2.5)
NumAPCourses	Normal(μ:0.0, σ:2.5)
USCitizen	Normal(μ:0.0, σ:2.5)
Gender	Normal(μ:0.0, σ:2.5)
StudentOfColor	Normal(μ:0.0, σ2.5)
isFirstGeneration	Normal(μ:0.0, σ:2.5)
DistanceFromHome	StudentT(ν:3.0, μ:0.0, σ:2.5)
isDivisionI	Normal(μ:0.0, σ:2.5)
isCampusWorkStudy	Normal(μ:0.0, σ:2.5)
WaitListed	Normal(μ:0.0, 9.3792)
EFC	StudentT(ν:3.0, μ:0.0, σ:2.5)
UnmetNeed	StudentT(ν:3.0, μ:0.0, σ:2.5)
HasLoans	StudentT(ν:3.0, μ:0.0, σ:2.5)
PellAmount	StudentT(ν:3.0, μ:0.0, σ:2.5)
Group-Level (Random) Effect	
AcademicYear (Random Intercept)	Normal(μ=0.0, $\sigma_{b_{\text{AcademicYear}}}$)
$\sigma_{b_{\text{AcademicYear}}}$	HalfNormal(σ=2.5)
School (Random Intercept)	Normal(μ=0.0, $\sigma_{b_{\text{School}}}$)
$\sigma_{b_{\text{School}}}$	HalfNormal(σ=2.5)

the models created. Convergence of the models was assessed using $\hat{R} < 1.01$ as the threshold for acceptable convergence (see the section below for details).

3.4 Model Quality and Performance Metrics

Several metrics are considered to measure the quality of the model:

- $\hat{R} = \sqrt{\frac{\frac{n-1}{n}\sigma_W + \frac{1}{n}\sigma_B}{\sigma_W}}$, also known as the Gelman-Rubin diagnostic, is a measure of the convergence of the MCMC algorithm. $\hat{R}$ computes and compares the variance within chains with the variance between chains. σ_B is the between-variance (the average of the variances of each of the chains) and σ_W is the within-variance, measuring the variability between the means of the chains. A value of $\hat{R}$ close to 1 is an indication of convergence.
- The high-density interval (HDI) summarizes the range of most credible values of a parameter within a certain probability mass. When 95%HDI includes zero, the regression coefficient is not statistically significant. The literature has also suggested the use 89%HDI, see (Kruschke, 2014) for example. But we prefer to use the 95% interval in the study, as it is more conservative and has an intuitive relationship with the standard deviation (Easystats, 2024).
- The percentage of 95% HDI within ROPE is a measure of the practical significance of the regression coefficients. ROPE (region of practical equivalence) corresponds to a null hypothesis for the model parameters and provides a range of values for the parameters in question that are considered good enough for practical matters. The so-called HDI+ROPE decision rule (Kruschke, 2014) determines the practical significance of the regression coefficients based on the overlap percentage between the HDI (95%HDI in this case) and the ROPE range. A percentage value of overlap closer to 0 implies that the parameter (regression coefficient in this case) is significant. The ROPE range is context-dependent.
- WAIC (Watanabe, 2010), which stands for widely applicable information criteria, is used both for model comparison and to measure the model's predictive performance (how well the model performs when making predictions on new data). WAIC uses the log-likelihood evaluated at the posterior distribution of the parameter values penalized by the variance in the log-predictive density across those posterior samples. It is formulated as $\text{WAIC} = -2 \cdot (\text{LPPD} - \text{penalty})$, where $\text{LPPD} = \sum_{i=1}^{n}(\log(\frac{1}{S}\sum_{s=1}^{S} P(y_i \mid \theta^s))$ is the log pointwise predictive density and $\text{Var}(\log P(y_i \mid \theta^s)))$ is the penalty term that accounts for model complexity. Note that n is the number of observations and S is the number of samples of the posterior distribution. WAIC's range is not bounded; higher values of WAIC are an indication of better model predictive performance.
- Pareto-smoothed importance sampling leave-one-out cross-validation, a mouthful of a name, and better referred to by the short acronym LOO, is a newer approach introduced by Vehtari et al. (Vehtari et al., 2017) to measure out-of-sample prediction accuracy from a fitted model. It uses a leave-one-out approach, fitting the model n times, each time with $n-1$ observations. but instead of re-fitting the model n times, the algorithm uses importance sampling to estimate the leave-one-out predictive density. Similar to the case of WAIC, its formulation is derived from approximating the posterior predictive distribution: $\text{LOO} = \sum_{i=1}^{n} \log\left(\frac{1}{S}\sum_{s=1}^{S}\frac{P(y_i|\theta^s_{-i})}{\hat{w}_i^s}\right)$. The $-i$ index in θ^s_{-i} is used to denote all samples except y_i; $\hat{w}_i^s$ are the Pareto-smoothed importance sampling weights. As in the case of WAIC, LOO is not bounded; higher values of LOO are indicative of better predictive performance. LOO has been described as being more robust than WAIC

in the presence of weak priors or influential observations.

4 STATISTICAL ANALYSIS AND RESULTS

We ran the hierarchical logistic regression model - equation (1)- with the prior distributions depicted in Table 2. Figure 1 displays, for several predictors, the trace plots with the sampling paths for each of the chains, along with the kernel density plots of the posterior distributions elicited from each chain (due to space restrictions we limited the number of predictors displayed in the figure to two). The trace plots show that the chains have "mixed" well, with each of the chains converging to the same posterior distribution of the regression coefficients of each predictor.

To answer Research Question 1 -measuring the influence of the predictors on freshmen attrition- we analyzed the results of the fixed effects of the logistic regression. Table 3 reports the regression coefficients derived from their posterior distributions together with statistical measures for each of the predictors to assess the quality and convergence of the posterior probability distributions of the regression coefficients of the predictors calculated by the MCMC (NUTS) computation and evaluate the fixed effects of the predictors on the log-odds of freshmen attrition.

- Mean and SD: The mean and standard deviation of the posterior distribution of the regression of each predictor, the coefficient measuring the average estimated effect (log-odds) of each predictor on the outcome (freshmen attrition), and the spread of the posterior distribution.
- HDI (2.5% and HDI 97.5%): The credible range of the regression coefficients, given by the lower and upper boundaries of the 95% Highest Density Interval (HDI).
- MCSE Mean: The Monte Carlo Standard Error of the Mean, which provides a measure of the variability of the mean estimate due to the finite sample size.
- MCSE SD: The Monte Carlo Standard Error of the Standard Deviation, a measure of the variability of the standard deviation due to limited sampling.
- ESS Bulk and ESS Tail: The Effective Sample Size (ESS) for the bulk and tail of the distribution, which measures the quality of the sampling of the bulk and tail regions of the posterior distribution of each regression parameter. The bulk is where most of the probability mass lies. The tail regions hold less probable values.
- $\hat{R}$: The convergence diagnostic. All reported $\hat{R}$ values were equal to 1.0, which suggests that the MCMC chains converged and therefore the posterior distributions and the estimates derived from the sampling process can be considered reliable.
- The percentage of 95%HDI within ROPE was computed for each of the regression parameters in the logistic regression models to ascertain the predictors that significantly impacted the log-odds of freshmen attrition. ROPE was fixed at [-0.2,0.2], following the recommendations by Kruschke (Kruschke, 2018) for binary logistic regression. The chosen ROPE range of [-0.2,0.2] is especially appropriate considering that we have a combination of binary predictors and numeric predictors scaled as z-scores. The ROPE can be seen then as a region of values of the regression coefficient around zero that are considered practically equivalent to having a negligible effect on the log-odds of the outcome. Hence, by setting the ROPE to [-0.2, 0.2], we establish the criterion of a negligible change in the log-odds.

We summarize the key findings from the posterior distributions of the fixed effects, focusing on predictors that were found to be statistically significant (95% HDI does not include 0) and practically significant using the percentage of 95% HDI within ROPE as a practical significance criterion.

Effective GPA at the End of the Academic Year, with Mean=-0.771 and 95% HDI=[-0.842, -0.698], is negatively associated with freshmen attrition: a higher GPA reduces the log-odds of attrition, making it a statistically and practically significant predictor.

High School GPA (HSGPA), with Mean=0.129 and 95% HDI=[0.044, 0.214], suggests that higher high school GPA values could be associated with higher attrition odds. However, a substantial percentage of the 95% HDI falls within ROPE, making its practical significance questionable. This result is counterintuitive and warrants further investigation.

Made Dean's List (isDeansList[1.0]), with Mean=0.571 and 95% HDI=[0.383, 0.756], is statistically significant and practically significant, though its practical impact remains unclear. Students who make the Dean's List appear to have a higher probability of attrition, a finding that requires deeper exploration (e.g. whether this institution was the students' second choice, who are seeking their first choice to continue their studies)

US Citizenship (USCitizen[1.0]), with Mean=-0.271 and 95% HDI=[-0.678, 0.127], suggests that

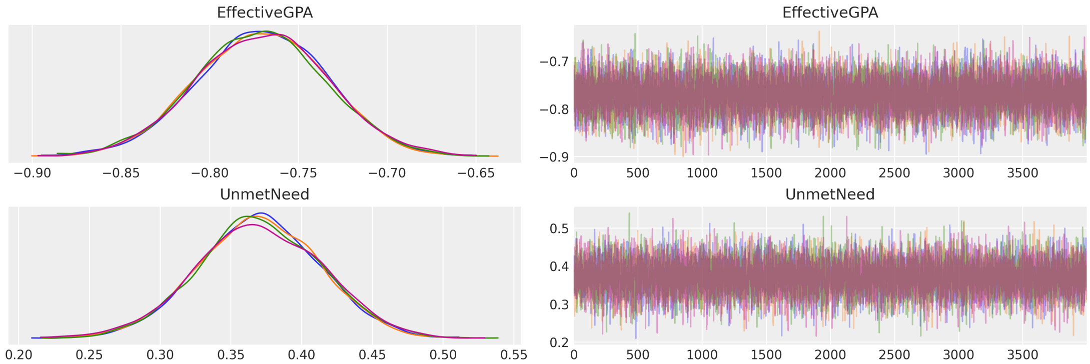


Figure 1: KDE plots of the posterior distributions and trace plots for each chain.

U.S. citizens may be less likely to drop out, but this effect is not statistically significant.

UnmetNeed, with Mean=0.370 and 95% HDI=[0.285, 0.456], is both statistically and practically significant. The positive coefficient indicates a strong association with freshmen attrition: students with higher unmet financial need face significantly increased odds of leaving the institution.

Gender (Male), with Mean=-0.234 and 95% HDI=[-0.382, -0.087], suggests that male students are less likely to leave the institution during or at the end of their freshman year compared to female students.

Has Loans (HasLoans[1.0]), with Mean=-0.210 and 95% HDI=[-0.357, -0.056], suggests that students with loans are slightly less likely to drop out.

Campus Work Study (isCampusWorkStudy[1.0]), with Mean=-0.612 and 95% HDI=[-0.833, -0.389], suggests that freshmen participating in the campus work-study program are significantly less likely to leave the institution. This result is both statistically and practically significant.

Division I (isDivisionI[1.0]), with Mean=-0.014 and 95% HDI=[-0.204, 0.179], suggests that Division I athletes do not exhibit a significant difference in attrition probability, as the 95% HDI includes 0, making this effect not statistically significant.

Pell Amount, with Mean=-0.181 and 95% HDI=[-0.262, -0.103], suggests that receiving a Pell Grant is associated with lower attrition, but the effect is relatively small and may not be practically significant.

Student of Color (StudentOfColor[1.0]), with Mean=0.165 and 95% HDI=[-0.037, 0.360], suggests that being a student of color is associated with slightly higher attrition odds, but the effect is not statistically significant.

Tutoring Class Count, with Mean=-1.802 and 95% HDI=[-2.732, -0.990], is both statistically and practically significant, indicating that students who attend tutoring sessions for their courses are much less likely to drop out. This suggests a strong protective effect against attrition.

We performed posterior predictive checks using ArviZ (Kumar et al., 2019) to measure out-of-sample predictive accuracy by computing the widely applicable information criteria (WAIC) measure, and the Pareto-smoothed importance sampling leave-one-out (LOO) measures. In the case of WAIC, the expected log pointwise predictive density (elpd_waic) is equal to -3239.53, with SE=66.66. The value is negative as it is a log-likelihood. Higher values (less negative) indicate better predictive performance. The effective number of parameters (p_waic), with a value of 28.35, measures model complexity. As the value is rather small, the measure signals that the model is not too complex. In the case of LOO, elpd_loo and p_loo and very close to the WAIC: elpd_loo=-3239.44), with SE=66.66, and p_loo=28.04. This makes the measures of posterior predictive accuracy and model complexity consistent. ArviZ also reports LOO κ diagnostics (see Table 4). The κ parameter refers to the shape parameter κ of the Pareto distribution. If all the records, like in this case, fall within the acceptable (”good”) range, it means that the importance sampling used when computing LOO is stable, and not influenced by particular observations, and therefore the metric is reliable.

To address Research Question 2, concerning the fluctuation in first-year student retention across schools and over time, We began by looking at the distribution of attrition by school and academic year, as depicted in Figure 2. Both charts display some differences in the attrition rate. For attrition distribution by academic year, year 2021 carries the larger percentage of attrition, probably since 2021 marked the end of the COVID pandemic. In the case of distribution of attrition by School, the attrition percentage is smaller in the School of Computer Science & Mathematics (CC), with larger values in Liberal Arts (LA).

We then proceeded to examine the variability in attrition rates by studying random effects for aca-

Table 3: Summary of Posterior Distributions.

Component	mean	sd	hdi_2.5%	hdi_97.5%	mcse_mean	mcse_sd	ess_bulk	ess_tail	r_hat	% 95% HDI within ROPE
Intercept	-2.341	0.270	-2.863	-1.808	0.002	0.002	11857.0	11173.0	1.0	0.000
EFC	0.010	0.036	-0.063	0.078	0.000	0.000	20261.0	11400.0	1.0	100.000
EffectiveGPA	-0.771	0.037	-0.842	-0.698	0.000	0.000	18953.0	13041.0	1.0	0.000
HSGPA	0.129	0.043	0.044	0.214	0.000	0.000	20415.0	12165.0	1.0	91.765
isDeansList[1.0]	0.571	0.095	0.383	0.756	0.001	0.000	20163.0	12725.0	1.0	0.000
NumAPCourses	-0.053	0.039	-0.129	0.024	0.000	0.000	24348.0	12380.0	1.0	100.000
USCitizen[1.0]	-0.271	0.205	-0.678	0.127	0.001	0.001	26992.0	10809.0	1.0	40.621
UnmetNeed	0.370	0.044	0.285	0.456	0.000	0.000	16588.0	12386.0	1.0	0.000
WaitListed[1.0]	0.110	0.116	-0.118	0.331	0.001	0.001	29026.0	11945.0	1.0	70.824
DistanceFromHome	0.122	0.028	0.067	0.176	0.000	0.000	29011.0	12249.0	1.0	100.000
Gender[M]	-0.234	0.075	-0.382	-0.087	0.001	0.000	22534.0	12650.0	1.0	38.305
HasLoans[1.0]	-0.210	0.077	-0.357	-0.056	0.001	0.000	21872.0	12397.0	1.0	47.841
isCampusWorkStudy[1.0]	-0.612	0.114	-0.833	-0.389	0.001	0.000	29701.0	11619.0	1.0	0.000
isDivisionI[1.0]	-0.014	0.099	-0.204	0.179	0.001	0.001	29319.0	11552.0	1.0	98.956
isFirstGeneration[1.0]	0.097	0.104	-0.105	0.300	0.001	0.001	24489.0	12078.0	1.0	75.309
PellAmount	-0.181	0.041	-0.262	-0.103	0.000	0.000	22582.0	13008.0	1.0	61.006
StudentOfColor[1.0]	0.165	0.101	-0.037	0.360	0.001	0.001	22907.0	12177.0	1.0	59.698
TutoringClassCount	-1.802	0.466	-2.732	-0.990	0.004	0.003	18164.0	9474.0	1.0	0.000

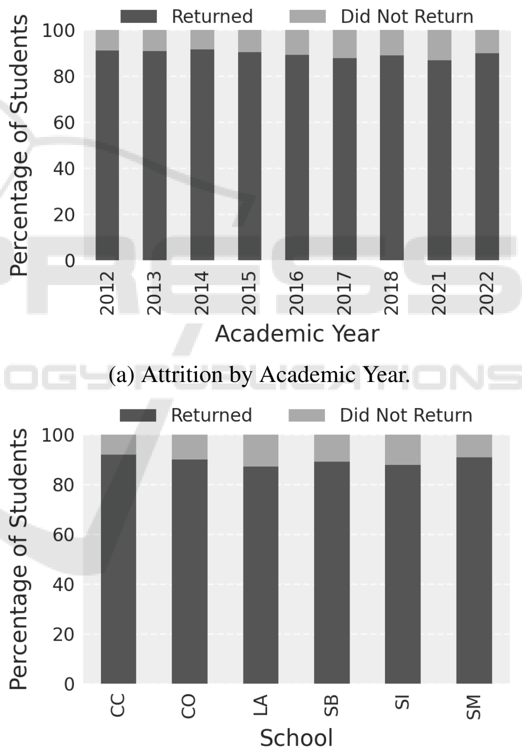
Table 4: Pareto κ Diagnostic Results.

Range	Count	Pct.
(-Inf, 0.70] (good)	10920	100.0%
(0.70, 1] (bad)	0	0.0%
(1, Inf) (very bad)	0	0.0%

demic year and school. As described in Equation (2), We used a varying-intercept model, with group intercepts tied to academic year and school. We did not nest the group effects; instead, we fitted the model in such a way that academic year and school were different group-level effects, independent of each other. This enabled the measurement of fluctuations in the outcome across each group while marginalizing over the effect of the other.

The average log-odds across all academic years is close to zero (-0.001108), which suggests that, on average, the effect of different academic years over freshmen attrition is small. However the standard deviation of 0.175093 points to a certain amount of fluctuation across multiple academic years, with the year 2021 exhibiting a higher-than-average attrition rate, probably due to the COVID pandemic, as noted in the above paragraphs. The 95%HDI was equal to [-0.196947, 0.241552], which presents a mixed scenario, with some years having lower attrition rates, and others somewhat higher values.

The forest plot in Figure 3 depicts the estimated random effects of Academic Year on freshmen attrition. The thick line represents the 50% HDI, and the thin line, the 95%HDI. The graph exposes fluctuations in attrition rates across different academic years. Although most years have probable intervals that include zero, there are some exceptions, such as in the aforementioned academic year 2021.



(a) Attrition by Academic Year.

(b) Attrition by School.

Figure 2: Distribution of Freshmen Attrition.

We conducted a similar examination of the variability in attrition rates across schools by considering random group effects by school and marginalizing the group effect of academic year. The analysis yielded an average log-odds value of -0.004483, with a standard deviation of 0.155275. This indicates that the effect on the log-odds of freshmen retention tied to schools is small on average, but

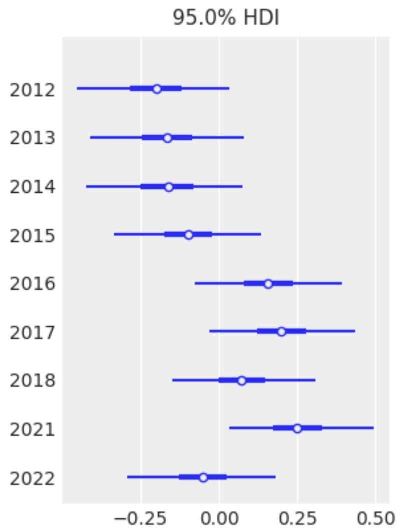


Figure 3: Random Effects of Academic Year on Freshmen Attrition Log-Odds.

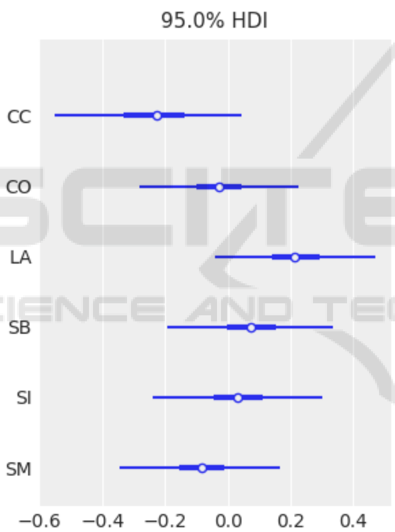


Figure 4: Random Effects of School on Freshmen Attrition Log-Odds.

there is non-negligible fluctuation. The 95%HDI =[-0.221347,0.200628], which also suggests a mixed scenario. The forest plot (see Figure 4) shows the variability of log-odds of freshmen attrition across all six schools.

Some schools, such as the School of Computer Science & Mathematics (CC), have mostly negative log-odds values, which point to a lower likelihood of students dropping out in or after their freshman year. Instead, freshmen students belonging or assigned to the School of Liberal Arts (LA) -and Behavioral Sciences (SB) to a much lesser extent- have a higher likelihood of leaving the institution. To quantify this difference, with a median log-odds value of -0.25 for CC and 0.2 for LA, the odds ratio between LA and CC is $e^{0.2}/e^{-0.25} = 1.57$, which means that LA freshmen are 1.57 times more likely to drop out than CC freshmen. These results underscore the need for additional analysis and potential targeted retention strategies: while Computer Science & Mathematics seems to be more sheltered from freshmen attrition concerns, the positive log-odds exhibited by Liberal Arts (LA) and, to some extent, Behavioral Sciences (SB) may require that the institution dig deeper into the underlying causes of these attrition rate values.

To answer RQ3 (Bayesian vs frequentist hierarchical models) we ran a frequentist mixed effects model (see Equation 3) with the same fixed effects predictors and group level intercepts for academic year and school.

$$
\begin{aligned}
\text{logit} &= b_0 + b_{\text{AcademicYear}} + b_{\text{School}} + \sum_{j=1}^{m} \beta_j x_j \\
b_{\text{AcademicYear}} &\sim \text{Normal}(0, \sigma_{\text{AcademicYear}}) \\
b_{\text{School}} &\sim \text{Normal}(0, \sigma_{\text{School}}) \\
p &= \frac{1}{1+\exp(-\text{logit})}
\end{aligned}
\tag{3}
$$

We used *pymer4* (Jolly, 2018), a Python library that provides an interface to *lme4*, the popular R GLMM package. The Pymer4 model produced similar significant logistic regression coefficients, validating the results in the Bayesian model, but there were some important differences:

EffectiveGPA, isDeansList, Gender, UnmetNeed, isCampusWorkStudy, and HasLoans remain strong predictors in both models.

HSGPA, DistanceFromHome and PellAmount are statistically significant in both the frequentist and the Bayesian models, but the Bayesian model outputs a very high overlap of the 95%HDI with ROPE (91.765%, 100%, and 61.006% respectively), which suggests limited practical impact.

The TutoringClassCount estimate is highly unstable in the Pymer4 model with a mean=-5.744 and 95% CI=(-105.501, 94.014). Instead, the Bayesian model shrinks the estimate to a more reasonable value: mean =-1.802, 95%HDI=(-2.732, -0.990). This is important for dealing with small sample sizes within some academic years and schools.

The frequentist intercept estimate has also a very high variance, with mean=-3.153 and 95%CI=(-24.075,17.769). In comparison, the Bayesian model shrinks the estimate to mean=-2.341, 95%HDI=(-2.8963,-1.808), reflecting the true baseline dropout rate much better.

The frequentist model gives a single estimate and confidence interval, assigning significance based

Table 5: Summary of Frequentist Model Estimates.

Component	Estimate	SE	2.5% CI	97.5% CI	OR	OR 2.5% CI	OR 97.5% CI	Z-stat	P-val	Significance
Intercept	-3.153	10.675	-24.075	17.769	0.043	0.000	5.214e+07	-0.295	0.768	
EFC	0.013	0.035	-0.056	0.083	1.013	0.946	1.086	0.376	0.707	
EffectiveGPA	-0.769	0.037	-0.841	-0.696	0.464	0.431	0.498	-20.854	0.000	***
HSGPA	0.127	0.043	0.042	0.212	1.136	1.043	1.237	2.926	0.003	**
isDeansList [1.0]	0.573	0.096	0.385	0.761	1.774	1.470	2.140	5.985	0.000	***
NumAPCourses	-0.054	0.039	-0.130	0.022	0.947	0.878	1.022	-1.405	0.160	
USCitizen [1.0]	-0.281	0.204	-0.680	0.118	0.755	0.507	1.126	-1.379	0.168	
UnmetNeed	0.370	0.044	0.285	0.456	1.448	1.330	1.577	8.507	0.000	***
WaitListed [1.0]	0.113	0.116	-0.114	0.340	1.120	0.893	1.405	0.978	0.328	
DistanceFromHome	0.124	0.028	0.069	0.179	1.132	1.072	1.196	4.431	0.000	***
Gender [M]	-0.240	0.076	-0.389	-0.091	0.787	0.678	0.913	-3.154	0.002	**
HasLoans [1.0]	-0.211	0.077	-0.362	-0.060	0.810	0.697	0.942	-2.741	0.006	**
isCampusWorkStudy [1.0]	-0.612	0.113	-0.833	-0.391	0.542	0.435	0.677	-5.425	0.000	***
isDivisionI [1.0]	-0.011	0.099	-0.206	0.184	0.989	0.814	1.202	-0.107	0.915	
isFirstGeneration [1.0]	0.105	0.103	-0.097	0.306	1.110	0.907	1.358	1.016	0.310	
PellAmount	-0.180	0.040	-0.258	-0.102	0.835	0.773	0.903	-4.536	0.000	***
StudentOfColor [1.0]	0.166	0.101	-0.033	0.365	1.180	0.968	1.440	1.635	0.102	
TutoringClassCount	-5.744	50.898	-105.501	94.014	0.003	0.000	6.756e+40	-0.113	0.910	
Random Effects	Academic Year: Var = 0.034, Std = 0.184		School: Var = 0.023, Std = 0.152							
Evaluation metrics	Log-likelihood: -3227.289		AIC: 6494.577							

purely on p-values, whereas the Bayesian model gives a full posterior distribution of the model parameters, which helps in gauging uncertainty in regression coefficients.

As for random effects, the frequentist model reports variance and standard deviation for each group (academic year and school), which makes it difficult to assess variability among groups when the group effects are small. Random effects in the Pymer4 model are estimated independently. This means that each group (academic year and school) gets its own separate estimated effect, without the effect being pulled towards the mean, even if there are a small number of observations per group. In contrast, the Bayesian model reports the full posterior for each group, which helps in detecting some significant deviation, as explained in the above paragraphs (see RQ2, Figure 3, and Figure 4). Additionally, in the Bayesian model, group effects are not treated independently, with partial pooling regularizing group effects.

5 CONCLUSION

The goal of this research work was to apply Bayesian hierarchical regression methods to analyze freshmen attrition. The paper provides a guideline on how to conduct the analysis and report the results and findings in the context of Bayesian methods and probabilistic programming. The Bayesian framework is a robust yet highly underutilized data-driven methodology in the academic analytics literature, especially regarding student academic performance and attrition analysis. The results presented in this paper identify college academic performance, financial need, gender, tutoring, and work-study program participation as having a significant effect on the likelihood of freshmen attrition. The study showed fluctuations across time and schools that deserve deeper investigation and potential customized intervention strategies. Also, a comparison was made with an equivalent frequentist mixed-effects model to highlight the differences between both approaches. The motivation of this research is not only to provide a guideline on the use of Bayesian methods for freshmen retention but also to serve as a proof of concept that encourages other researchers and practitioners to apply the Bayesian framework in this domain. The practical implications of this work go beyond the methodological approach presented on the use of Bayesian inference and probabilistic programming for student attrition analysis. We believe that the analysis provides actionable findings to stakeholders, administrators, and decision-makers in higher education.

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