

# Analytic Study on High-Performance Concrete Column with High-Strength Reinforcement Under Moment-Compress Loads

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**Abstract:** According to analytic calculation in RC column of high-performance concrete and high-strength reinforcement, these rules on geometric deformation compatibility, physical constitutive relation and static equilibrium were applied, the model of analytic calculation was confirmed and some equations on static equilibrium were for reinforced high-performance concrete column with high-strength reinforcement. Some calculation formulas on reinforcement and bearing capability were presented for moment-compressed column by the KADAN formula and simplified calculation method. The research shows that good agreements and small errors are observed by comparing the experimental data of bearing capability and the result from proposed formulas. However some larger errors are observed from the formula of current code, the error becomes larger with the strength promotion of reinforcement.


## 1 INTRODUCTION

Reinforced concrete compressed members are one of the most common basic components in building structures, the calculation on bearing capability and reinforcement of columns became some important parts of structural design. Some codes of structure design took these calculation methods on bearing capability and reinforcement for normal RC column (GB50010, 2010; ACI318, 2010).

In recent years, high-performance concrete (HPC) and high-strength reinforcement (HSR) were widely applied in RC building structures. Some experiment studies and numerical analysis were presented for high-performance concrete column with high-strength steel bars. Monotonic eccentric loading tests on eight reinforced concrete columns with 630MPa grade steel bars were carried out, 630MPa reinforced concrete biased columns can be calculated according to the bearing capacity formula in specification GB 50010-2010. (Gao, 2023; Ma et al., 2024). Textile-reinforced concrete (TRC) were replaced by high-strength high ductile concrete (HSHDC) and some columns were tested under eccentric loading, a calculation formula considering strain-lag behavior was presented to

predict the maximum load of the column strengthened with TR-HSHDC (Ding et al., 2024). Axially-compressed behaviour of concrete columns reinforced with novel high-strength steel-rebar materials were studied by some tests, the corresponding finite element (FE) model of the axially-compressed RC-NHHSRs column was established. (Shen et al., 2024). The influence of eccentric axial loads on the lateral low-velocity impact behaviors of RC columns was studied by the numerical simulation approach (Jia et al., 2024).

However few analytic studies on bear capability and reinforcement were taken for compressed column of high-performance concrete and high-strength reinforcement. The behaviors of symmetrical reinforcement of concrete column with HRB600 steel bars were analyzed under eccentric compression, the basic formula under large or small eccentric compression and the corresponding section design calculation methods were given based on the existing *code for design of concrete structures* (GB50010—2010) (Zhang et al., 2021).

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## 2 BASIC FORMULA ON BEARING CAPABILITY OF RC COLUMN

### 2.1 Overview on Calculation Method

Three kinds of internal force analysis were certified for RC column and were shown as follows:

- Compressed column under axial load. For the short-column under strength failure mode, the capability bearing was equated to some limit compressed forces from concrete and steel bar. For the slender-column or moderate-column under buckling mode or strength-buckling mode, the bearing capability of short-column was reduced by the stability coefficient of buckling.
- Compressed column under larger eccentric load. It was named as moment member under tensile failure mode, the bearing capability and reinforcement were calculated by the method on bearing capability of beam and axial compressed column.
- Compressed column under small eccentric load. It was named as compressed member under compress failure mode, the bearing capability was equated to compressed forces from concrete and steel bar.

The bearing capability of RC column could be calculated according to these codes of structure design as GB50010-2010 or ACI318-19.

### 2.2 Method on Bearing Capability of HPC-HSR Column

Number simulation analysis, analytical method and testing method have been applied to analyze the bearing capability of RC column with high-performance concrete and high-strength steel bars. These research methods were verified each other and shown as follows:

- Finite element method of number simulation analysis. The softwares of Aboqus or Anasys and so on were applied to analyze the bearing capability and reinforcement of HPC-HSR column, however these analysis modes might be modified for RC column with high-performance concrete and high-strength steel bars.
- Formula analytic method of mathematical method. According to the failure modes and basic assumed conditions, some analytical formulas of RC column were presented in research literature, however some results from theoretical formulas had more errors than some

test data. Many calculation formulas on bearing capability of RC column were applied in some structural design codes

- Testing method. Few testing on bearing capability of HPC-HSR columns were conducted, however some expensive fees, prolonged process and many limit conditions were the shortcomings in these testing methods.

### 2.3 Formulas for Bearing Capability of HPC-HSR Column

#### 2.3.1 Static Equations from Analysis Mode

According to some tests and analysis mode bases on basic assumes, the ultimate limit state of HPC-HSR column was analyzed, some simply method and factors were applied from *the code for design of concrete structure* (GB50010-2010). So the normal formulas of bearing capability for HPC-HSS column with rectangle section were presented and were shown as follow:

$$N = \alpha_1 f_c b \xi h_0 + f_y' A_s' - \sigma_s A_s \quad (1)$$

$$Ne = \alpha_1 f_c b \xi h_0^2 \left(1 - \frac{\xi}{2}\right) + f_y' A_s' (h_0 - a_s') \quad (2)$$

In these formulas, the former was derived from static balance on axial force, the later was determined from moment balance. In these equations,  $N$  was force perimeter or bearing capability;  $\alpha_1$  was the shape coefficient, the value is 1 as concrete strength is not more than C50, the value is 0.94 as concrete strength is not less than C80, otherwise the value is an interpolation between C50 and C80;  $f_c$  is compressed strength of concrete; for a rectangle section,  $b$  is width,  $h_0$  is effective height,  $\xi$  is relative height; for steel bar,  $f_y'$  is compressed strength and  $A_s'$  is section area of compressed yield failure,  $\sigma_s$  is stress and  $A_s$  is section area of tensile or compress;  $e$  is eccentric distance;  $a_s'$  is a distance between force point of compress area and compress edge of section.

#### 2.3.2 Applicable Conditions in Analytic Formulas

According to the conduct processes on equations (1) and equation (2), some applicable conditions of analytic equations were presented for HPC-HSR column and were shown as the following:

- Bearing capability of axial compressed column. In a word, the bearing capability of eccentric compressed column was not more than an bearing capability of axial compressed column. For slender or moderate column, these factors as buckling and second-order effect took

effect on the bearing capability, so the bearing capability ( $N$ ) of axial compressed column was reduced by a buckling coefficient( $\varphi$ ) and was shown as the following:

Note:  $f_c$  was compressed strength of high-performance concrete,  $A_c$  was section area of concrete;  $f_y$  was compressed strength of high strength steel bars.  $A_s$  was section area of steel bars.  $\varphi$  was the buckling coefficient of axial-press bar and defined according to the buckling coefficient from GB50010-2010.

- Bearing capability of larger eccentric compressed column. For rectangle section column of HPC-HSS, a relative height of  $\xi$  was not more than  $\xi_b$  and the tension stress  $\sigma_s$  of steel bars was yield strength of  $f_y$  in ultimate limit state. So the equation (1) was simplified as the equation (4), equation (2) and equation (4) were applied for larger eccentric compressed column of HPC-HSS.

$$N = \alpha_1 f_c b \xi h_0 + f_y A'_s - f_y A_s \quad (4)$$

- Bearing capability of small eccentric compressed column. For rectangle section column of HPC-HSS, a relative height of  $\xi$  was more than a limit relative height of  $\xi_b$ , the stress  $\sigma_s$  of steel bars was not more than yield strength in ultimate limit state and shown as the following:

$$-f'_y \leq \sigma_s = f_y \frac{\xi - \beta_1}{\xi_b - \beta_1} \leq f_y \quad (5)$$

Note:  $\beta_1$  was shape coefficient of high-performance concrete, the value is 0.8 as concrete strength is not more than C50, the value is 0.74 as concrete strength is not less than C80, otherwise the value is an interpolation between C50 and C80.

### 3 PRACTISE CALCULATION METHOD OF BEARING CAPABILITY

#### 3.1 Simplified Method of GB50010-2010

Due to the variability and randomness of structural loads, the RC members are primarily designed with symmetric reinforcement. So  $f_y = f'_y$  or  $A_s = A'_s$  were applied in eccentric compressed column, the equations (1), (3) for larger eccentric compressed column could be solved directly according to GB50010-2010, however an equation (6) for small

eccentric compressed column was derived from these formulas and shown as the following:

$$Ne \frac{\xi_b - \xi}{\xi_b - \beta_1} = \alpha_1 f_c b h_0^2 \left( \xi - \frac{\xi^2}{2} \right) \frac{\xi_b - \xi}{\xi_b - \beta_1} + (N - \alpha_1 f_c b h_0 \xi)(h_0 - a'_s) \quad (6)$$

As the equation (6), a cubic equation of  $\xi$  was determined and could not be solved directly. According to GB50010-2010, the second order section of  $\xi(1 - 0.5\xi)$  was simplified as 0.43, so a cubic equation was simplified as line equation, the relative height  $\xi$  of compressed area and reinforcement areas were shown as the following:

$$\xi = \frac{N - \alpha_1 f_c b \xi_b h_0^2}{\frac{Ne - 0.43 \alpha_1 f_c b h_0^2}{(\xi_b - \beta_1)(h_0 - a'_s)} + \alpha_1 f_c b h_0} + \xi_b \quad (7)$$

$$A_s = A'_s = \frac{Ne - \alpha_1 f_c b \xi h_0^2 (1 - \frac{\xi}{2})}{f'_y (h_0 - a'_s)} \quad (8)$$

#### 3.2 Precised Method from Cardano's Formula

Let:  $\bar{N} = \frac{N}{\alpha_1 f_c b h_0}$ ;  $\bar{M} = \frac{Ne}{\alpha_1 f_c b h_0^2}$ , then equation (6) can be expressed as

$$\xi^3 - \xi^2(2 + \xi_b) + 2\xi \left( \xi_b - \left(1 - \frac{a'_s}{h_0}\right)(\xi_b - \beta_1) + \bar{M} \right) + 2\bar{N} \left(1 - \frac{a'_s}{h_0}\right)(\xi_b - \beta_1) - 2\xi_b \bar{M} = 0 \quad (9)$$

The simplify calculation was derived from equation (9) by substituting parameters and shown as:

- Determine some coefficients  $b, c, d$ .

$$b = -(2 + \xi_b); \quad c = 2 \left( \xi_b - \left(1 - \frac{a'_s}{h_0}\right)(\xi_b - \beta_1) + \bar{M} \right);$$

$$d = 2\bar{N} \left(1 - \frac{a'_s}{h_0}\right)(\xi_b - \beta_1) - 2\xi_b \bar{M} \quad (10)$$

- Calculate some parameters  $p, q$ .

$$p = -\frac{b^2 - 3c}{3}; \quad q = \frac{9bc - 2b^3 - 27d}{27}; \quad (11)$$

- Calculate the relative height  $\xi$  of compressed area.

$$\xi = \sqrt[3]{-\frac{q}{2} + \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} + \sqrt[3]{-\frac{q}{2} - \sqrt{\frac{q^2}{4} + \frac{p^3}{27}}} - \frac{b}{3} \quad (12)$$

Then the symmetric reinforcement area were calculated by equation (12) and equation (8).

#### 3.3 Simplified Method from Reduction Order

According to simplified method of equation (6) from GB50010-2010, the second order section of  $\xi(1 - 0.5\xi)$  was reduced to the first order part of  $0.25(1 + \xi)$ , so the equation (6) was reduced as an second-order

equation, then the relative height  $\xi$  of compressed area was derived as:

$$\xi = 2(B - \sqrt{B^2 + C}) \quad (13)$$

In the equation (13), some parameters of B or C were shown as;

$$B = \bar{M} + \frac{\xi_b - 1}{4} - \left(1 - \frac{a'_s}{h_0}\right)(\xi_b - \beta_1); C = \bar{N} \left(1 - \frac{a'_s}{h_0}\right)(\xi_b - \beta_1) - \bar{M}\xi_b \quad (14)$$

Then the symmetric reinforcement area were calculated by equation (14) and equation (8).

## 4 VERIFICATION PROCESS ON FORMULA OF BEARING CAPABILITY

### 4.1 Evaluated Method on Calculated Formulas of Bearing Capability

Some compressed tests on typical small eccentric compression specimens were selected to verify the formulas accuracy, including different reinforcement strengths, concrete strengths, and geometric parameters.

First, the axial compressed force  $N_t$  was a bearing capability from some tests, the relative height  $\xi$  of compressed area were calculated according to the formulas of the papers and sample parameters, then the bearing capability  $N_c$  was calculated according to the relative height  $\xi$  of compressed area, section parameters of concrete and reinforcement, strength parameters of construction materials, finally the precision of calculation formulas was evaluated by the ratio  $N_t/N_c$  between the test bearing capability and the calculated bearing capability. As the ratio was closed to 1, it means the high precision of calculation formulas, otherwise it means the less precision of calculation formulas.

### 4.2 Evaluated Examples of HPC-HSR Column

#### 4.2.1 HPC-HSR Column of HRB500/HRBF500

Some members with C40 or C60 concrete and HRB500/HRBF500 reinforcement were tested by small eccentric compressed load. The bearing capabilities of members were calculated by precise method, simplified method of the code (GB50010-2010) and simplified method of reduced order. The formula accuracy on bearing capability of small eccentric compressed specimens of

HRB500/HRBF500 reinforcement were analyzed as Figure 1. the maximum deviation between simplified method of the code and precise method exceeded 40%, however the relative deviation between simplified method of reduced order and precise method was smaller, some local deviations exceeded 20% for  $\bar{N} > 1$ . The calculation error of C40 specimens is smaller than that of C60 specimens. (Mao, 2008).

#### 4.2.2 HPC-HSR Column of HRB630

Some members with C40 concrete and HRB630 reinforcement were tested by small eccentric compressed load. The formula accuracy on bearing capability of small eccentric compressed specimens of HRB630 reinforcement were analyzed as Figure 2, the ratios of the test results and results from precise method or simplified method of reduced order were all around 1, the maximum ratio was 1.05. Then the accuracy of the formulas meet the engineering requirements. However, the maximum deviation of the results calculated by the code formula exceeds 40%, the safety margin of the code formula was larger. (Luo, 2013)

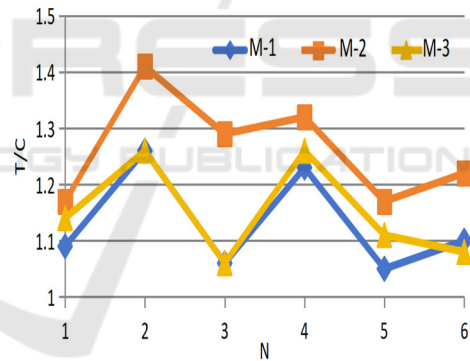


Figure 1: Column of HRB500 reinforcement.

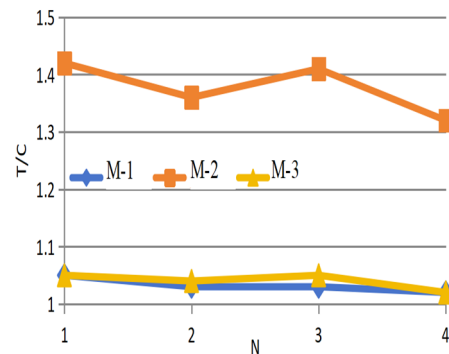


Figure 2: Column of HRB500 reinforcement.

Note:  $T$  was the result from tests,  $C$  was the result from calculation;  $N$  was number of samples;  $M-1$  was the result from precised method of Cardano's formula;  $M-2$  was the result from the simplified method of GB50010-2019;  $M-3$  was the result from the simplified method of reduced-order.

## 5 CONCLUSION

### 5.1 Formulas on Capability Bearing and Reinforcement of HPC-HSS Column

Two kinds of calculation formulas on capability bearing and reinforcement are presented for HPC-HSS column under small eccentric load. The former is based on the Cardano formula for solving cubic algebraic equations, and the calculated results have a good degree of agreement with experimental results. The latter is based on the reduction processing of the original equation, the calculation formula is simpler than the precise formula, but the calculation accuracy is slightly worse. Both methods proposed in this paper are consistent with the experimental results and meet the accuracy requirements for engineering calculations.

### 5.2 Influences of Slender Ratio and Material Strength

Comparing the results of calculation formulas with the experimental results of specimens with different strength reinforcement, the precision of the calculation results is significantly related to the material strength. With the increase of material strength, the bearing capability might be increased for short HPC-HSS column. The optimal length of HPC-HSS column and the minimum of bearing capability were discussed for buckling and slender ratio of slender or moderate column.

### 5.3 Deficiency and Reflection

Some influences were simplified and neglected, the formulas might induce some deficiencies and inaccuracy from test data. The formulas were referenced to engineering design and analyzed to some characters. The range of slender ratio should be confirmed for the formula for buckling. The calculation model of bearing capability might should be more accurate on column with high-performance concrete and high-strength reinforcement.

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## REFERENCES

- GB50010-2010. *Code for design of concrete structures*. China Construction Press, Beijing. pp. 45-47. (In Chinese)
- ACI 318-19. 2019. *Building Code Requirements for Structural Concrete*. American Concrete Institute, Michigan. pp. 155-158.
- Zhang, B., Qiu, L., Chen, X., et al., 2021. Study on symmetric reinforcement calculation of rectangular section column with HRB600 reinforcing bars under eccentric compression. *Building Structure* 51(16): 116-120. (In Chinese)
- Gao, Z., 2023. *Experimental study on force performance of 630 MPA class high strength reinforced concrete column*. Hebei University of Technology, Tianjin. (In Chinese)
- Shen, Q., Wu, T., Wang, F., et al., 2024. Axially-compressed behaviour of concrete columns reinforced with novel high-strength steel-rebar materials: Testing, modelling and design. *Structures* 70: 107841p1-18.
- Ding, W., Deng, M., Li, T., et al., 2024. Experimental study on eccentric compressive performance of RC columns strengthened with textile-reinforced high-strength high ductile concrete. *Construction and Building Materials* 457: 139382p1-13.
- Jia, P. C., Wu, H., Peng, Q., et al., 2024. Dynamic behaviors of eccentrically loaded RC column under lateral low-velocity impact. *Engineering Structures* 309: 118031p1-21.
- Ma, L.-C., Shi, Q.-X., Wang, P., et al., 2024. High strength reinforced column monotonic load test eccentric compression performance design method. *Engineering Mechanics* 41(3): 82-90. (In Chinese)
- Mao, D., 2008. *Research on mechanical properties of reinforced concrete compressive members at 500Mpa reinforcement*. Zhengzhou University, ZHENGZHOU. (In Chinese)
- Luo, S., 2013. *Experimental study on the mechanical properties of eccentrically compressed members of reinforced concrete with 600Mpa grade reinforcement*. Southeast University, Nanjing. (In Chinese)