Prediction Using Kriging Surrogate Model Based on the Formalization of Excavation Deformation Characteristics

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Abstract: The deformation characteristics of a tunnel during the advancement of the excavation face are crucial for determining the excavation support scheme. Traditional simulation analysis methods often involve a substantial workload and lengthy computation times. In this study, we propose a method to formulate the tunnel excavation deformation characteristic curve. By combining Latin Hypercube Sampling techniques with the Kriging surrogate model, we introduce a rapid prediction method for tunnel excavation deformation characteristics based on the surrogate model. Case studies demonstrate that this method offers good applicability and high prediction accuracy. Compared to traditional simulation analysis methods, this approach is significantly more efficient.

1 INTRODUCTION

Hydraulic tunnels, as a critical component of hydraulic infrastructure systems, exhibit deformation characteristics during excavation that are of paramount concern to engineers (Zhang et al., 2017). These characteristics serve as a crucial basis for determining the stability and safety of the surrounding rock, as well as for designing support measures and selecting the timing of such support (Ren et al., 2021; Liu et al., 2023). Consequently, the ability to rapidly predict the deformation characteristics of the surrounding rock under various excavation schemes is essential.

Su Kai et al. (2019) analyzed the deformation patterns of the surrounding rock during the advancement of a tunnel face through numerical simulation. They introduced the concept of displacement completion rate and applied it to determine the timing of initial support installation. However, numerical simulation methods are labor-intensive and time-consuming. Zhou Shuoan (2014) developed a surrogate model based on neural networks to predict the deformation characteristics of tunnels, using parameters such as rock mass deformation, strength, and depth ratio as inputs. However, this model is limited to predicting deformation at a specific moment and cannot forecast the progression of deformation over time.

To address these issues, this paper proposes a rapid prediction method for the deformation characteristics curve during tunnel excavation. First, by thoroughly analyzing the spatial effects of tunnel excavation and the trend of the tunnel deformation characteristics curve, a parametric representation method for the deformation characteristics curve of the surrounding rock is proposed. Then, by integrating Latin Hypercube Sampling with the Kriging surrogate model, a rapid prediction of the tunnel excavation deformation characteristics curve is achieved. Case studies have demonstrated the effectiveness of this method.

2 METHOD

2.1 Formulated Deformation Characteristic Curve of Tunnel Excavation

Tunnel excavation refers to the process of removing geotechnical materials from the predetermined location of the tunnel using a specified excavation method. During the advancement of the excavation

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face, the rock mass structure surrounding the tunnel continuously changes, leading to deformation within the excavation disturbed zone. The magnitude and distribution characteristics of the deformation in the surrounding rock mass are not only related to the excavation at the current location but are also influenced by excavation activities within a certain range both ahead and behind the face. Consequently, rock mass deformation is gradually completed during a specific stage and is subject to spatial effects.

Since the deformation within the excavation disturbed zone gradually decreases from the excavation boundary to the deeper surrounding rock, the tunnel excavation design primarily focuses on the development and evolution characteristics of rock mass deformation at key points on the excavation boundary during construction. This is to adequately evaluate the stability of surrounding rock deformation and design the support scheme. Therefore, in this study, the vertical displacement at the tunnel crown of a typical section is denoted as '*u*,' the distance between this section and the excavation face is denoted as '*L*,' and the curve of '*u*' varying with '*L*' is termed the 'tunnel excavation deformation characteristic curve.' The tunnel excavation deformation characteristic curve is rapidly predicted using a surrogate model.

Since the surrogate model cannot output a continuous tunnel excavation deformation characteristic curve, it is necessary to formulate the tunnel excavation deformation characteristic curve. This involves representing the curve with an equation that contains a finite number of undetermined coefficients. Based on the observed variation characteristics of the tunnel excavation deformation characteristic curve, a function like the one shown in Equation (1) is selected to express the relationship between u(mm) and L(m).

$$u = \frac{u_{\max}}{1 + ae^{-kL}} \tag{1}$$

where $u_{\max}$ represents the maximum deformation during excavation, k denotes the rate of change of the deformation rate in the tunnel excavation deformation characteristic curve, a indicates the proportional relationship between the deformation at L=0 and $u_{\max}$.

In summary, due to the limited output data from the Kriging model, predicting a continuous curve presents a challenge. Formula 1 proposed in Part 2 can represent the deformation characteristic curve of tunnel excavation, which implies that the prediction of the tunnel excavation deformation characteristic curve can be transformed into predicting the values of the three undetermined coefficients. This avoids the difficulty of directly predicting a continuous curve using a surrogate model, representing one of the innovations of this paper.

2.2 Kriging Surrogate Model

The Kriging model is an interpolation model that assumes the response value at any point xxx in the input space can be expressed as a linear weighted sum of the known sample response values, as follows:

$$\hat{y}(x) = \sum_{i=1}^{n} \lambda_i(x) y(x_i) \tag{2}$$

where $\lambda_i(x)$ represents the weighting coefficient for the *i*-th sample's true response value, and $y(x_i)$ is the true response value corresponding to the *i*-th known sample point x in the model's input space. Together, x_i and $y(x_i)$ form the *i*-th sample's input-output data pair. n is the number of known sample points.

To determine the weighting coefficients, the Kriging model treats the unknown function to be fitted as the realization of a Gaussian stationary stochastic process:

$$Y(x) = \mu + Z(x) \tag{3}$$

where μ is an unknown constant representing the mean of $Y(x)$, and $Z(x)$ is a stationary stochastic process with a mean of 0 and variance σ^2. The covariance of this process is given by:

$$Cov\left[Z(x), Z(x')\right] = \sigma^2 R(x, x') \tag{4}$$

where $R(x,x')$ is the correlation function, used to define the correlation between any two points x and x'. A common choice is the Gaussian correlation function (Han, 2016):

$$R(x, x') = exp\left\{-\sum_{i=1}^{n} \theta_i \left(x_i - x_i'\right)^2\right\} \tag{5}$$

In this equation, θ_i is a hyperparameter that measures the influence of the *i*-th input variable x_i on the model and must be optimized using maximum likelihood estimation (Kaymaz, 2005; Dong et al., 2024).

To minimize the mean squared error (MSE) of the predictions and ensure unbiasedness, the optimal weighting coefficients must be determined. This is done by minimizing the MSE:

$$MSE\left[\hat{y}(x)\right] = Var(\hat{y}(x)) \tag{6}$$

while satisfying the unbiasedness condition:

$$\sum_{i=1}^{n} \lambda_i(x) = 1 \tag{7}$$

Applying the Lagrange multiplier method, the best linear unbiased prediction (BLUP) for $\hat{y}(x)$ can be obtained (Wan et al., 2023):

$$\hat{y}(x) = \mu + r^T R^{-1}\left(Y_S - \mu F\right) \tag{8}$$

where $\mu = \left(F^T R^{-1} F\right)^{-1} F^T R^{-1} Y_S$, r^T is the correlation vector between the prediction point x and the known sample points, R^{-1} is the $n\times n$ correlation matrix composed of the correlation function values between all known sample points, Y_S is the n-dimensional vector of known sample true response values, and F is an $n\times 1$ column vector of ones.

2.3 Excavation Deformation Characteristics Curve Prediction Method Based on the Kriging Surrogate Model

Based on the formulation of the tunnel excavation deformation characteristic curve, the Kriging surrogate model is established to achieve rapid prediction of the tunnel excavation deformation characteristic curve. The inputs to this model are the primary influencing factors of tunnel excavation deformation characteristics, and the outputs are the formulation coefficients of the curve (i.e., u_{max}, k, and a). This process primarily involves the selection of influencing factors, generation of the sample set, training of the surrogate model, and prediction.

In this study, the deformation modulus E, Poisson's ratio μ, internal friction angle φ, cohesion c, and depth H are selected as the influencing factors, which serve as the inputs to the surrogate model.

Given the advantage of Latin Hypercube Sampling (LHS) in reflecting overall variability with a smaller sample size (McKay et al., 2012), LHS is employed to sample the surrogate model inputs. The corresponding model outputs for each sampled point are obtained through a combination of finite element analysis and Equation (1).

A total sample set is then formed, which is divided into a training sample set for training the surrogate model and a testing sample set for validating the model's prediction accuracy.

Based on these training and testing samples, the Kriging surrogate model is constructed and its prediction accuracy is verified.

For any given set of surrogate model input data (i.e., values of deformation modulus E, Poisson's ratio μ, internal friction angle φ, cohesion c, and depth H), the u~L functional relationship curve describing the tunnel excavation deformation characteristics can be obtained using Equations (8) and (1). Figure 1 illustrates the implementation process of this method.

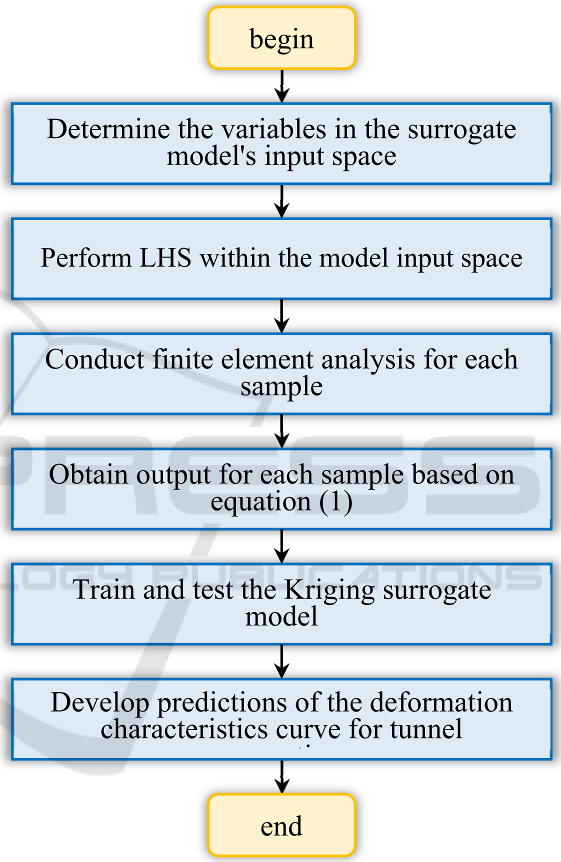

Figure 1: Prediction of Tunnel Excavation Deformation Characteristics Implementation Procedure.

3 EXAMPLE

3.1 Project Overview

A certain water conveyance tunnel has a total length of approximately 3.5 km, with a burial depth ranging from 260 to 460 meters. The rock primarily consists of dolomite and limestone. The tunnel has a circular cross-section, with an excavation diameter D of 6.8

meters and an advance rate of 2 meters per round. The range of rock mass mechanical parameters is provided in Table 1.

Table 1: Range of rock mass mechanical parameters.

ρ (kg/m^3)	φ (°)	c (MPa)	E (GPa)	μ
2650	35~45	0.5~1	5~10	0.24~0.28

3.2 Training and Testing of Kriging

Figure 2 illustrates the finite element mesh in the central portion of the model, extending approximately five times the excavation advance length. The model dimensions are X×Y×Z=74.8m×204m×74.8m, with the tunnel located at the center of the model. A uniformly distributed vertical compressive stress is applied to the top of the model, while normal displacement constraints are applied to the bottom and sides.

After completing the finite element simulation analysis for all training sample points, the surrogate model output data, namely u_{max}, k, and a, can be obtained based on Equation (1) and the results of the simulation analysis. On the basis of obtaining the training sample set, the Kriging surrogate model for predicting the deformation characteristics of tunnel excavation can be trained according to the principles described in Section 3. Table 2 compares the surrogate model predicted values and the fitted values of the undetermined coefficients for three test sample points (VS1, VS2, VS3), showing that the maximum relative error does not exceed 2%, indicating that the constructed surrogate model has good predictive accuracy.

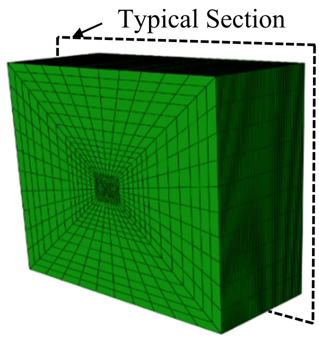

Figure 2: Finite Element Mesh Diagram.

3.3 Analysis of Predicted Results

The Kriging surrogate model was used to predict the excavation deformation characteristics of three typical sections of the water conveyance tunnel (Section 1, Section 2, Section 3). Table 3 lists the burial depths and mechanical parameters of the surrounding rock for the predicted sections. Additionally, to validate the prediction accuracy, finite element simulation analyses were conducted for the three sections mentioned above in Figure 3.

Table 3: Typical hole depth and surrounding rock mechanical parameters.

Section number	φ (°)	c (MPa)	E (GPa)	μ	H (m)
1#	38.7	0.60	9.00	0.25	400
2#	41.0	0.67	9.14	0.25	431
3#	43.2	0.76	9.77	0.24	446

Table 2: Comparison between the predicted and fitted values of the undetermined coefficients for the test samples.

Test Sample	u_{max}(mm)			a			k		
	Fitted value	Projected value	Relative error	Fitted value	Projected value	relative error	Fitted value	Projected value	Relative error
TS_1	12.61	12.69	0.63%	1.74	1.73	0.57%	0.59	0.60	1.69%
TS_2	12.78	12.81	0.23%	1.85	1.84	0.54%	0.59	0.60	1.69%
TS_3	12.94	12.95	0.07%	1.68	1.69	0.6%	0.63	0.64	1.58%

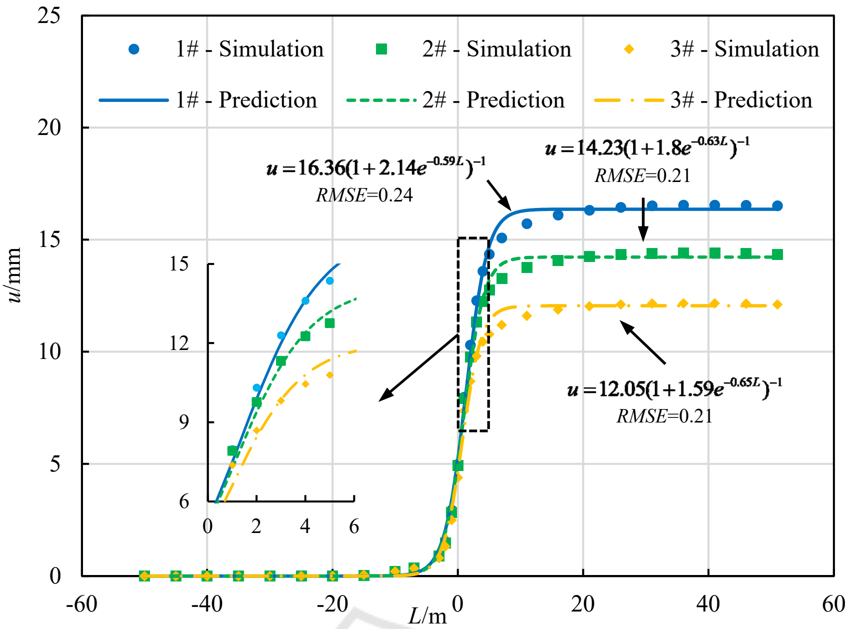

Figure 3: Comparison of prediction results and simulation analysis results.

The comparison of the surrogate model prediction curves with the finite element simulation results for Sections 1, 2, and 3 reveals the following: (1) For sections with different burial depths and varying mechanical parameters of the surrounding rock, the surrogate model consistently provides excavation deformation prediction curves that align well with the simulation analysis results, indicating reliable prediction accuracy and strong adaptability. (2) The excavation deformation in different sections primarily occurs during the excavation process of the rock mass in front of and behind the observation profile (-4m < L < 6m). Considering the need for deformation control and the excavation cycle length (2m), the initial support should be installed as soon as possible after excavation, with the lag distance behind the excavation face not exceeding 4m. (3) The differences in excavation deformation characteristics between sections are mainly reflected in two aspects: the deformation rate and the maximum deformation. From Section 1 to Section 3, as the burial depth gradually increases, both the deformation rate and the maximum deformation decrease. This is primarily because when the difference in burial depth is small, the mechanical parameters of the rock are the main factors influencing the deformation rate and deformation magnitude.

4 CONCLUSION

To achieve rapid and accurate prediction of tunnel excavation deformation characteristics, this study proposes a rapid prediction method based on a surrogate model, building on the formulation of the excavation deformation characteristic curve. A case study analysis was conducted to validate the method. The main research conclusions are as follows:

(1) A formula containing three undetermined coefficients was proposed. The trend of the curve generated by this formula aligns with the growth pattern of surrounding rock deformation during tunnel excavation. This approach transforms the prediction target from a continuous curve to independent undetermined coefficients, thereby reducing prediction difficulty.

(2) The constructed Kriging surrogate model can predict the deformation characteristics of the tunnel for given model inputs. This method is more efficient than traditional simulation analysis methods, significantly reducing computation time.

(3) The case study analysis verified the prediction accuracy and applicability of this method. The predicted excavation deformation characteristic curve closely matches the results obtained from simulation analysis.

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