The Role of Dark Matter in Galaxy Formation and Evolution

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Abstract: At the forefront of contemporary astrophysics, it is widely accepted that galaxy formation is directly attributed to the gravitational pull of dark matter halos. As the universe ages, dark matter not only observes the inception and growth of galaxies but also significantly impacts their evolutionary paths and morphological transformations. This paper seeks to explore how these enigmatic substances act as cosmic architects, shaping and propelling the galaxy formation process. Additionally, we will explore how dark matter orchestrates the large-scale structural network of the universe, including the formation and evolution of galaxy clusters and superclusters, highlighting its pivotal role in the grand scheme of the cosmos. Additionally, the article will delve into the various approaches currently employed by the scientific community to detect dark matter and develop its theoretical models. This ranges from direct detection experiments conducted in precise laboratories to indirect detection utilizing astronomical observation techniques, including the gravitational lensing effect.

1 INTRODUCTION

Dark matter is a mysterious form of matter in the universe, whose existence and properties are essential for understanding the structure, evolution, and composition of the cosmos. It refers to a form of matter that cannot be directly observed via electromagnetic radiation, but its presence can be inferred indirectly from its gravitational effects. In long-term observations of celestial motions, astronomers have discovered that the motion and gravitational structures of many celestial bodies do not adhere to existing gravitational models. Hence, they postulate the existence of an invisible substance pervasively distributed across numerous galaxies, star clusters, and the broader universe, contributing to gravitational forces and influencing the evolution of cosmic structures across various scales. Its mass significantly exceeds the total of all visible celestial bodies in the universe. By integrating anisotropic observations of cosmic microwave background radiation with standard cosmological models, it is determined that dark matter comprises 85% of the universe's total mass and 26.8% of its total energy (Ade, 2016). Currently, scientists continue to investigate dark matter and its properties using methods such as gravitational lensing and computer simulations.

2 FUNDAMENTAL AND PROPERTIES OF DARK MATTER

Although the properties of dark matter remain poorly understood, scientists continue to explore and study it using various methods. Regarding the properties of dark matter, since it cannot be directly observed, we generally agree that it neither emits nor reflects light and therefore cannot be observed through optical means. This explains why there are no effective methods to detect dark matter. Among the current methods used to observe the universe, only gravitational wave detection involves capturing electromagnetic radiation. Fortunately, dark matter possesses gravity, being one of the primary sources of gravitational force in the universe, significantly influencing the structure and dynamical behavior of celestial systems like galaxies and galaxy clusters. Additionally, dark matter has mass and therefore inertia. Consequently, its presence can be indirectly

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inferred through the observation of gravitational lensing effects. Although the mass of individual dark matter particles remains undetermined, this is due to the uncertain particle properties of dark matter itself. Dark matter is exceedingly stable and rarely transforms into other forms, changing only under extreme conditions such as during the universe's early formation or its eventual contraction. Notably, in addition to gravity, particle-type dark matter may engage in weak interactions, but it largely lacks participation in electromagnetic or strong interactions. This restricts the interactions between dark matter and visible matter, which may contribute to the remarkable stability of dark matter itself. Furthermore, the prevailing view within the scientific community is that the velocity of dark matter significantly exceeds the speed of light.

2.1 Evidence for Dark Matter from Observational Studies

In 1922, astronomer Jacobs Kaptin first proposed the concept of the possible existence of "dark matter", who suggested that the motion of celestial systems could indirectly infer the presence of unseen matter surrounding the celestial bodies. Although dark matter has not yet been directly detected, substantial evidence indicates its widespread presence in the universe, including from galaxy rotation curves and velocity distributions. dispersion In 1933 astrophysicist Fritz Zwicky employed spectral redshifts to measure the velocities of galaxies within the Coma Cluster relative to the cluster itself (Zwicky, 1933). Observations of visible matter mass distributions in spiral galaxies and calculations based on the law of universal gravitation suggest that outer celestial bodies should orbit the galaxy's center more slowly than those near the center. However, measurements of the rotation curves of numerous spiral galaxies reveal that outer celestial bodies orbit at velocities nearly equal to those of inner celestial bodies, significantly higher than expected. This implies the existence of massive unseen matter within these galaxies. Using the potential force theorem, the distribution of matter within a galaxy can be deduced from the dispersion velocity distribution of its visible celestial bodies.

There are currently three mainstream methods for determining the mass distribution of galaxies or galaxy clusters. (1) Analysis the motions of galaxies or clusters using gravitational theory. In 1939, astronomer Horace W. Babcock examined the spectra of the Andromeda Nebula, revealing that the rotational speeds of celestial bodies in the outermost

regions were significantly greater than those predicted by Kepler's law, indicating a higher massto-light ratio (Babcock, 1939). This implies the presence of substantial dark matter within the galaxy. (2) Observing the X-rays emitted by galaxy clusters. Galaxy clusters commonly contain hot gases that emit X-rays. Once these gases achieve hydrodynamic equilibrium within the cluster's gravitational field, they will emit different temperatures due to uneven distribution of mass (3) The gravitational lensing effect, a subtle prediction of general relativity, demonstrates that the light from the back of a galaxy cluster is bent as it passes through massive clusters. By examining the extent and pattern of this light bending, scientists can infer the distribution of matter within the galaxy cluster, even though much of this matter does not emit visible light.

At the cosmic scale, the total amount of dark matter in the universe can be determined by precisely measuring the anisotropy of cosmic microwave background radiation. Observations reveal that 26.8% of the universe's total energy is attributed to dark matter, compared to only 4.9% from traditional matter comprising celestial bodies and interstellar gases(Ade, 2016). Large-scale computer simulations of cosmic evolution using n-body gravity modeling highlight the critical importance of dark matter particle properties for their aggregation behavior (Melott, 1983). The simulations reveal that slowmoving dark matter particles tend to clump together more readily under gravity, thereby facilitating the formation of large-scale structures (White, 1983). Conversely, particles moving at high speeds, such as neutrinos, tend to disrupt or smooth out these structures due to their rapid motion (White, 1987), and thus are not considered the primary candidates for dark matter.

2.2 Theoretical Models of Dark Matter

Dark matter has been proposed since 1933, and astrophysicists and particle physicists have raised considerable possibilities for having models about dark matter, such as the weakly interacting massive particles, the Axion model, Graitino model, the neutrino model, the sterile neutrino model and so on. For the convenience of organization, based on their inherent velocity dispersion and temperature differences, different types of particle models are classified into (1) cold dark matter, namely WIMPs model, Axion model. (2) warm dark matter, represented with sterile neutrino and Graitino models. (3) hot dark matter, such as the neutrino which have own mass.

3 THE IMPACT OF DARK MATTER ON GALAXY FORMATION

In the modern physics cosmological model, dark matter is crucial for galaxy formation and evolution. Following decades of meticulous observations and simulations, the dark matter model has been solidly established. During the first second of the inflationary epoch, fluctuations in matter distribution began to influence gravity, leading to a gradual decrease in its stability. Over time, the originally well-mixed dark matter and gas separate due to the fluctuating gravity, with the gas gradually descending toward the center of the dark matter halo. Some dark matter halos are so large that the gas can cool significantly at their core, enabling the formation of stars and eventually a proto-galaxy. As dark matter halos merge, the energy dynamics within the galaxy influence the formation of future stars.

3.1 Dark Matter Halos



Figure 1: Composite image (of the full TNG100-1 box) All the gravitationally collapsed structures (in orange/white) are surrounded by successive shock surfaces (blue) which encode their formation histories.

Due to the fact that dark matter only participates in interactions with gravity, it collapses and aggregates to form large-scale structures resembling fibers compared to other baryonic matter. We will transform these elliptical spherical dark matter structures with deep potential wells into dark matter halos.

3.2 Halo Mass Function and Its Implications for Galaxy Formation

The halo mass function plays a pivotal role in astrophysics and cosmology. Given a halo's redshift and known mass, its density distribution can be derived from its mass function (Jeremy, 2008).

Press-Schechter Formalism (PS theory) is one of the earliest models used to describe the halo mass function. Based on Gaussian random field theory, this model considers the smoothed density perturbation field as a Gaussian random field with a mean of 0 and a variance of $\sigma^2(M)$, predicting the formation of dark matter halos by identifying regions where the perturbation exceeds the critical density δc .

Extended Press-Schechter Formalism (EPS theory): An extension of the PS theory that incorporates the merging history of halos, allowing for a more precise description of the evolution of the halo mass function.

Sheth-Tormen Formalism (TS theory): A refinement of the PS theory that incorporates additional parameters to more accurately align with numerical simulation results.

As the seed of galaxy formation, the mass distribution and abundance of dark matter halos directly determine the rate and quantity of galaxy formation. In the early universe, small mass dark matter halos first formed and gradually merged to form larger mass halos as the universe expanded and evolved, triggering the formation of galaxies.

The halo mass function also regulates the properties of galaxies by influencing internal physical processes such as star formation, gas cooling, feedback mechanisms, etc. For example, high mass halos typically retain more gas for star formation, while low mass halos may lose most of their gas due to feedback, leading to a decrease in star formation efficiency. By studying the halo mass function under different redshifts, we can understand the evolution of galaxies in different cosmic periods. For example, as the universe expands and cools, the merging activity of dark matter halos becomes more frequent, and galaxies also undergo complex processes such as mergers, interactions, and evolution.

3.3 Hierarchical Structure Formation

In the standard cold dark matter (Λ CDM) cosmological model, the complex structures in the universe are evolved from tiny quantum fluctuations that gradually grow and evolve due to gravitational forces during the period of cosmic inflation. This

theoretical framework provides a theoretical basis for the hierarchical growth of dark matter halos.

In the standard cold dark matter (ACDM) cosmological model, the complex structures in the universe are derived from tiny quantum fluctuations that progressively grow and evolve through gravitational forces during cosmic inflation. This theoretical framework underpins the hierarchical growth of dark matter halos. After the Big Bang, dark matter was nearly evenly distributed throughout the universe, albeit with minor density variations. These slight density variations gradually coalesce under gravitational influence, forming small dark matter clumps known as the initial dark matter halos. As the universe expands and cools, the density fluctuations of dark matter halos intensify, transitioning into a nonlinear growth phase.

During this phase, gravity predominates, leading to the further collapse of less dense dark matter halos into denser ones in response to fluctuations in density. Small dark matter halos evolve into larger ones through the merging and accretion of other halos or dark matter particles, following a bottom-up evolutionary model where lower-mass dark halos form first and then coalesce into higher-mass ones. Typically, smaller halos form and nest within larger ones. This structure resembles the growth of trees, with small branches evolving into larger ones. Within this hierarchical structure, dark matter halos of varying masses have distinct formation epochs and evolutionary histories. Generally, lower-mass halos form earlier while higher-mass ones form later.

This is attributed to the smaller density fluctuations required for lower-mass halos, which facilitate their formation in the early universe, whereas higher-mass halos necessitate greater density fluctuations and longer evolutionary periods.

3.3.1 Hierarchical Structure Formation's Impact on Galaxy Formation

As previously mentioned, dark matter halos create gravitational potential wells for normal matter, such as gas and dust, enabling their aggregation and cooling, which in turn leads to the formation of stars and galaxies. Without the gravitational influence of dark matter halos, stable structures of normal matter would be difficult to form in the universe.

The shape and distribution of dark matter halos influence the morphology of galaxies. For instance, an irregular or asymmetrically structured dark matter halo could potentially stretch the galactic disk, resulting in the distortion or deformation of the galaxy's shape. Moreover, mergers of dark matter halos can trigger interactions and collisions between galaxies, thereby further impacting their morphology and dynamical properties.

The hierarchical merging process of dark matter halos significantly drives galaxy evolution. Through merging and accretion, dark matter halos grow and accumulate mass, thereby influencing star formation rates, gas distribution, and the dynamical states within galaxies. Additionally, the evolution of dark matter halos interacts with feedback processes, such as stellar winds and supernova explosions in galaxies, collectively shaping the course of galaxy evolution.

On larger scales, the hierarchical merging of dark matter halos also facilitates the formation of galaxy clusters and supermassive galaxies. The merging and interaction of multiple dark matter halos can create large galaxy clusters comprising hundreds or even thousands of galaxies. Within these clusters, galaxies gravitationally interact and merge, further enhancing galaxy formation and evolution.

3.4 The Formation of Clusters and Super Clusters

As described in section 3.3.1, the existence of dark matter allows star clusters to retain a stable structure. While significant, the gravitational interactions between visible matter, such as stars and gas, within star clusters, are insufficient to fully maintain their stability. The gravitational pull of dark matter compensates for this deficiency, enabling star clusters to preserve their shape and structure over extended periods. Dark matter facilitates star formation through gravitational dynamics. During the formation of star clusters, dark matter accumulates in the central regions, creating dense dark matter halos. The gravitational pull of these halos accelerates the collapse and cooling of nearby gas, which in turn initiates the star formation process.

Just as in star clusters, the stability of super clusters also relies on the gravitational pull of dark matter. The gravitational interactions between galaxies and galaxy clusters in super clusters, while crucial, are insufficient to fully sustain their stability. The gravitational influence of dark matter fills this role, allowing super clusters to maintain their overall shape and structure over long periods. Dark matter further influences the evolutionary process of super clusters. Using observational methods such as gravitational lensing, scientists can deduce the distribution and evolution of dark matter within super clusters. These observational findings enhance our understanding of the evolutionary history and future trends of super clusters.

4 DARK MATTER AND GALACTIC EVOLUTION

Before the discovery of dark matter, it was commonly assumed that all visible matter constituted the entire mass of the universe; this misconception resulted in substantial discrepancies between observed and theoretical estimates of cosmic mass. Dark matter constitutes an invisible yet massive component of galaxies, with a mass significantly exceeding that of visible matter like stars and gas. Consequently, a higher proportion of dark matter directly increases the total mass of the galaxy. The gravitational pull of dark matter is one of the primary gravitational forces in galaxies. As the proportion of dark matter rises, the gravitational pull of galaxies intensifies significantly, which in turn influences the motion and distribution of matter within them.

As discussed in section 3.3.1, dark matter halos serve as the structural framework for galaxy formation and morphology. Thus, dark matter stabilizes stars and gas within galaxies via its gravitational pull, ensuring their sustained orbital motion. In other words, a higher proportion of dark matter leads to increased galaxy rotation speeds, further enhancing their stability. Dark matter is more uniformly distributed than ordinary matter, and a greater proportion of it causes the galaxy's center of gravity to be more aligned with the center of dark matter distribution. This leads to alterations in the galaxy's shape, resulting in a flatter and more extended configuration. The growing proportion of dark matter influences the distribution and movement of gas and dust within galaxies. For instance, the gravitational influence of dark matter may concentrate gas and dust closer to the galaxy's center, which in turn impacts the formation and evolution of stars within these galaxies.

The galaxy rotation curve illustrates the relationship between the velocities of stars and their orbital distances within the galaxy. Normally, as one moves farther from the galaxy's center, the velocity of stars should decrease as the gravitational pull from the center diminishes. However, observations reveal that many galaxies' rotation curves do not conform to this expectation, particularly in the outer regions where the stellar rotational velocities do not significantly decrease and even show an upward trend in some cases. This phenomenon is referred to as "rotation curve anomaly" or "high-velocity rotating galaxies". The presence of dark matter offers a plausible explanation for this phenomenon (Rubin, 1980). Since dark matter does not emit electromagnetic radiation, it cannot be directly

observed, but it influences the stars and gas in galaxies through gravitational forces. In the outer regions of galaxies, where the mass of visible matter is minimal, the mass of dark matter remains substantial, exerting sufficient gravitational force to sustain high stellar velocities, resulting in a flat or even rising rotation curve, it is generally believed that baryonic matter, which is difficult to detect, does contribute to some of the dark matter effects, but evidence suggests that it comprises only a small fraction (Graff, 1996, Najita, 2000).

In galaxy clusters, the gravitational pull of dark matter acts to counterbalance the centrifugal forces, thereby maintaining their stability. Likewise, within galaxies, the gravitational force of dark matter supports their stability, preventing their disintegration from the centrifugal motion of internal components. Uneven distribution of dark matter creates imbalances in the tidal force fields within galaxies. However, it is this uneven tidal force field that, to some extent, helps stabilize galaxy structures. It helps prevent massive tidal disruption of galaxies when disturbed by external forces, such as the gravitational pull from neighboring galaxies.

4.1 Feedback Mechanisms

Two primary and frequently employed feedback mechanisms in simulation analysis are supernova feedback and ANG feedback Supernova feedback primarily harnesses the immense energy and materials expelled during supernova explosions. These energies and materials heat and disperse the adjacent baryonic gas, creating supernova winds. These winds can subsequently influence the gas distribution and star formation rate within the galaxy. Additionally, supernova explosions can trigger intergalactic gas flows and the formation of galaxywide winds, thereby impacting the distribution of baryonic matter on a larger scale. Active galactic nuclei (AGNs), powered by supermassive black holes at the centers of galaxies, are intense radiation sources. AGNs heat, disperse, or expel the baryonic matter within galaxies by emitting massive amounts of energy in the form of jets and radiation (Risa, 2018) Similar to supernova feedback, this mechanism profoundly influences the gas dynamics, star formation rate, and morphology of galaxies. For instance, the energy emitted by AGNs can inhibit star formation in the central regions of galaxies while promoting gas cooling and star formation in the outer regions.

5 OBSERVATIONS OF DARK MATTER IN GALAXIES

Observations of dark matter within galaxies represent a pivotal area of research in contemporary astronomy and physics. Since dark matter neither emits electromagnetic waves nor interacts strongly with ordinary matter electromagnetically, it cannot be directly observed using optical or electromagnetic methods. Nevertheless, scientists have devised multiple indirect methods for observing and detecting dark matter, which are especially critical for observations at the galaxy scale.

Scientists infer the presence of dark matter in galaxies by observing how the rotational speed of stars, gas, and other substances varies with distance from the galaxy's center. Based on Newton's law of gravity and dynamical theory, if a galaxy contained only the mass of its visible matter, the rotational speed of material distant from the center should decrease. However, empirical observations reveal that the rotation curves of many galaxies in the universe remain flat beyond the central region, suggesting the presence of additional mass, specifically dark matter, as noted in section 4.

Another notable example is the gravitational lensing effect. This phenomenon was predicted by Albert Einstein's theory of general relativity and independently confirmed by Arthur Eddington in 1919 through the observation of star displacements during a total solar eclipse. As light from distant galaxies passes through foreground galaxies or clusters, it is bent by the gravity of the dark matter, creating the gravitational lensing effect. Scientists deduce the distribution and mass of dark matter in galaxies or clusters by analyzing the bent light.

5.1 Simulations and Modeling

The numerical simulation of galaxy formation using dark matter is an important field in cosmology, which relies on complex computer simulations to understand and predict the formation and evolution of large-scale structures in the universe, such as galaxies, galaxy clusters, etc.

Firstly, a theoretical framework for simulation must be established based on general relativity and cosmological principles. The standard cosmological model (ACDM model) is typically employed as it encompasses dark matter, dark energy, ordinary matter, and radiation, enabling comprehensive simulation across all scales. Simulations often begin with observational data from cosmic microwave background radiation shortly after the Big Bang, which offer insights into the early state of the universe.

Similarly, the initial amplitude and distribution of density fluctuations, as described in section 3.3, are among the key initial parameters for simulations. Nbody simulation is the most prevalent method among scientists, where dark matter particles interact and evolve into large-scale structures under the influence of gravity. Besides N-body simulations, grid methods are also utilized to simulate the dynamics of dark matter and gases.

The simulation process involves extensive gravity calculations, commonly employing the Fast Fourier Transform (FFT) or tree-based algorithms, such as the Barnes-Hut algorithm, to compute gravitational interactions among particles. Another significant factor influencing simulation outcomes is the feedback among celestial bodies. Although dark matter does not engage in interactions other than gravity, the feedback effects of gas, stars, and black holes in simulations, such as supernova explosions, galaxy winds, and active galactic nuclei, can impact the structure and evolution of dark matter halos.

After the simulation is completed, it is also important to extract data from the simulation and analyze information such as the formation time, mass, morphology, and kinematic characteristics of the galaxy. This work primarily facilitates the comparison between simulation results and observational data, such as galaxy redshift surveys and cosmic microwave background radiation observations, and verifies the model's accuracy. Some individuals employ visualization tools to render simulation results as images or animations, aiming to provide a more intuitive understanding of galaxy formation and evolution.

6 CHALLENGES AND OPEN QUESTION

An an important concept in cosmology, dark matter is currently facing a lot of problems and challenges in many aspects.

6.1 Small-Scale Structure Problems and the Missing Satellites Problem

After extensive numerical simulations, it was found that dark matter forms numerous halos and substructures in the universe. However, the number of galaxies observed by scientists today is significantly lower than that predicted by simulation results, a phenomenon known as the 'small-scale problem'. Similarly, the Milky Way is expected to be surrounded by a multitude of satellite galaxies. However, the actual number of observed satellite galaxies is markedly lower than theoretical predictions, a discrepancy known as the 'missing satellite problem'.

6.2 The Impact of Baryonic Physics on Dark Matter Distribution

At scales less than 1h-1Mpc (million parsecs), baryonic processes significantly influence the clustering of dark matter. This influence is especially pronounced at smaller scales, like 0.1h-1Mpc, where gas adiabatic processes can substantially boost dark matter clustering. Moreover, baryonic processes including radiative cooling, star formation, and dynamical supernova feedback in the universe can modify the mass distribution of dark matter halos, particularly under the influence of active galactic nucleus feedback mechanisms, which amplify this effect. For instance, the feedback from active galactic nuclei discussed in 4.1 might inhibit the formation of massive dark matter halos. Theoretically, baryonic matter also influences the morphology of dark matter halos. As the universe evolves, the spatial morphology of dark matter halos transitions gradually from flat to rounded. The presence of baryonic matter hastens this transformation, further rounding the shapes of dark matter halos. However, feedback from active galactic nuclei can mitigate this acceleration.

7 PLANNED TASKS AND EXPECTATIONS FOR THE FUTURE

The Large Synoptic Survey Telescope (LSST) is a planned ground-based telescope designed for deep and wide-field continuous observations of the sky, aimed at unveiling the large-scale structure of the universe, the nature of dark matter and dark energy, and the behavior of transient celestial objects. LSST will feature an unprecedented wide field of view, enabling it to cover extensive regions of the sky. Additionally, by utilizing its high sensitivity to observe extremely faint celestial objects, LSST will reveal deeper mysteries of the universe. In terms of its mission, LSST is designed to perform long-term continuous observations to capture the dynamic changes in celestial bodies. (2) The James Webb Space Telescope (JWST), a next-generation spacecraft developed by NASA and space agencies in other countries, was successfully launched in 2021. It has now superseded the Hubble Telescope as a pivotal tool for exploring the universe's depths. JWST is outfitted with state-of-the-art instruments like NIRCam and MIRI, which facilitate advanced spectral and imaging analyses.

In the future, it is necessary to continue to improve the sensitivity and accuracy of direct and indirect detection techniques for dark matter particles, in order to obtain more direct evidence about dark matter.

Based on new observational data and experimental results, continuously improve and revise existing dark matter theoretical models to better explain dark matter phenomena in the universe.

8 CONCLUSIONS

In general, dark matter plays an indispensable role in the formation and evolution of galaxies. Dark matter, in its high-density form, generates substantial gravitational forces that attract adjacent matter, including visible substances like gas and dust. These gravitational forces cause surrounding matter to gradually accumulate, forming primitive nebulae that evolve into celestial entities such as stars and galaxies. Additionally, because dark matter does not interact with electromagnetic radiation, its distribution is uniformly spread across vast spatial scales. This distribution pattern ensures that galaxies experience consistent gravitational forces during their formation, leading to relatively regular and symmetrical structures. The gravitational influence of dark matter not only fosters the formation of galaxies but also sustains their stability. Its presence allows the internal matter within galaxies to maintain a relatively stable state of motion, preventing excessive contraction or disintegration due to internal gravitational forces. In larger-scale systems such as galaxy clusters, the gravitational influence of dark matter also facilitates interactions and mergers among galaxies. These processes profoundly influence the formation and evolution of galaxy clusters.

Through the study of dark matter, we can deepen our understanding of the universe's structure and evolution. This avenue of research also advances the fields of fundamental and particle physics, unveiling novel physical phenomena and laws. In summary, the exploration of dark matter and its halos represents a pivotal area of development in astrophysics. IAMPA 2024 - International Conference on Innovations in Applied Mathematics, Physics and Astronomy

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