Portfolio Optimization Based on Prospect Theory

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Keywords: Prospect Theory, Portfolio Optimization, Decision-Making.

Abstract: This paper investigates the application of prospect theory in the context of portfolio optimization and presents a model based on the mean absolute deviation and on Prospect Theory. By analyzing historical returns from assets of three critical sectors traded on B3 (Brazilian Stock Exchange) and over an eight-year period, a prospect optimization approach was implemented and its results were compared to those obtained from the Conditional Value at Risk (CVaR) approach. An additional application was held regarding one of the most relevant sector of assets in terms of contribution to the S&P500's composition with the purpose to test the new model under different market conditions. Such results revealed the effectiveness of prospect theory in optimizing portfolios since those results were considered similar to the CVaR's, but at higher returns. Both models were compared through different portfolio performance metrics and, notably, the prospect model exhibited competitive results in most cases. However, the study also identified opportunities for further refinements. Overall conclusions herein suggests the promise of prospect theory in addressing the needs of decision makers in portfolio management, delivering a singular approach that balances the possibility of gains and losses under different scenarios.

1 INTRODUCTION

In the early 1950s, (Markowitz, 1952) contradicted the thesis that investors should maximize or anticipate expected returns and stated that investors should consider the expected return as something desirable and the variance as something undesirable. Markowitz developed an optimization model to minimize the portfolio risk, considering the variance of portfolio returns as the risk measure, subjected to an expected return, which resulted in a quadratic optimization model. Moreover, Markowitz presented the concept of efficient frontier for portfolios, and his contributions are considered the basis of Modern Portfolio Theory (MPT).

Since Markowitz's contribution, subsequent researches have been published with the purpose of developing new approaches for portfolios or reducing the computational cost in obtaining the solution for the optimization problem.

A few decades later, (Konno and Yamazaki, 1991) analyzed Markowitz's model and replaced the variance - the second moment of probability distribution of portfolio returns, by the mean absolute deviation (MAD) - the first moment of probability distribution of portfolio returns, with the intent of reducing the computational time of the quadratic model.

The variance and the mean absolute deviation are statistical measures traditionally applied in the context of portfolio problems. However, these measures cannot identify the anomalies in the probability distribution of portfolio returns nor the extent of potential losses in a given portfolio. Thus, in the mid-1990s, the concept of Value at Risk (VaR) (Group of Thirty, 1993) was proposed in the JP Morgan G30 publication.

The VaR is defined by (Jorion, 2006) as the maximum loss of a portfolio under ordinary market conditions and at a given confidence level. VaR is therefore related to the percentiles of the probability distribution of the losses at a predetermined confidence level. Recognized as a metric for risk, VaR has become a widely useful tool for the financial market and also an important regulatory measure.

Despite being widely disseminated, VaR has been considered unsuitable when losses do not follow a normal distribution, which occurs in most cases. Often, the probability distributions of returns exhibit "heavy tails", making VaR inefficient for identifying extreme risks. In that context, (Rockafellar and Uryasev, 2002) stated that due to the fact that VaR does not

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Ribeiro, C. O. and Santos, A. T.
Portfolio Optimization Based on Prospect Theory.
DOI: 10.5220/0013331900003956

In *Proceedings of the 7th International Conference on Finance, Economics, Management and IT Business (FEMIB 2025)*, pages 51-60
ISBN: 978-989-758-748-1; ISSN: 2184-5891

evaluate losses that exceed the predetermined confidence level, it may show an optimistic tendency rather than a conservative one, which should prevail in risk management.

In 2000's a new risk measure called Conditional Value at Risk (CVaR) was proposed by (Rockafellar and Uryasev, 2000). CVaR is the average of the expected returns that exceed VaR's for a given confidence level. CVaR consists of the expectancy of the tail values of the probability distribution that represent the worst-case return scenarios (Rockafellar and Uryasev, 2000). Accordingly, CVaR is able to measure the "tail losses" under a more robust approach. Additionally, CVaR exhibits superior properties compared to VaR, since (Artzner et al., 1999) stated that CVaR can be considered as a coherent risk measure by having a group of properties which includes subadditivity: $\text{CVaR}_\alpha(R_1+R_2) \leq \text{CVaR}_\alpha(R_1)+\text{CVaR}_\alpha(R_2)$ - that is a property that VaR does not meet.

The objective functions comprehended in optimization problems originated from MPT can be placed within the concept of utility functions, especially since the predominance of Utility Theory as the main method of decision-making under uncertainty at that time. Utility Theory is based on the assumption of the rational investor, and has served as the primary lens through which the behavior of economic agents has been interpreted. However, an alternative method has been disseminated in decision-making analysis since (Kahneman and Tversky, 1979). That paper introduced Prospect Theory, fundamentally altering the decision-making analysis by demonstrating that people weight losses more heavily than gains in general situations, contradicting the assumption of the rational investor.

Prospect Theory diverges from Utility Theory by considering that decisions are affected by other biases instead of the assumption taken in the concept of rational investor. (Kahneman and Tversky, 1979) posited that the value function in Prospect Theory is characterized by a deviation from a reference point, exhibiting concavity for gains and convexity for losses and having greater weight on the convex part than on the concave one. Further refinements by (Kahneman and Tversky, 1992) incorporated nonlinear preferences, a concept of loss aversion and a cumulative function which allows applications for continuous variables.

Other researches in the literature have tested the effectiveness of Prospect Theory in explaining investor behavior, as well as in explaining portfolio returns or even in understanding market tendencies.

Subsequent researches have extensively tested the applicability of Prospect Theory in explaining investor behavior and portfolio returns across various market sectors. For instance, (Benartzi and Thaler, 1995) examined the equity premium puzzle — why American stocks outperformed bonds throughout the 20th century — by presenting two main arguments. The first argument is based on Prospect Theory, asserting that investors are more sensitive to losses than to gains. In other words, investors tend to treat the possibility of losses more severely and seek greater possibilities of gains. The second argument addresses a distinct concept known as "myopic loss aversion", which means that people tend to show more concern about their portfolios in a short-period and do not show the same concern for long-term results. The paper presented a piecewise linear optimization model accordingly and the results defined a period of investor's indifference towards their portfolios that proper justified the "equity premium puzzle" from the authors' perspective.

Further studies, such as those by (Benartzi and Thaler, 1995) and (Barberis et al., 2001) have demonstrated that asset prices that were influenced by loss aversion were closely aligned with historical data while showed minimal correlation with consumption growth. The broad dissemination of Prospect Theory influenced (Barberis and Thaler, 2003) to critique and highlight the theoretical distinctions between rational and non-rational investor profiles, identifying challenges in arbitrage limits and noting a lack of practical applications at that time.

A few years later, (De Giorgi et al., 2010) argued that the financial market would not need to adopt the equilibrium hypotheses if agents had heterogeneous preferences in accordance with Cumulative Prospect Theory (Kahneman and Tversky, 1992). The paper contradicted traditional financial models in which this concept of equilibrium was fundamental.

Specifically in portfolio problems, Prospect Theory has been applied as an alternative approach to explain returns, risk and the overall decision-making by investors in different scenarios. (Best and Grauer, 2016) proposed a multi-period problem for maximizing returns, in which the concept of loss aversion was applied and the loss aversion coefficient from (Kahneman and Tversky, 1992) was used in a portfolio with different assets and rates. The paper also considers the concept of *kink*, that is the non-differentiable segment that connects the gain-curve to the loss-curve of the Prospect Theory's value function. Relevant opportunities in optimization have arisen regarding the concept of *kink*. There has been particular interest in this application, specially by the fact it has been quite challenging. (Best and Zhang, 2011) and (Best et al., 2014) are examples of this application.

(Best and Grauer, 2017) compared three portfolio optimization approaches—power utility, mean-variance, and prospect theory—analyzing static and dynamic contexts with varying borrowing and lending rates. The paper showed that the prospect theory model performed well for risk aversion coefficients between 2 and 2.25, but results were considered inconsistent otherwise. The power utility and mean-variance models performed reasonably for risk-averse investors, though less risk-averse ones relied heavily on borrowing.

(Wang et al., 2021) applied Prospect Theory to analyze the behavior of Chinese investors during an external regulatory shock. The study concluded that Prospect Theory had a strong predictive power to explain stock returns, particularly for small-cap companies, with the theory effectively explaining investor decision-making.

(Zhong et al., 2022) proposed the ”Three-way decision model”, an optimization approach based on Prospect Theory that addresses challenges of time and monetary variables. The model delivered three outcomes as options to the investor: ”accept”, ”reject”, or ”not accept nor reject”, and it aimed to maximize monetary value while the variable time was also taken into account. Finally, some improvements were suggested, particularly when treating monetary outcomes and time as independent variables, which does not necessarily happen. The authors also noted that certain model parameters require refinement.

Fostering the discussion of decision-making under risk, Prospect Theory has been increasingly applied to different fields with diverse objectives. This paper therefore seeks to contribute to further advancing by leveraging Prospect Theory as a decision-making method through a particular application to portfolio problems. This approach consists in a new optimization model developed from the mean absolute deviation (Konno and Yamazaki, 1991) in addition with concepts from Prospect Theory.

Studies in literature that consider Prospect Theory in the context of portfolio optimization, have provided comparisons with the classic Portfolio Theory (Markowitz, 1952), that is the case of (Pfiffelmann et al., 2016). However, a few applications were presented regarding other risk measures. Therefore, this paper provides a comparison of the new optimization model proposed along with Conditional Value at Risk (CVaR) (Rockafellar and Uryasev, 2000), under different confidence levels.

The methodology herein provides a contribution to portfolio optimization, specially on the application of prospect theory as a framework for decision-making under risk, justified by the development of a new optimization model which is compared against the traditional CVaR model. Additionally, the mathematical approach developed utilizes two linearization techniques, the first one is based on (Júdice et al., 2003), where auxiliary variables were applied on the model and the second one, was presented in (Asghari et al., 2022) since the function has non-linear terms comprehended in the piecewise function.

The paper is structured, as: Section 2 presents the new optimization model and elaborates the methodology. Section 3 provides an analytical comparison of the portfolio models, presents the results and elaborates an argumentative discussion about these results. Section 4 provides concluding remarks and identifies potential opportunities for further researches.

2 PORTFOLIO OPTIMIZATION MODEL

In this paper, a new approach is presented based on Prospect Theory and on the mean absolute deviation, that was linearized in accordance with (Júdice et al., 2003). Firstly, a new risk measure is introduced where potential losses grow non-linearly by being weighted by two different coefficients. For potential portfolio gains, just one coefficient is applied.

Consider positive real numbers ζ, $\beta_1 \dots \beta_I$ with $0 < \zeta \leq \beta_1 \leq \beta_2 \leq \dots \leq \beta_I$. Let $w \in \mathbb{R}^n$ be the portfolio composition with $u \in \mathbb{R}$. The proposed risk measure is an extension of MAD, given as:

$$f(u,w) = \begin{cases} \zeta u & \text{if } u^+ \geq 0 \\ \beta_i u & \text{if } d_{i-1} \leq u^- \leq d_i \; i \in \{1 \dots I\} \end{cases} \tag{1}$$

with:

$d_0 \leq d_1 \leq d_2 < d_I$ as negative real numbers

As posited in Prospect Theory, the investor generally weights losses more heavily than gains and therefore, the risk measure should reproduce this relation. Moreover, the risk measure emulates two scenarios for potential losses, one more severe so that it is weighted more heavily than the another. This condition produces a piecewise linear function comprehended by two functions with different slopes.

Assume $u = u^+ - u^-$, $u^+ \geq 0$ *and* $u^- \geq 0$. Due to the complementary conditions $u^+ \cdot u^- = 0$ and $c_i = -d_i \;\; i \in \{1 \dots I\}$, $f(u,w)$ can be rewritten as:

$$f(u,w) = \begin{cases} \zeta u^{+} & \text{if} \quad u^{+} \geq 0 \\ \beta_i u^{-} & \text{if} \quad c_{i-1} \leq -u^{-} \leq c_i \quad i \in \{1 \dots I\} \end{cases} \tag{2}$$

As observed, $f(u,w)$ is a piecewise linear function with $c_0 \leq c_1 \leq c_2 \leq c_I$ being the breakpoints for the function.

Let consider:

$$g(u^{-}) = \left\{ \beta_i u^{-} \quad \text{if} \quad c_{i-1} \leq -u^{-} \leq c_i \quad i \in \{1 \dots I\} \right. \tag{3}$$

Adopting the approach of (Asghari et al., 2022), consider $Y_i \in \{0,1\}$ $\;i \in \{1 \dots I\}$ so that $\sum_i^I Y_i = 1$ and $z_{Y_i} = Y_i \cdot g(u^{-})$ be the auxiliary variables. It is easy to verify that:

$$f(u,w) = \sum_{i}^{I} z_{Y_i} + \zeta \cdot u^{+} \tag{4}$$

subject to:

$$\sum_{i=1}^{I} c_{i-1} Y_i \leq u_t^{-} \leq \sum_{i=1}^{I} c_i Y_i \tag{5}$$

$$\sum_{i=1}^{I} Y_i = 1 \tag{6}$$

$$g(u_t^{-}) - (1 - Y_i)M \leq z_{Y_i} \quad \forall t \in \{1 \dots T\} \tag{7}$$

$$g(u_t^{-}) + (1 - Y_1)M \geq z_{Y_1} \quad \forall t \in \{1 \dots T\} \tag{8}$$

$$-Y_i M \leq z_{Y_i} \leq Y_i M \quad \forall t \in \{1 \dots T\} \tag{9}$$

$$Y_i \in \{0,1\}, \quad j = 1 \dots n \tag{10}$$

$$z_{Y_i} \in \mathbb{R} \tag{11}$$

Based on this framework, let introduce the parameter V^{*} as the breakpoint of the function $g(u^{-})$ and, in theoretical terms, the *kink* as postulated by Prospect Theory.

Let $t \in \{1, \dots, T\}$ denote the time horizon and let revisit (Júdice et al., 2003), where the mean absolute deviation (MAD) model was linearized as $\phi(u) = \sum_{t=1}^{T} \left(u_t^{+} + u_t^{-}\right)$. By the complementary condition $u^{+} \cdot u^{-} = 0$, and through the relation in $u_t^{+} - u_t^{-} = \sum_{j=1}^{n} (R_j^t - \overline{R_j}) w_j, \;\; \forall t \in \{1, \dots, T\}$, a fundamental part of the constraints set in the reformulated optimization model is defined.

Let M be a sufficiently large number to bound the auxiliary variables $z_{Y_i} = Y_i \cdot g(u^{-})$. This adjustment is necessary since $g(u^{-})$ has been reallocated to the constraint set, while z_{Y_i} remains in the objective function, according to the process of linearization.

Consequently, the reformulated optimization model, introducing a new risk measure in the objective function, is now expressed as follows:

$$\text{Minimize}_{(w,u,z)} \Psi(u,z) = \sum_{t=1}^{T} (\zeta u_t^{+}) + \sum_{t=1}^{T} \sum_{i=1}^{I} (z_{tY_i}) \tag{12}$$

subject to:

$$\boldsymbol{w} = \left\{ w \in \mathbb{R}^n \;\middle|\; \begin{array}{l} \sum_{j=1}^{n} w_j = 1; \; \sum_{j=1}^{n} \overline{R_j} w_j \geq R_0; \\ w_j \geq 0, \; \forall j \in \{1 \dots n\} \end{array} \right\} \tag{13}$$

$$V^{*} Y_2 \leq u_t^{-} \leq V^{*} Y_1 + M Y_2 \tag{14}$$

$$\sum_{Y_i=1}^{I} Y_i = 1 \tag{15}$$

$$g(u_t^{-}) - (1 - Y_i)M \leq z_{Y_i} \quad \forall t \in \{1 \dots T\} \tag{16}$$

$$g(u_t^{-}) + (1 - Y_i)M \geq z_{Y_i} \quad \forall t \in \{1 \dots T\} \tag{17}$$

$$-Y_i M \leq z_{Y_i} \leq Y_i M \quad \forall t \in \{1 \dots T\} \tag{18}$$

$$u_t^{+} - u_t^{-} = \sum_{j=1}^{n} (R_j^t - \overline{R_j}) w_j \quad \forall t \dots T \tag{19}$$

$$M \in \mathbb{R}, \quad z_{Y_i} \in \mathbb{R}$$

$$Y_i \in \{0,1\}, \quad j = 1 \dots 10$$

$$u_t^{+}, u_t^{-} \geq 0$$

$$0 < \zeta < \beta_1 < \beta_2, \quad t \in \{1 \dots T\}$$

Where:

R_j^t is the historical return of asset j with $j = 1 \dots n$ in the period t with $t = 1 \dots T$.

w_j is the percentage allocated to asset j.

$\overline{R_j}$ is the average return of asset j.

R_0 is the minimum return required by the investor.

M is a sufficiently large real number.

$Y_1, Y_2, \dots Y_I$ are binary variables responsible for the assignment of the expressions in $g(u_t^{-})$.

i corresponds to the set of equations of the piecewise linear function with $i = 1 \ldots I$ and, in this case, with $I = 2$.

z_{tY_i} are the auxiliary variables.

V^* is a parameter based on the idea of the *kink* arising from Prospect Theory and its value denote in which expression of $g(u_t^-)$ the variable u_t^- lies on.

Observe that this is a mixed-integer programming model in which the number of binary variables does not depend on the number of observations of the time series.

In the model, the Equation (12) is the objective function that minimizes the total portfolio risk, weighting differently gains and losses. Equation (13) is the set of constraints defined in Markowitz's model. Equation (14) represents the domain of u_t^-, i.e., the lower and upper limits for u_t^- in each expression, respectively. Equation (15) ensures the characteristic of the variables as binary type. Equations (16) to (18) establish a lower and upper bound for z_{tY_i}. Equation (19) establishes an exact relationship between the difference of returns above and below the average.

The prospect model composed by a new risk measure turned the piecewise linear function into a linear function and consequently the model into a Linear Programming (LP) problem which is less complex in terms of computational time.

2.1 Portfolio Assessment

The performance of each portfolio optimization model is measured through the set of metrics based on (Ramos et al., 2023). These metrics are described as follows.

The volatility of the portfolio returns can be measured through the standard deviation. With respect to the negative deviations, that comprehends the portfolio returns below the expected returns, the metric used is the semi-deviation. These metrics can be expressed as follows:

$$\sigma = \sqrt{E\left[(R - E[R])^2\right]} \tag{20}$$

$$Sd^- = \sqrt{E[((R - E[R])^-)^2]} \tag{21}$$

Where: $E[R]$ is the expected return of the portfolio.

While standard deviation σ denotes the portfolio risk, semi-deviation denotes only the deviations below the expected returns of the portfolio (Ramos et al., 2023).

The traditional VaR is used considering the historical returns, or in other words, it considers the non-parametric approach.

$$VaR_\alpha(w) = -Q_\alpha\left(\sum_{j=1}^{n} R_j w_j\right) \tag{22}$$

The significance level considered is 95%, quite common in the literature.

The widely known Sharpe Ratio consists of the excess-return ratio per the standard deviation and measures the portfolio performance.

$$\text{Sharpe} = \frac{E[R - r_f]}{\sigma} \tag{23}$$

Where: r_f is the risk-free rate.

In (Sortino and Satchell, 2001) , the standard deviation contained in the Sharpe Ratio's formula is replaced by the Semi Deviation Sd^-, comprehending therefore the negative deviations.

$$\text{Sortino} = \frac{E[R - r_f]}{Sd^-} \tag{24}$$

The STARR Ratio is another metric utilized and it is the expected excess of portfolio returns per the CVaR of the excess of portfolio returns. STARR Ratio also derives from Sharpe Ratio, but takes into account the reward for each CVaR's value and is therefore considered a tail-risk-reward (Ramos et al., 2023). An example of such approach can be found in (Mainik et al., 2015).

$$\text{STARR} = \frac{E[R - r_f]}{CVaR_\alpha(R - r_f)} \tag{25}$$

For the methodology of Systematic Risk, an approach similar to (Ramos et al., 2023) is adopted where a coefficient β is used and obtained through the Ordinary Least Squares (OLS) Regression, such as follows:

$$E[R] = r_f + \beta(E[B] - r_f) + \varepsilon \tag{26}$$

where:

r_f is the risk-free rate.

$(E[B] - r_f)$ is the excess return (market risk premium), where $E[B]$ is the benchmark return (IBOV index for the Brazilian study and S&P500 for the American study, both presented herein).

β is the coefficient of the regression or in better words, the sensibility of the portfolio to the market risk premium.

ε is an error considered.

Additionally, β's results represent the sensibility of the portfolio returns to the market risk premium so that returns above and below the expected market returns are commonly called bull or bear market and therefore, were called $\beta^{(+)}$ and $\beta^{(-)}$, respectively. The Brazilian benchmark index adopted was the IBOV for the Brazilian application and S&P500 for the American's.

3 RESULTS

3.1 Data and Application

The application of the methodology previously discussed considered three pivotal sectors from the Brazilian Stock Exchange (Brasil, Bolsa, Balcão - (B3, 2025)). These sectors were: Oil & Gas, Financial/Banking, and Electricity sector. A total of 27 assets were chosen, comprising 7 from the Oil & Gas and 10 from each of the remaining sectors. The selection criteria prioritized assets with historical returns that cover all time horizon and focused on prominent Brazilian companies. Observe in Table 1 the set of assets belonging to each of the sectors considered.

These sectors also were selected due to their substantial influence on the fluctuations of IBOV. This index is also utilized as a reference in one of the metrics previously detailed.

Analogously, an additional application was held to test the methodology proposed herein under different market conditions. Therefore, an important sector from the American stock market was selected to run both models under the same method, criteria and period. The sector selected was Technology with 10 stocks from the most valuable companies open in Nasdaq (American Exchange), as shown in Table 2.

The analysis for both studies spans approximately eight years, from January 2015 to December 2023, justified not only for its duration but also for encompassing the COVID-19 pandemic, a period of considerable volatility to the portfolio returns. This time horizon with some instability is critical for testing the robustness of the proposed portfolio model.

In order to consider a balanced temporal dynamics, the eight-years period was subdivided into moving windows, as proposed by (Best and Grauer, 2016). This approach facilitates the identification of trends and documents performance variations across these periods within the time horizon. Each window spans the same number of 319 days, ensuring consistency in temporal analysis.

The application is conducted for two portfolio optimization problems: the CVaR optimization model and the newly prospect optimization model, derived from mean absolute deviation (MAD) and Prospect

Table 1: Companies By Sector - B3-Brazil.

Code	Electricity
CPLE3	Cia Paranaense De Energia-Copel
CMIG3	Cia Energética De Minas Gerais-Cemig
ELET3	Centrais Elet Bras S.A-Eletrobras
CPFE3	Cpfl Energia S.A
LIGT3	Light S.A.
EQTL3	Equatorial Energia S.A
EGIE3	Engie Brasil Energia S.A
ENGI11	Energisa S.A.
CSLC3	Centrais Elet De Santa Catarina S.A
TAEE11	Transmissora Aliança de En. Elétrica S.A
Code	**Banking/Financial**
BPAN4	Banco Pan S.A
BEES3	Banestes S.A-Banco do E. do E. Santo
BRSR6	Banrisul S.A-Banco do E. do R. G. Do Sul
BBDC4	Banco Bradesco S.A
BBAS3	Banco Brasil S.A
BAZA3	Banco Amazônia S.A
ITUB4	Itaú Unibanco Holding S.A
BMEB4	Banco Mercantil Do Brasil S.A
BNBR3	Banco Nordeste Do Brasil S.A
SANB11	Bco Santander (Brasil) S.A
Code	**Oil & Gas**
ENAT3	Enauta Participações S.A
RPMG3	Refinaria De Petróleos Manguinhos S.A.
PETR3	Petróleo Brasileiro S.A - Petrobras
PRIO3	Petro Rio S.A
UGPA3	Ultrapar Participações S.A
LUPA3	Lupatech S.A
OSXB3	Osx Brasil S.A

Theory. The CVaR problem is assessed across a few confidence levels, whereas the prospect optimization model is assessed under different weighting parameters.

Each optimization model and sector was analyzed with respect to risk, return, and with respect to the portfolio assessment metrics described herein. The time horizon was divided into seven moving windows across the three selected sectors.

The CVaR is evaluated under a few confidence levels, that are 90%, 95% and 99% whereas the prospect model was evaluated under a group of a few parameters, based on Prospect Theory and being subdivided into 9 problems, as shown on the next table.

The variations in the coefficients of the Prospect Optimization Model were designed to assess the model under different gains and losses scenarios, con-

Table 2: Companies By Sector - Nasdaq-USA.

Code	**Technology**
AAPL	Apple Inc.
MSFT	Microsoft Corporation.
NVDA	NVIDIA Corporation.
AMZN	Amazon.com Inc.
GOOGL	Alphabet Inc.
META	Meta Platforms Inc.
TSLA	Tesla Inc.
AVGO	Broadcom Inc.
CSCO	Cisco Systems, Inc.
ADBE	Adobe Inc.

Table 3: Parameters for each Prospect model.

Prospect Model	ζ	β_1	β_2
Prospect 1.1	1.10	0.50	1.25
Prospect 1.2	1.10	0.50	2.25
Prospect 1.3	1.10	0.50	3.25
Prospect 2.1	1.10	1.50	1.25
Prospect 2.2	1.10	1.50	2.25
Prospect 2.3	1.10	1.50	3.25
Prospect 3.1	1.10	2.50	1.25
Prospect 3.2	1.10	2.50	2.25
Prospect 3.3	1.10	2.50	3.25

trasting with CVaR performance, and both of their performances evaluated by the defined portfolio metrics.

The parameter V^*, which is based on the idea of the *kink*, was quantified as an average of the CVaR's results.

The results for the metrics mentioned are presented along three tables specifically for each sector, but considering the whole time series. The full period was applied for a matter of enough data to calculate the portfolio performance metrics and to organize a reasonable comparison. Each metric is compared separately in each column of each table. Therefore, colors in green represent positive results to a specific metric in a column from each table. Colors in red mean negative results. Colors in yellow mean intermediate results. Occasional column with only yellow results means that the results are too close to each other so that they are all considered intermediate.

3.2 Discussion

The performance of the models for the Brazilian case is exhibited in Table 4, which contains results for the electricity sector and in Tables 5 and 6, which contain results for the Financial and the Oil & Gas sector, respectively.

The Table 4 exhibited properties of higher risk for prospect models with exception to CVaR 90%. Notably, the standard deviation σ and the semi-deviation $Sd(-)$ are higher along this period demonstrating higher volatility of portfolio returns. When rechecking the data, it was concluded that there were considerable losses within that period, and it was confirmed through the analysis of the index of electricity of this sector (B3-IEE, 2025). The exception occurred for VaR, in which prospect model achieved low risk. With respect to the reward-risk metrics, Sharpe, Sortino and STARR, CVaR's models produced a better relation to risk-return. With respect to the betas, which represent the portfolio's sensitivity in tracking the market premium risk, the findings were not particularly significant, with stand out to the prospect model. It is important to note that, according to (Ramos et al., 2023), betas below 1 indicate periods of market downturn, which suggests that the results highlighted in green reflect a lower sensitivity of the portfolios in following this trend.

Table 5 presents very similar returns between the two models, with a marginally notable performance by the CVaR 99% model in terms of accumulated return. Regarding the risk metrics σ and $SD(-)$, the CVaR models stood out, except for the CVaR 99%, which exhibited higher risk values. As for the metric VaR, the prospect model was prominent, repeating the relation observed in the previous table. Concerning the risk-return reward measures, the CVaR model outperformed across all three indices—Sharpe, Sortino, and Starr. For the betas, the prospect model stood out in both the benchmark and the bull market, whereas the CVaR model excelled in the bear market.

The Table 6 revealed a different relationship for the models in some performance metrics compared to the previous tables. The results for accumulated return, standard deviation, and the Sharpe ratio were superior for the prospect model. However, for the remaining metrics in the table, including the betas at the end of the columns, the CVaR model achieved superior results.

Additionally to the main case herein, an application was held to the American market and an important sector, which is the Technology, was selected to test the model under different market conditions. Such results are exhibited in Table 7 that showed superior results for the prospect model.

Notably, the prospect model achieved better results in Table 7 in terms of returns, all risk-return reward metrics and in betas. On the other hand, it may be considered more volatile considering the results of $Sd^{(-)}$ and VaR's. It's also important to notice that despite CVaR has been superior in a few metrics, It lost its superiority in the risk-return reward metrics, result that was observed in last three Tables regard-

Table 4: Performance Metrics for Electricity Sector.

Electricity Sector with 10 assets											
Risk	**Cum. Ret.**	**Avrg. Ret**	**σ**	**Sd(-)**	**VaR**	**Sharpe**	**Sortino**	**STARR**	**β**	**β(+)**	**β(−)**
CVaR 90%	0.660043	0.000296	0.004005	0.003093	0.005198	0.074487	0.096447	0.021228	0.063460	0.021962	0.034169
CVaR 95%	0.721675	0.000323	0.003750	0.002445	0.005553	0.086910	0.133334	0.024408	0.058796	0.031897	0.017675
CVaR 99%	0.716388	0.000321	0.004031	0.002544	0.006466	0.080274	0.127206	0.023626	0.063441	0.034894	0.029544
Prospect 1.1	0.674502	0.000302	0.005146	0.004577	0.004899	0.059230	0.066595	0.019269	0.044670	0.013594	0.009173
Prospect 1.2	0.675335	0.000302	0.005186	0.004619	0.004904	0.058848	0.066068	0.019247	0.044737	0.014486	0.010847
Prospect 1.3	0.670603	0.000300	0.005161	0.004593	0.004855	0.058719	0.065979	0.019129	0.044491	0.013958	0.008974
Prospect 2.1	0.679653	0.000304	0.005282	0.004714	0.004759	0.058144	0.065154	0.019197	0.044561	0.009642	0.009642
Prospect 2.2	0.675522	0.000303	0.005286	0.004713	0.004878	0.057745	0.064772	0.019102	0.044797	0.012970	0.010225
Prospect 2.3	0.671259	0.000301	0.005193	0.004618	0.004844	0.058412	0.065689	0.019117	0.044337	0.013861	0.009103
Prospect 3.1	0.675058	0.000302	0.005234	0.004673	0.004826	0.058287	0.065288	0.019173	0.044373	0.013902	0.009264
Prospect 3.2	0.675102	0.000302	0.005254	0.004685	0.004814	0.058065	0.065116	0.019164	0.044457	0.013718	0.009633
Prospect 3.3	0.671439	0.000301	0.005202	0.004628	0.004840	0.058337	0.065574	0.019119	0.044684	0.014160	0.011265
Average	0.680548	0.000305	0.004894	0.004158	0.005070	0.063955	0.078935	0.020148	0.048900	0.017420	0.014126
Minimum	0.660043	0.000296	0.003750	0.002445	0.004759	0.057745	0.064772	0.019102	0.044337	0.009642	0.008974
Maximum	0.721675	0.000323	0.005286	0.004714	0.006466	0.086910	0.133334	0.024408	0.063460	0.034894	0.034169

Table 5: Performance Metrics for Banking / Financial Sector.

Banking / Financial Sector with 10 assets											
Risk	**Cum. Ret.**	**Avrg. Ret**	**σ**	**Sd(-)**	**VaR**	**Sharpe**	**Sortino**	**STARR**	**β**	**β(+)**	**β(−)**
CVaR 90%	0.6749598	0.0003023	0.0044868	0.0032494	0.0064019	0.0679822	0.0938703	0.0200801	0.123820	0.101417	0.081619
CVaR 95%	0.6759288	0.0003027	0.0045429	0.0032063	0.0065380	0.0672376	0.0952662	0.0200886	0.131423	0.106065	0.085652
CVaR 99%	0.6923757	0.0003101	0.0048577	0.0033469	0.0073117	0.0643964	0.0934663	0.0202511	0.136302	0.131051	0.131051
Prospect 1.1	0.6749052	0.0003022	0.0045383	0.0036612	0.0063434	0.0672044	0.0833051	0.0192861	0.120924	0.083417	0.122114
Prospect 1.2	0.6749052	0.0003022	0.0045408	0.0036617	0.0063840	0.0671671	0.0832939	0.0192926	0.121436	0.091364	0.109767
Prospect 1.3	0.6749052	0.0003022	0.0045401	0.0036680	0.0063258	0.0671783	0.0831497	0.0192946	0.120832	0.083619	0.122911
Prospect 2.1	0.6749052	0.0003022	0.0045358	0.0036593	0.0063818	0.0672409	0.0833469	0.0193093	0.122353	0.084633	0.096643
Prospect 2.2	0.6749052	0.0003022	0.0045357	0.0036539	0.0063761	0.0672424	0.0834720	0.0193022	0.122592	0.091526	0.098825
Prospect 2.3	0.6749052	0.0003022	0.0045382	0.0036652	0.0063537	0.0672064	0.0832134	0.0193072	0.121781	0.085380	0.096273
Prospect 3.1	0.6749052	0.0003022	0.0045415	0.0036686	0.0063279	0.0671571	0.0831374	0.0192895	0.120903	0.083235	0.095485
Prospect 3.2	0.6749052	0.0003022	0.0045403	0.0036658	0.0063827	0.0671743	0.0831990	0.0192909	0.120822	0.084256	0.095347
Prospect 3.3	0.6749052	0.0003022	0.0045411	0.0036734	0.0063378	0.0671634	0.0830276	0.0193008	0.122256	0.090480	0.087357
Average	0.676451	0.000303	0.004562	0.003565	0.006455	0.067029	0.085979	0.019508	0.123787	0.093037	0.101920
Minimum	0.674905	0.000302	0.004487	0.003206	0.006326	0.064396	0.083028	0.019286	0.120822	0.083235	0.081619
Maximum	0.692376	0.000310	0.004858	0.003673	0.007312	0.067982	0.095266	0.020251	0.136302	0.131051	0.131051

Table 6: Performance Metrics for Oil & Gas Sector.

Oil & Gas Sector with 7 assets.											
Risk	**Cum. Ret.**	**Avrg. Ret**	**σ**	**Sd(-)**	**VaR**	**Sharpe**	**Sortino**	**STARR**	**β**	**β(+)**	**β(−)**
CVaR 90%	0.6641466	0.0002974	0.0091415	0.0066263	0.0122230	0.0328367	0.0453010	0.0132606	0.308324	0.181852	0.466155
CVaR 95%	0.6573600	0.0002944	0.0091305	0.0065635	0.0122558	0.0325433	0.0452713	0.0131186	0.315649	0.190211	0.463402
CVaR 99%	0.6572365	0.0002943	0.0109428	0.0071091	0.0136906	0.0271487	0.0417893	0.0121112	0.317266	0.201762	0.458171
Prospect 1.1	0.6669770	0.0002987	0.0088806	0.0070480	0.0124028	0.0339440	0.0427702	0.0128963	0.339118	0.223844	0.579172
Prospect 1.2	0.6824364	0.0003056	0.0088822	0.0070543	0.0123860	0.0347175	0.0437133	0.0131909	0.338976	0.223784	0.576523
Prospect 1.3	0.6743679	0.0003020	0.0088779	0.0070511	0.0123787	0.0343271	0.0432208	0.0130430	0.339160	0.223198	0.577457
Prospect 2.1	0.6808568	0.0003049	0.0088809	0.0070452	0.0124080	0.0346429	0.0436695	0.0131690	0.338882	0.222711	0.575979
Prospect 2.2	0.6824364	0.0003056	0.0088829	0.0070409	0.0123409	0.0347145	0.0437968	0.0131996	0.338450	0.222083	0.577830
Prospect 2.3	0.6742435	0.0003019	0.0088805	0.0070465	0.0124059	0.0343110	0.0432409	0.0130474	0.339153	0.222908	0.577640
Prospect 3.1	0.6723612	0.0003011	0.0088903	0.0071144	0.0123583	0.0341783	0.0427098	0.0129719	0.340836	0.226336	0.577777
Prospect 3.2	0.6723612	0.0003011	0.0088808	0.0070555	0.0123709	0.0342150	0.0430663	0.0129987	0.338825	0.223488	0.574337
Prospect 3.3	0.6739684	0.0003018	0.0088798	0.0070591	0.0124090	0.0342997	0.0431463	0.0130376	0.339567	0.224591	0.578087
Average	0.671563	0.000301	0.009096	0.006984	0.012469	0.033490	0.043475	0.013004	0.332850	0.215564	0.548544
Minimum	0.657236	0.000294	0.008878	0.006563	0.012223	0.027149	0.041789	0.012111	0.308324	0.181852	0.458171
Maximum	0.682436	0.000306	0.010943	0.007114	0.013691	0.034717	0.045301	0.013261	0.340836	0.226336	0.579172

Table 7: Performance Metrics for Technology Sector - Nasdaq-USA.

Technology Sector with 10 assets.											
Risk	**Cum. Ret.**	**Avrg. Ret**	**σ**	**Sd(-)**	**VaR**	**Sharpe**	**Sortino**	**STARR**	**β**	**β(+)**	**β(−)**
CVaR 90%	1.029705	0.000461	0.005352	0.003718	0.008104	0.060200	0.086651	0.002834	0.001390	-0.000772	0.001539
CVaR 95%	1.033281	0.000463	0.005450	0.003757	0.008420	0.059408	0.086177	0.002849	0.001055	-0.000936	0.000915
CVaR 99%	1.024239	0.000459	0.005610	0.003863	0.009021	0.056996	0.082763	0.002808	0.001798	-0.000760	0.001108
Prospect 1.1	1.062510	0.000476	0.005289	0.003889	0.008359	0.063689	0.086619	0.002960	0.001670	-0.000437	0.000803
Prospect 1.2	1.060851	0.000475	0.005290	0.003877	0.008400	0.063537	0.086693	0.002955	0.001644	-0.000470	0.001762
Prospect 1.3	1.060538	0.000475	0.005288	0.003882	0.008376	0.063530	0.086539	0.002953	0.001646	0.000705	0.000904
Prospect 2.1	1.062882	0.000476	0.005299	0.003898	0.008434	0.063605	0.086466	0.002962	0.001629	-0.000466	0.001744
Prospect 2.2	1.061604	0.000475	0.005291	0.003879	0.008406	0.063585	0.086745	0.002958	0.001609	-0.000499	0.001719
Prospect 2.3	1.063218	0.000476	0.005291	0.003883	0.008428	0.063728	0.086829	0.002964	0.001649	-0.000453	0.001760
Prospect 3.1	1.062743	0.000476	0.005298	0.003904	0.008413	0.063602	0.086323	0.002961	0.001692	-0.000417	0.000828
Prospect 3.2	1.063239	0.000476	0.005295	0.003891	0.008310	0.063676	0.086662	0.002964	0.001581	-0.000510	0.001698
Prospect 3.3	1.061254	0.000475	0.005288	0.003875	0.008442	0.063597	0.086779	0.002956	0.001652	-0.000455	0.001765
Average	1.053839	0.000472	0.005337	0.003860	0.008426	0.062429	0.086270	0.002927	0.001585	-0.000456	0.001379
Minimum	1.024239	0.000459	0.005288	0.003718	0.008104	0.056996	0.082763	0.002808	0.001055	-0.000936	0.000803
Maximum	1.063239	0.000476	0.005610	0.003904	0.009021	0.063728	0.086829	0.002964	0.001798	0.000705	0.001765

ing the Brazilian study. These findings suggest that the prospect model has tendency to seek more gains in profitable markets, getting therefore a better performance, but also under higher volatility. When the scenario is characterized by losses it may be quite conservative, changing this feature. This can be justified by the fact that the stocks in Technology sector have delivered higher returns and have been quite profitable during the time horizon considered, or over almost 8 years. Differently from Brazilian sectors that had returns nearly 30% less over the same time horizon.

In some of the results, it is possible to notice that the prospect model exhibited a propriety of higher volatility as observed in (Pfiffelmann et al., 2016). This feature can be observed through Table 4 and partially in Table 5 and 7.

Finally, both models were considered suitable for this application and the prospect model delivered results suitable and comparable to CVaR's in many instances. It's possible to infer that in some occasions discussed, prospect model delivered more volatility and very similar results in all sectors, specially in the financial sector for Brazilian study (Table 5) and for the American study in Table 7. On the other hand, CVaR delivered a better relation of reward return-risk in all Brazilian sectors.

Next section presents overall conclusions and suggests opportunities to continue this study.

4 CONCLUSIONS

This paper has elucidated the significant potential of utilizing prospect theory in the context of portfolio optimization. By the interpretation of the results from the proposed prospect model, an alignment with the outcomes of a traditional optimization model was observed in many instances.

The primary advantage of integrating prospect theory into portfolio optimization lies in its ability to assign different weightings to gains and losses, thereby accommodating specific needs of the decision makers. This approach also enables the weighting for losses in distinct segments, thus allowing simulate different scenarios.

The results of this paper were analyzed using portfolios composed of assets from three different sectors for Brazilian case and one more sector within the American case. This approach was crucial for evaluating both models when selecting assets under distinct conditions, thereby avoiding undesirable biases. The alignment of these results demonstrated the consistency of the new model as an alternative tool for decision-making analysis under uncertainty in the context of investments. This was the primary goal of this paper, that is, to provide an alternative mathematical model with a level of efficiency comparable to well-established mathematical approaches in the specialized literature.

While the proposed model exhibited similarities to CVaR's in most of instances, it also demonstrated inconsistencies in some of the results. These findings underscore the necessity for further refinements. Future researches should focus on calibrating all parameters, particularly the break-point of the function based on the idea of the *kink*). Besides future researches could propose more break-points for the functions testing different parameters for the betas. This would increase the number of binary variables and might provide opportunities of refinements.

In conclusion, the application of prospect theory to portfolio optimization problems holds promise, but it also presents challenges that require attention and ongoing refinement to fully achieve its potential.

ACKNOWLEDGEMENTS

The first author received financial support from CNPq (Brazilian National Research Council), grant 308909/2021-6.

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