






# Estimation of Hydrodynamic Dispersion Coefficient Under Saturated and Unsaturated Conditions

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**Abstract:** In the study of soil salinization, the hydrodynamic dispersion coefficient is an important parameter for solute transport. However, dispersion coefficients for soils require a great deal of time and effort, especially for silts and clays, which can be complicated and prolonged to measure. In this study, the hydrodynamic dispersion coefficients of silt and clay were determined by laboratory experiments and numerical analysis under different saturation states. By comparing with the conventional method, the applicability of proposed method of quickly obtaining the unsaturated dispersion coefficient was verified. Additionally, by investigating the relationship between the hydrodynamic dispersion coefficient and the soil pore water velocity, the empirical formula of the soil hydrodynamic dispersion coefficient obtained.


## 1 INTRODUCTION


Soil salinization has become a common concern in countries around the world. About 8.7 percent of the world's land is threatened by salinization, and this number continues to rise (FAO, 2021). China is one of the countries with serious salinization and wide distribution, mainly distributed in arid, semi-arid, and coastal areas. In order to remove salinity from soil, it is important to understand the transport process of solutes in soil, which has been well demonstrated in many research fields. The hydrodynamic dispersion coefficient ( $D$ ) is an important parameter for controlling soil solute transport. The  $D$  plays a crucial role in the simulation and optimization of solute flux and subsequent desalination methods.


For the measurement method of unsaturated  $D$ , the traditional soil column method has a good effect on sand, but it is time-consuming and labor-intensive. In contrast, centrifugation method is fast, but the equipment is expensive. The past developed suction


method can also be used to measure the dispersion coefficient of sand. Although relatively cheap, it takes a long time to reach a steady state, and the flow needs to be adjusted. Also, due to the high cost and time-consuming study of clay, silt, and loam, most of the past studies have focused on the determination of sand, highly permeable soils. In this paper, a simplified quasi-steady-state suction method is proposed to measure the unsaturated  $D$  of silty and clay soils. Also, the  $D$  under the saturated state was measured. In addition, by collecting the dispersion coefficients of different types of soil in various references, an empirical formula was obtained that can be used to estimate the hydrodynamic dispersion coefficient from the soil pore water velocity.

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## 2 METHOD

### 2.1 Hydrodynamic Dispersion Coefficient

Solute transport can be measured using the following convective–dispersive equation:

$$C \frac{\partial \theta}{\partial t} + \theta \frac{\partial C}{\partial t} = \frac{\partial Q}{\partial z} \quad (1)$$

where  $C$  is the salt concentration ( $\text{kg}/\text{m}^3$ );  $\theta$  is the volumetric water content ( $\text{m}^3/\text{m}^3$ );  $t$  is the elapsed time (s),  $z$  is position or depth (m); and  $Q$  is the solute flux ( $\text{kg}/(\text{m}^2 \cdot \text{s})$ ), which can be defined as the sum of convective and hydrodynamic dispersive fluxes:

$$Q = qC - \theta D \frac{\partial C}{\partial z} \quad (2)$$

where  $q$  is the Darcy flow velocity ( $\text{m}/\text{s}$ ), and  $D$  is the hydrodynamic dispersion coefficient ( $\text{m}^2/\text{s}$ ), which is expressed as the sum of molecular diffusion coefficient ( $D_c$ ) and the mechanical dispersion coefficient ( $D_m$ ):

$$D = D_c + D_m \quad (3)$$

In this study, the value of  $D_c$  was set to  $1.65 \times 10^{-9} \text{ m}^2/\text{s}$  (Castillo et al., 1993).  $D_m$  is represented by

$$D_m = \lambda v^\alpha \quad (4)$$

where  $\lambda$  is the dispersivity (m),  $v$  is the average pore water velocity ( $\text{m}/\text{s}$ ), and  $\alpha$  is generally equal to 1.

## 2.2 Experimental Design

### 2.2.1 Experiment for Unsaturated Soils

Three experimental cases using fluvo-aquic soil (a silty soil collected from Jiangsu, China) and kaolin clay soil were considered. The experimental setup is shown in Figure 1. The inner diameter of the soil column was 25 mm, and the height was 70 mm, which was divided into two layers. A TDR sensor (TRIME-MUX6) was inserted near the surface of the lower column to measure the variation in salinity. Water suction was achieved using a vacuum pump (DA60-D). The suction flask, gas jar, and a small column attached at the end (filled with silica gel) were used to collect water and water vapor. The following experimental steps were followed: (1) uniformly mix the soil with fresh water, and fill in the lower soil column to a certain  $\theta$  value; likewise, fill in the upper soil column with soil mixed with saline water ( $C = 0.5\%$ ) to same  $\theta$  value; (2) start water suction using the vacuum pump; (3) measure the drainage water mass using electrical balance, and monitor the output of the TDR sensor. To investigate the quasi-steady-state condition, both the upper and

lower soil columns were mixed with fresh water, and a preliminary test was conducted before the experiment.

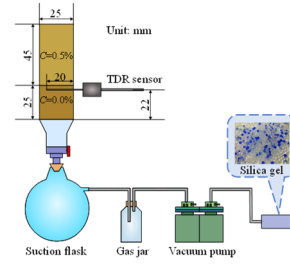


Figure 1: Experimental setup.

### 2.2.2 Experiment for Saturated Soils

Five experimental cases using fluvo-aquic soil and Tohoku paddy soil (collected from Natori, Japan) were considered. As shown in Figure 2, the experimental apparatus comprises a soil column made of vinyl chloride, with an inner diameter of 0.107 m, pump (WP1000, Welco Co., Ltd), water tank, funnel, electronic balance (PB4002-S, Mettler Toledo Co., Ltd), Four-electrode salinity sensor, and data logger (CR1000, Campbell Scientific, Inc.). The depths of the sensors from the soil surface were 110 mm, 190 mm, 270 mm, and 350 mm. The experiment was conducted by adopting the following procedure: (1) The air-dried soil was screened and evenly filled into the soil column (bulk density of  $1.6 \text{ g}/\text{cm}^3$ ); (2) Kept the head of the water tank constant and continuously supplied the fresh water from bottom of the column to attain saturation by capillary; (3) After saturation, the NaCl solution with  $C$  of  $10 \text{ kg}/\text{m}^3$  supplied from top of the column and kept the water level constantly; (4) Changes in the electrical conductivity (EC) value of the sensor were observed; (5) When the EC value of the bottom sensor was constant, the experiment was terminated.

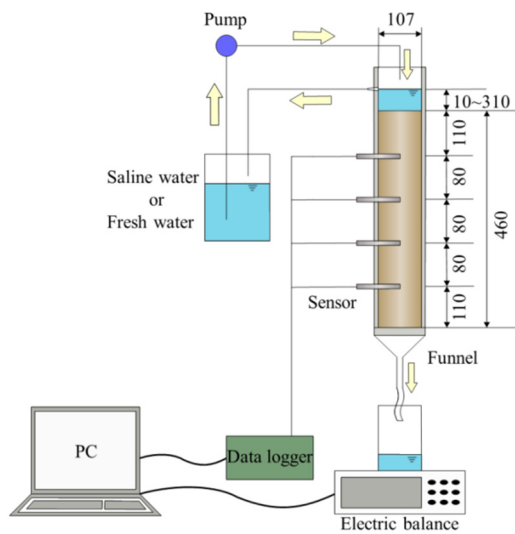


Figure 2: Experimental equipment (Unit: mm).

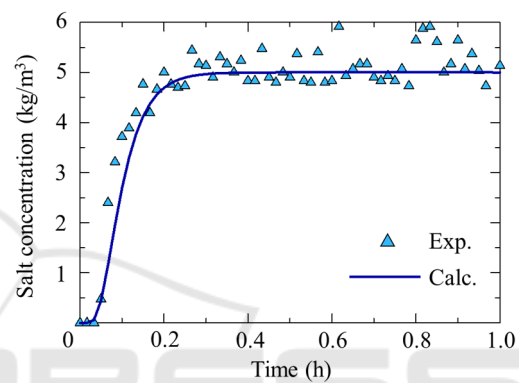
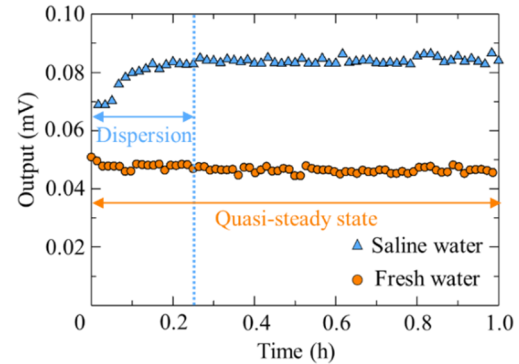
### 3 RESULTS AND DISCUSSION

#### 3.1 Hydrodynamic Dispersion Coefficient of Unsaturated Soil

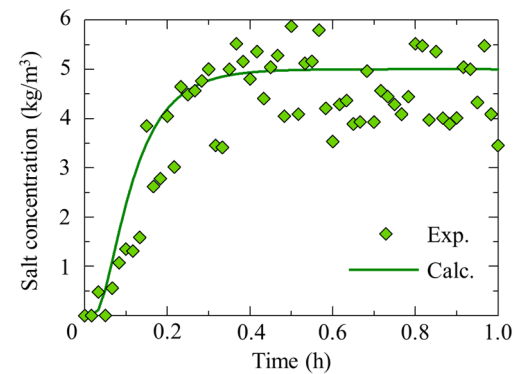
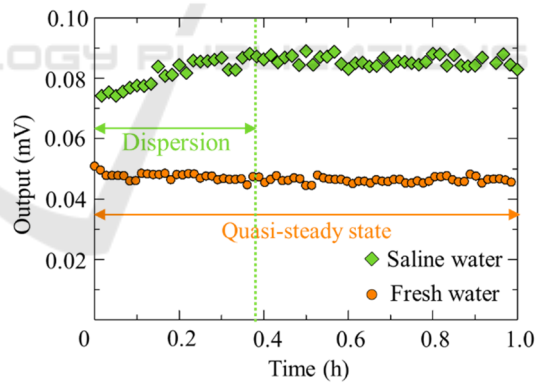
Figure 3 shows the time variation in the output values of the TDR sensor and salt concentration in Case1–3. Cases 1 and 2 correspond to fluvo-aquic soil at two flow rates. Case 3 corresponds to kaolin clay soil. No significant changes in the TDR sensor output values were observed in the freshwater test until 1.0 h after the start of the experiment, which is considered a quasi-steady state. In contrast, the TDR sensor output value in the saline water experiment increased for approximately 0.25 h, after which it showed a constant value. Further,  $\theta$ ,  $C$ , and the TDR sensor output obtained from the preliminary test revealed that the salinity at the sensor position reached 0.5% after 0.25 h.

The pore water velocity was calculated from the time variation in the drainage mass during the quasi-steady-state condition. The  $v$  values of the fluvo-aquic soil were approximately  $4.45 \times 10^{-6}$  m/s and  $3.93 \times 10^{-6}$  m/s. The salt concentration variation curve was obtained by converting the sensor output value, and the dispersion coefficients were determined by fitting the breakthrough curve for the experimental and calculated values. As can be seen from the breakthrough curve in Figure 3, the calculated and experimental values have good fitness. The  $D$  values of the fluvo-aquic soil were approximately  $2.50 \times 10^{-9}$  m<sup>2</sup>/s and  $5.00 \times 10^{-9}$  m<sup>2</sup>/s, and that of kaolin clay was  $1.25 \times 10^{-9}$  m<sup>2</sup>/s.

Noticeably, the simplified suction method proposed in this study yields quick results.



a) Case1– Fluvo-aquic soil



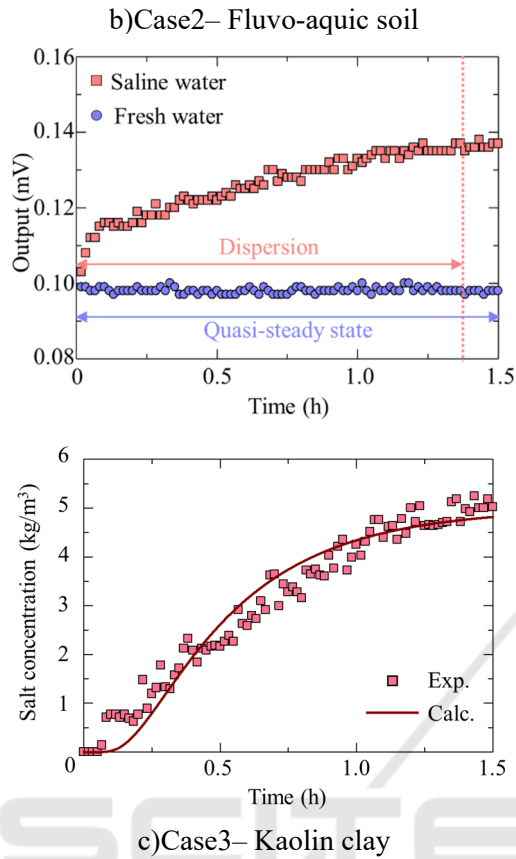


Figure 3: Time variation of sensor output and breakthrough curve of Case 1–3.

### 3.2 Hydrodynamic Dispersion Coefficient of Saturated Soil

Case 1–4 of fluvo-aquic soil and Case 5 of Tohoku paddy soil determined their corresponding  $D$  at different  $v$  as shown in Table 1. Figure 4 shows the breakthrough curve of Case 3 and Case 5 under the average value of  $8.04 \times 10^{-7}$  m/s ( $v$ ), by fitting the experimental and calculated values. It can be seen that the  $C$  changes with respect to time and space. After adding the NaCl solution,  $C$  increased gradually until it approached  $10 \text{ kg/m}^3$  of the  $C$ . Calculated results showed sound agreement. Due to the scale dependency of  $D$  (Moradi et al., 2020), the values calculated separately for each sensor from top to bottom.

Table 1: Dispersion coefficient of each sensor at different pore water velocity in Case 1–5.

No.	Soil type	$v$	$D$ of different depth ( $\text{m}^2/\text{s}$ )			
		(m/s)	110 mm	190 mm	270 mm	350 mm
Case 1	Fluvo-aquic soil	3.18E-06	-	3.50E-09	3.50E-09	3.50E-09
Case 2		1.61E-06	9.50E-10	1.00E-09	1.00E-09	1.00E-09
Case 3		8.19E-07	4.50E-09	5.40E-09	5.50E-09	3.50E-09
Case 4		1.23E-06	3.50E-09	3.80E-09	6.00E-09	5.50E-09
Case 5	Tohoku paddy soil	1.77E-06	7.00E-09	-	1.80E-08	2.30E-08

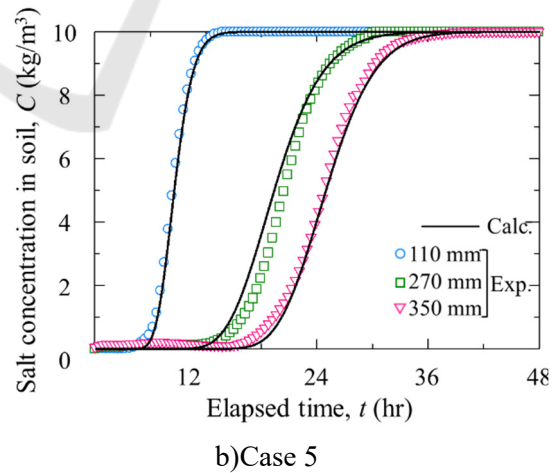
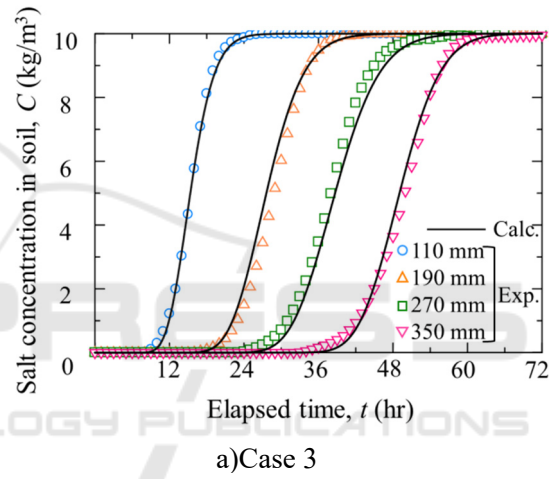


Figure 4: Breakthrough curve of the saturated soil.

### 3.3 The Relationship between $D$ and $v$

As the  $D$  of each type of soil is different, and the measurement is complicated, the  $D$  of different types

of soil in past studies are collected and summarized in Figure 5. The relationship between the  $D$  and  $v$  obtained in this experiment and those in the references (Shikanuma et al., 2003; Kobashi et al., 2004; Cho et al., 1981; Matsubayashi et al., 1997; Taikoku et al., 1997; Kinoshita et al., 2003; Sasaki et al., 1986; Shao et al., 2002; Takahashi et al., 2005). The experimental values obtained in this study generally conformed to the  $D$ - $v$  liner line, which is represented by:

$$D = 0.0095v^{1.15} \quad (5)$$

where the value of dispersivity is approximately 0.0095 m, and  $\alpha$  is approximately equal to 1.15.

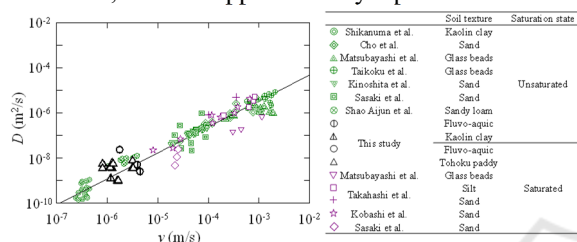


Figure 5: Relationship between  $D$  and  $v$  in saturated and unsaturated soil.

## 4 CONCLUSIONS

In this study, the hydrodynamic dispersion coefficients of fluvo-aquic soil and kaolin clay under unsaturated state and fluvo-aquic soil and Tohoku paddy soil under saturated state were investigated. Proposed simplified suction method in this study yields quicker results than the conventional methods. Additionally, by collecting the different types of soil in past studies and combining the values in this study, the dispersivity of different saturation conditions were measured at 0.0095 m. Therefore, when the soil dispersion coefficient cannot be experimentally measured under limited conditions, it can be calculated using the dispersivity obtained in this study when the soil pore water velocity is known. However, the measurements are relatively few. In the future, we will accumulate experimental data and compare that with data from other measurement methods to verify its accuracy.

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