

Portfolio Design and Optimization Based on the CAPM Model

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Keywords: Portfolio Optimization, Asset Allocation, Risk Management, Capital Asset Pricing Model (CAPM), Conditional Value at Risk (CVaR).

Abstract: In the evolving landscape of global financial markets, traditional portfolio management approaches face challenges due to the rise of new asset classes and increasingly complex investment environments. This study examines the optimization of portfolio returns and risks by integrating traditional assets with emerging ones. This paper explores the optimization of portfolio returns and risks by combining traditional and emerging assets. The research uses data from assets (e.g., Apple, crude oil, Bitcoin, and SPY options), employing models including CAPM, the mean-variance model, and CVaR to determine the most efficient asset allocation. The results reveal that a portfolio consisting of 50% Apple, 10% crude oil, 30% SPY, and 10% Bitcoin achieves an expected annualized return of 8.32% with an annualized volatility of 8.46%. This allocation achieves a strong balance between risk and return, offering a solid foundation for optimizing portfolio strategies. This research highlights the significance of strategic asset allocation and sophisticated risk management, offering key insights for investors aiming for stable, long-term growth. Future research could further improve portfolio performance by incorporating real-time data and machine learning models, allowing for more adaptive and responsive investment strategies in the face of market uncertainties.

1 INTRODUCTION

Investors in the world's financial markets with a little bit of investment philosophy put their core focus on the asset allocation and portfolio management part of the equation (Fama & French, 2004; Jagannathan & McGrattan, 1995). Traditional theories of financial markets and traditional investment assets such as stocks have always been the mainstay of the investment market. Recent developments in the world have brought with them a diversification of financial instruments and ever closer international market relations. Modern asset portfolio theory (MPT) and capital asset pricing models have also begun to be gradually applied, but they have encountered certain limitations in dealing with the emergence of new asset classes and the complexity of the contemporary market environment (Markowitz, 1991). More recently, the growth of new asset classes, such as cryptocurrencies and futures, and their inclusion in investment portfolios have further raised questions about the applicability of these traditional financial theories. MPT offers a theoretical framework for asset allocation, primarily through mean-variance optimization. However, this theory hinges on the

assumptions of normally distributed asset returns and a stable covariance matrix, which may not hold true in markets characterized by non-linearity and extreme volatility. To address these shortcomings, the CAPM model was introduced, which evaluates the systematic risk and anticipated return of individual assets. Nevertheless, as market complexity escalates, particularly with the incorporation of new assets like cryptocurrencies and futures, traditional models like CAPM are increasingly challenged by issues such as non-linear data and the need for advanced risk management strategies.

In response to these challenges, researchers have developed several enhanced models and approaches, including the CVaR model and genetic algorithms. These innovations seek to address the shortcomings of traditional CAPM. For example, the CVaR model provides a more comprehensive risk management strategy by considering tail risks in extreme market conditions, while genetic algorithms excel in handling complex, non-linear optimization challenges.

The objective of this research is to investigate and assess how to optimize portfolio returns and risks by integrating various models within portfolios that

combine both traditional and emerging assets. With the rise of new assets (e.g., cryptocurrencies), traditional portfolio theory can no longer fully meet the needs of modern investors. Thus, by incorporating advanced tools such as the CAPM, the mean-variance model, and the CVaR model, this study aims to offer new insights and approaches for portfolio optimization both in theory and practice.

The study begins with a comprehensive literature review, examining research on portfolios containing both traditional and emerging assets, analysing their theoretical underpinnings and practical limitations. Following this, the data sources and methodologies employed are detailed, including the specific application and computational processes of each model. The study then proceeds to an empirical analysis to evaluate the performance of these models in portfolio optimization, discussing their validity and any limitations encountered. The findings are ultimately summarized, with recommendations and directions for future research provided.

2 DATA AND METHOD

The data sources for this study cover a number of important financial markets and trading platforms, and the time span for data selection is from August 1, 2019 to August 1, 2024, a total of five years. The data are presented on a weekly basis to ensure data diversity and accuracy. Specifically, for legacy asset Apple Inc (APPL) Stock Data: daily closing prices, trading volume, and other relevant data for Apple Inc stock are obtained through Yahoo Finance. A five-year time horizon provides ample historical data for yield and volatility analysis. For WTI Crude Oil Futures Data. The daily price data from CME Group. WTI crude oil futures, as the most critical energy commodity globally, are utilized to evaluate how fluctuations in the energy market influence investment portfolios. For emerging asset, Bitcoin (BTC) Data used daily closing price and volume data for Bitcoin via CoinMarketCap. A recent five-year time horizon was chosen to reflect the long-term trends and volatility of the Bitcoin market. SPY Options data collected from Yahoo Finance, covering a variety of strike price and expiration date data for S&P 500 ETF (SPY) options. This data is used for risk management and yield optimization. This data will be used to construct a diversified portfolio containing both traditional and emerging assets. The underlying assets are selected based on their market impact, liquidity, and risk-return characteristics. By introducing these assets, the study can analyze the

synergies between different asset classes and their impact on overall portfolio risk and return.

When conducting portfolio optimization, the selection of the objective function is vital, as it significantly influences risk-reward trade-offs the optimization process. The study uses the following objective functions to evaluate and optimize the portfolio:

- Maximize Sharpe ratio. The objective of this study is to optimize the Sharpe ratio in order to identify the portfolio with the highest return for a given level of risk. The Sharpe ratio is a risk-adjusted return measure that compares an asset's excess return to its volatility.
- Minimizing Risk (Variance). Risk minimization is one of the core objectives of modern portfolio theory. This study will create a low-risk portfolio by focusing on minimizing the portfolio's variance. This method is particularly well-suited for conservative investors, aiming to mitigate the effects of asset price fluctuations on the portfolio.
- Minimizing Conditional Value at Risk (CVaR). CVaR is an important metric used to assess the maximum loss under extreme market conditions. By minimizing CVaR, this study will construct portfolios that are less risky under extreme market conditions, which is especially important for portfolios that contain highly volatile assets such as Bitcoin.
- Maximize Return (Expected Return). In some cases, investors may be more concerned with the expected return of a portfolio. By maximizing expected returns, this study will evaluate the performance of high-yield portfolios in different market environments for aggressive investment strategies.

These objective functions will be applied in different models and methods, and optimization analysis will be performed by tools such as CAPM, mean-variance model and CVaR model. Through these optimization methods, this study will explore the optimal portfolio construction strategies and their effects under different investment objectives.

In analyzing asset returns, this study uses the symbols and definitions displayed in Table 1. The specific definitions and units of these symbols are listed in Table 1 for easy understanding and use.

Table 1: Symbol description.

Notation	Meaning
i	Asset
$E(R_i)$	Expected rate of return on asset i
R_f	Risk – free rate
β_i	β – coefficient for asset i
$E(R_m)$	Expected return on the market portfolio
R_i	Rate of return on assets
R_m	Market rate of return
P_t	Closing price of the index in week t
$Var(R_{market})$	Variance of market returns

In this study, data were collected for four assets: shares of Apple Inc (APPL), Crude Oil WTI Futures, S&P 500 ETF (SPY), and Bitcoin. Please refer to Appendix 1 for the specific data. In order to guarantee the accuracy of the data and the uniformity of the analysis, the following preprocessing steps were undertaken. First, the raw data was imported from various data sources, and relevant information such as Time (Date), Close and Volume were extracted, and other irrelevant columns were removed. For each asset data, the column names were standardized as Date, Close and Volume to facilitate subsequent data integration. The datasets for each asset were merged (outer join) by date (Date). During the merging process, it was ensured that all the data had the same time range and were aligned by Date. The resulting integrated data frame contains the closing price and trading volume for each asset on the same date. A small number of missing values appear in the merged data frame due to possible differences in trading times across assets. To fill in these missing values, a stepwise interpolation method is used. The missing data are first filled using the forward fill (pad) method, followed by applying the backfill method to the portions where missing values still exist, ensuring data integrity. To be able to reasonably compare the performance of different assets, the closing prices of each asset are normalized using the Min-Max normalization method. The standardized data converts the closing prices of each asset into the range of [0, 1] so that they can be compared and analyzed on the same scale. The above preprocessing steps result in a complete and standardized dataset that can

support subsequent analysis and research. Please refer to Appendix 2 for details.

For model assumption, it is assumed that the market operates efficiently, with all available information already incorporated into asset prices, so that there is no systematic information advantage. Assets are priced rationally, and investors cannot obtain excess returns through information asymmetry. The study assumes that market conditions are relatively stable over the study period, with no major structural changes or sudden systemic risks. This implies that historical data is effective in predicting future market behavior and that the risk and return characteristics of assets remain constant over the study period. If the asset's returns are normally distributed, the mean-variance optimization method can be applied. While returns may occasionally exhibit fat tails or skewness, this model assumes a normal distribution for simplicity. It is assumed that the correlation between different assets remains constant and exhibits minimal variation over time. As a result, the correlation parameters within the covariance matrix stay unchanged throughout the study. The study presumes that no transaction costs or tax liabilities are involved in portfolio adjustments, enabling investors to modify their asset allocation without the influence of transaction fees. The assumption of rational investor behavior: It is assumed that all investors are rational and that their investment decisions are based on an analysis of risk and return aimed at maximizing their utility function. This assumption excludes the impact of investor behavioral biases on market prices and portfolios.

3 RESULTS AND DISCUSSION

3.1 Calculation of Basic Indicators

To comprehensively evaluate the risk and return features of various assets, one computed and examined key financial metrics across several assets.

Table 2: Data preprocessing results

	Annualized mean return	Annualized variance	Annualized volatility	Sharpe ratio	Maximum retracement	CVaR (95%)
APPL	-4.74524	34.32520	5.85877	-0.809939	1.99435	-0.83951
Crude Oil WTI Futures Historica	13.75573	413.13013	20.32560	0.67676	36.32113	-0.75141
SPY	-13.17218	546.78701	23.38347	-0.56331	2.92006	-1.46487
Bitcoin	3.09303	25.18650	5.01861	0.61631	3.30931	-0.54200

Specifically, the following measures were calculated:

- Annualized mean return: Calculates the annualized average return for each asset, highlighting its performance over time.
- Annualized variance: The risk level is determined by evaluating the fluctuations in asset returns.
- Annualized volatility: As a measure of risk, denotes the standard deviation of return volatility.
- Sharpe ratio: Measure the performance of returns at the same level of risk under the assumption that the risk-free rate is 0%.
- Maximum retracement: An assessment of the maximum loss experienced by the asset during a retracement, showing the potential downside risk to the asset.
- CVaR (95%): Assesses potential losses under extreme market conditions to further characterize risk.

By computing and evaluating these metrics, one can gain a broader insight into the performance of each asset and establish a foundation for future investment decisions. Specific charts are shown in Table 2.

3.2 CAPM Modelling

The CAPM is a commonly employed tool in finance to assess the anticipated return of an individual asset or portfolio. It calculates the expected return by incorporating the market risk premium and connects the asset's systematic risk (beta) to the broader market's volatility. The fundamental principle of the CAPM is to assess an asset's exposure to market risk using the beta coefficient and subsequently calculate a fair expected return. In this study, the CAPM serve as a tool to analyze the potential losses that could be incurred by you in the market of individual assets and provide basic data for subsequent portfolio optimization. The CAPM model exhibits different validity under different market conditions, which is reflected in the results of this paper (Fama & French, 2004). To construct the CAPM model, this study uses weekly data for the past five years, including historical weekly return data for several assets, including Apple (APPL), WTI crude oil futures, SPY options, and Bitcoin (BTC). The historical returns of the market portfolio are represented using the S&P 500 index (S&P 500).

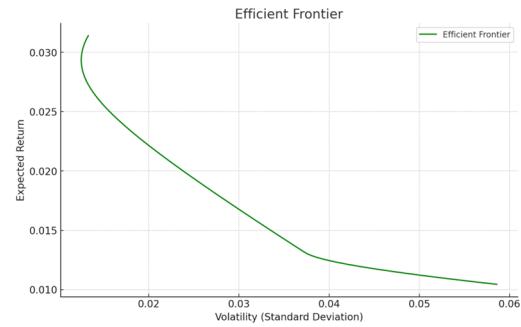


Figure 1: Effective Frontier (Photo/Picture credit: Original).

A 4.61% risk-free rate was selected, derived from the current yield on the 10-year U.S. Treasury bond, as shown in Appendix 3. In the CAPM model, the risk-free rate serves as a reference return, representing the minimum return investors would anticipate in the absence of market risk. Fig. 1 illustrates the portfolio's expected return across various risk levels.

The beta coefficient is utilized to assess an asset's systematic risk, reflecting its sensitivity to market-wide fluctuations. To derive the beta coefficient, one used historical S&P500 data from August 1, 2019, to August 1, 2024, with the source detailed in Appendix 3. The calculated market volatility was 1.33%, and the market's expected return was 3.14%:

$$R_{market} = \frac{P_t - P_{t-1}}{P_{t-1}} \quad (1)$$

$$\sigma_{market} = \sqrt{Var(R_{market})} \quad (2)$$

To assess the systematic risk of different assets relative to the market, this study calculates the beta coefficients of APPL, Crude Oil WTI Futures, SPY, and Bitcoin. The beta coefficients were calculated using the following formula:

$$\beta = \frac{Cov(R_{asset}, R_{market})}{Var(R_{market})} \quad (3)$$

Table 3 shows the beta coefficients of the four selected assets. The results show that APPL has a beta of 1.184, indicating higher volatility than the market, while Crude Oil WTI Futures has a beta of 0.897, indicating lower volatility than the market, and SPY and Bitcoin have a beta of 0.941 and 0.900, respectively, indicating that the volatility of the two is closer to the market.

Table 3: The β – coefficients.

Asset	Beta
APPL	1.184
Crude Oil WTI Futures Historical	0.897
SPY	0.941
Bitcoin	0.9

This study uses the Capital Asset Pricing Model (CAPM) to calculate the expected rate of return for each asset. The formulae for the CAPM model are as follows:

$$E(R_i) = R_f + \beta_i \times (E(R_m) - R_f) \quad (4)$$

Here, $E(R_i)$ is expected rate of return on asset i . R_f is risk-free rate. β -coefficient for asset i , indicates the systematic risk of the asset relative to the market. $E(R_m)$ is expected return on the market portfolio, $E(R_m) - R_f$ is the market risk premium.

In the previous section, one calculated the market return to be 3.14% and noted that the risk-free rate is 4.61%. Using the following formula, one derived a risk premium of -1.77%. This indicates that the expected market return is lower than the risk-free rate, suggesting that investors believe the risk associated with holding market assets (e.g., stocks) is not sufficiently compensated and may even result in a loss. Using the data above and the beta coefficients, one applied the CAPM formula to compute the annualized expected return for each asset. The results of these calculations are presented in Table 4.

Table 4: Expected returns.

Asset	Expected Return (%)
APPL	1.0443%
Crude Oil WTI Futures	1.5523%
SPY	1.4744%
Bitcoin	1.5470%

The analysis of the CAPM model indicates that an asset's expected return is directly linked to its beta coefficient, suggesting that a higher beta results in a higher expected return. This outcome aligns with financial theory, which posits that investors anticipate greater returns when taking on higher systematic risk. In this research, the market risk premium is negative, indicating the current market's pessimistic outlook on future returns. Under such market conditions, the expected return derived from the CAPM model is lower than the risk-free rate, implying that investors might favor risk-free assets to mitigate market risk.

By calculating the beta coefficients of each asset, the study reveals the differences in risk exposure of different assets to market volatility. For assets with

high beta coefficients (e.g., APPL), they have higher market risk and therefore higher expected returns. While for assets with low beta coefficients (e.g., Crude Oil WTI Futures), their market risk is relatively low and therefore the expected return is lower. By utilizing the CAPM model, this study uncovers both the expected return and market risk attributes of each asset, laying a strong foundation for subsequent portfolio optimization. Future research could integrate the mean-variance optimization model to delve deeper into constructing a risk-adjusted optimal portfolio under prevailing market conditions.

3.3 Mean-variance Optimization Model

To maximize the expected return for a given level of risk or minimize the risk for a given return target, this study adopts Mean-Variance Optimization (MVO) for portfolio optimization. Mean-Variance Optimization is the core method of modern portfolio theory, which helps investors construct optimal portfolios by optimally allocating the weights of different assets. The core concept of mean-variance optimization is to maximize expected returns by reducing the portfolio's variance. Diversification across multiple asset classes is effective in reducing overall portfolio risk (Goetzmann et al., 2005). In optimizing portfolios, this study aims to achieve an optimal trade-off between risk and return by employing a dual-objective strategy that focuses on maximizing the Sharpe ratio while minimizing volatility. However, a strategy based solely on Sharpe ratio maximization often tends to concentrate on a few high-return assets, resulting in an under-diversified portfolio. Therefore, this study further introduces diversification constraints to ensure that the portfolio is reasonably allocated among different assets.

To enhance portfolio diversification, minimum weight and maximum weight constraints are set for each asset in the study, i.e., the weight of each asset in the portfolio should not be less than 10% and not more than 50%. This constraint aims to prevent the portfolio from being overly dependent on a single asset and reduce concentration risk. The mean-variance model, despite its widespread use, exhibits limitations in dealing with nonlinear market behavior (Elton et al., 2007). After optimization, the portfolio weight allocation after considering the diversification constraint is as shown in Table 5.

Table 5: Portfolio weightings 1.

Asset	Data
APPL	30%
Crude Oil WTI Futures	50%
SPY	10%
Bitcoin	10%

The optimization results show that under the mandatory diversification constraint, the portfolio has an expected return of 0.2385%, a volatility of 3.34%, and a Sharpe Ratio of -1.399. Despite the low Sharpe Ratio, the overall volatility is effectively controlled due to the diversified allocation of the portfolio among different assets. With the introduction of the diversification constraint, the portfolio is no longer concentrated in just a single asset but has a reasonable allocation across multiple assets. While this allocation reduces the Sharpe ratio, it improves the stability of the portfolio and helps to reduce the impact of extreme market volatility in long-term investments. The choice of assets has some limitations at the same time. Bitcoin is a decentralized asset and can be highly volatile. Cryptocurrencies, especially Bitcoin, are considered speculative assets due to their high volatility (Baur et al., 2018). Meanwhile, commodities such as crude oil are key variables in investment portfolios due to their volatility (Cheung & Miu, 2010). It is their combination that makes this investment more possible.

3.4 Risk Management Optimization

In portfolio management, an effective allocation of asset weights is essential to strike a balance between risk and return. As described by Rockafellar and Uryasev (2000), CVaR models provide a more reliable risk management tool under extreme market conditions. To reduce the downside risk of the portfolio in a volatile market environment, this study optimizes the asset weights of four key assets by incorporating two critical risk measures, Maximum Drawdown (MDD) and Conditional Value at Risk (CVaR). This paper analyzes the maximum drawdown and CVaR data for four assets: Apple Inc. stock (APPL), Crude Oil WTI Futures, S&P 500 ETF (SPY), and Bitcoin. The initial weights are set to 30% for Apple stock, 50% for WTI Crude Oil Futures, 10% for SPY, and 10% for Bitcoin.

Effective risk management is key in financial markets (Jorion, 2006). To prevent the portfolio from being overly concentrated in any single asset and to maintain a balanced allocation across assets, the study

imposes constraints, setting a minimum weight of 10% and a maximum weight of 50% for each asset. This is intended to optimize portfolio diversification while controlling risk. In this study, an optimization method based on the SLSQP (Sequential Least Squares Programming) algorithm is used, with the objective function being to minimize the weighted sum of the portfolio's maximum retracement and CVaR. By adjusting the weights of each asset, the optimal portfolio allocation is obtained. In the weight optimization process, the final optimal weight allocation is given by considering the weight constraints of at most 50% and at least 10% for each asset as given in Table 6.

Following the optimization, the portfolio achieves an annualized expected return of 8.32% and an annualized volatility of 8.46%. This allocation balances risk management and returns, enabling the portfolio to perform more consistently across varying market conditions. By incorporating weight constraints in the optimization process, this study not only achieves effective risk control, but also ensures portfolio diversity. The optimized portfolio allocation can strike a good balance between risk and return and is suitable for investors seeking solid returns. Future research can further explore the strategy of dynamically adjusting weights under different market environments to cope with more complex market fluctuations.

Table 6: Portfolio weightings 2.

Asset	Proportion
APPL	50%
Crude Oil WTI Futures	10%
SPY	30%
Bitcoin	10%

3.5 Performance Indicators of the Portfolio

Besides analyzing individual assets, the portfolio's overall performance was assessed using several key financial indicators, including portfolio return, Sharpe ratio, and Calmar ratio. Table 7 presents a detailed overview of these indicators, offering insights into the portfolio's risk and return profile.

Table 7: Overall profile and descriptions.

Metric	Value	Description
Annualized Return	8.32%	Annualized return of the optimized portfolio
Sharpe Ratio	-1.399	Risk-adjusted return based on the portfolio volatility
Calmar Ratio	0.231	Return-to-risk ratio considering maximum drawdown.
Annualized Volatility	8.46%	Standard deviation of the portfolio's returns.
Maximum Drawdown	36.32%	Maximum observed loss from a peak to a trough
CVaR (95%)	-1.464	Expected loss in extreme market conditions

3.6 Limitations and Prospects

The limitations of this study are mainly in several aspects. First, Due to under dynamic market conditions, traditional static models may not adequately reflect actual market volatility (Campbell & Viceira, 2002). The study operates under the assumption that market conditions remain relatively stable and does not thoroughly account for the effects of dynamic factors like market sentiment and macroeconomic changes on investment portfolios. Second, the historical data used may not fully reflect future market volatility and risk, and thus the results of the study may be subject to a certain degree of uncertainty in practical application. Moreover, the analysis is concentrated on a narrow set of asset categories, excluding other investment instruments like bonds and real estate, which could reduce the overall diversification and risk management of the portfolio. Future research could further enhance portfolio performance by introducing real-time data analytics and machine learning models to adjust portfolios more dynamically in response to market changes and uncertainties.

4 CONCLUSIONS

To sum up, the goal of this study is to construct an optimized portfolio that maximizes risk-adjusted returns. By applying modern portfolio theory, including mean-variance optimization, Capital Asset Pricing Model (CAPM), Conditional Value-at-Risk (CVaR), and Maximum Drawdown Analysis, one determines the optimal asset allocation: 50% for Apple, 10% for Crude Oil, 30% for SPY, and 10% for Bitcoin. The portfolio has an expected annualized return of 8.32% and annualized volatility of 8.46%,

effectively balancing risk and return. The use of the CAPM model in asset pricing is widely supported. While this study provides a solid framework for portfolio optimization, it is limited by assuming static market conditions and excluding dynamic factors such as market sentiment. Modern portfolio theory plays an important role in asset allocation. Future research could explore the integration of real-time data and machine learning models to further improve portfolio performance. Although there are certain limitations, the research sheds light on the critical role of strategic asset distribution and effective risk management in securing consistent, long-term returns.

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APPENDIX

- Appendix 1: Original Data: <https://docs.google.com/spreadsheets/d/1W1QmlLgwWWc5PAI4q1RFyHpN2H8M5f7l/edit?usp=sharing&ouid=102167542405241696605&rtopf=true&sd=true>
- Appendix 2: Data cleaning: <https://docs.google.com/spreadsheets/d/1-RRsvqImCkYFhWrln5OI3lEj0ubl2cT/edit?usp=sharing&ouid=102167542405241696605&rtopf=true&sd=true>
- Appendix 3: Trespure data: https://ycharts.com/indicators/10_year_treasury_rate