# Frequency Fitness Assignment: Optimization Without Bias for Good Solution Outperforms Randomized Local Search on the Quadratic Assignment Problem

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Abstract: The Quadratic Assignment Problem (QAP) is one of the classical $\mathcal{NP}$-hard tasks from operations research with a history of more than 65 years. It is often approached with heuristic algorithms and over the years, a multitude of such methods has been applied. All of them have in common that they tend to prefer better solutions over worse ones. We approach the QAP with Frequency Fitness Assignment (FFA), an algorithm module that can be plugged into arbitrary iterative heuristics and that removes this bias. One would expect that a heuristic that does not care whether a new solution is better or worse compared to the current one should not perform very well. We plug FFA into a simple randomized local search (RLS) and yield the FRLS, which surprisingly outperforms RLS on the vast majority of the instances of the well-known QAPLIB benchmark set.

## 1 INTRODUCTION

The Quadratic Assignment Problem (QAP) is a challenging and very important combinatorial optimization problem (Koopmans and Beckmann, 1957; Burkard et al., 1998; Loiola et al., 2007). Here, the goal is to assign a set of $n$ facilities to a set of $n$ locations. Such an assignment can be represented as a permutation $s$ of the first $n$ natural numbers, where $s[i]$ specifies the location where facility $i$ should be placed. For each QAP, a distance matrix $A$ is given, where $A_{pq}$ specifies the distance from location $p$ to location $q$, as well as a flow matrix $B$, where $B_{ij}$ is the amount of material flowing from facility $i$ to facility $j$. The objective function $f$ subject to minimization then rates a permutation $s$ as follows:

$$f(s) = \sum_{i=1}^{n} \sum_{j=1}^{n} A_{s[i]s[j]} B_{ij} \tag{1}$$

The QAP has a wide range of applications including, e.g., building layout (Elshafei, 1977; Çubukçuoğlu et al., 2021; Krarup and Pruzan, 1978), keyboard layout (Burkard and Offermann, 1977), circuit design (Eschermann and Wunderlich, 1990), wiring (Steinberg, 1961), and scheduling (Soroush, 2011). While there has been notable success in applying exact methods to the QAP (Drezner et al., 2005), QAPs are $\mathcal{NP}$-hard (Sahni and Gonzalez, 1976; Dréo et al., 2006) and thus are often solved with heuristic algorithms such as simulated annealing (Thonemann and Bölte, 1994; Wilhelm and Ward, 1987), tabu search (Skorin-Kapov, 1990; Taillard, 1991; Misevičius, 2005; Misevičius, 2008), iterated local search (Stützle, 2006), evolutionary methods (Horng et al., 2000; Taillard and Gambardella, 1997), memetic algorithms (Fleurent and Ferland, 1993; Merz and Freisleben, 1999), estimation of distribution algorithms (Zhang et al., 2006), ant colony optimization (Gambardella et al., 1999; Talbi et al., 2001; Taillard and Gambardella, 1997), or even particle swarm optimization (Hafiz and Abdennour, 2016).

*All* such heuristic approaches that have been applied to the QAP have one design principle in common: Their (iterative) search procedure is biased towards good solutions. Regardless of whether they employ diversity strategies or methods to increase exploration, on average over time, they do prefer (to

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exploit) better solutions (in terms of their objective value) over worse ones. Indeed, this is maybe the most fundamental concept of metaheuristic optimization.

In (Weise et al., 2014), a mechanism called Frequency Fitness Assignment (FFA) was proposed, which was later shown to render optimization processes invariant under all injective transformations of the objective function value (Weise et al., 2021b) and, as a result, removing the bias towards better solutions (Weise et al., 2023). By replacing the objective value $f(s)$ of a solution $s$ with its encounter frequency $H[f(s)]$ in all selection decisions of a heuristic, FFA offers this new concept of optimization, which breaks with the existing ideas upon which all metaheuristics are built. The only algorithms that have similar properties are random walks, random sampling, and exhaustive enumeration – none of which are ranked as good approaches to the QAP. FFA has been shown to improve the performance of a randomized local search (RLS) on the Max-Sat problem (Weise et al., 2021b; Weise et al., 2023), the Job Shop Scheduling Problem (JSSP) (Weise et al., 2021a; de Bruin et al., 2023), and on Traveling Salesperson Problem (TSP) instances (Liang et al., 2022; Liang et al., 2024).

However, whether it can improve algorithm performance on a wide set of QAP instances has not yet been studied. In this work, we do not aim to outperform any of the related heuristics listed above. It instead is our goal to establish that FFA is indeed a suitable technique for the QAP. Our **first contribution** is to conduct the first large experiment of FFA on the QAP involving all instances from the QAP benchmark set QAPLIB by (Burkard et al., 1997). We publish all of our code, results, as well as the scripts used for generating the tables and figures in an immutable online archive at https://doi.org/10.5281/zenodo.13324662. As a **second contribution**, we show that, if plugged into a simple RLS, FFA yields a significant improvement in the quality of the discovered results. We show that, despite using a computational budget 100 times smaller than in prior works on FFA, this tangible improvement can be observed.

Finally, our **third contribution** is to provide lower bounds $m$ for the numbers $M$ of possible different objective values for all instances of the QAPLIB. While lower bounds $lb$ for the objective function $f$ exist (Peng et al., 2010; de Klerk and Sotirov, 2010; Drezner et al., 2005), we are the first to investigate $m$ on the QAPLIB instances. This lower bound $m$ can give us an impression about other aspects that may be relevant for optimization and may be related to the amount of neutrality present.

The rest of our work is structured as follows. In Section 2, we discuss related works both on FFA and the QAP before defining the algorithms used in our study in Section 3. In Section 4, we present the results of our experiment before concluding the paper in Section 5 with a summary and outlook on future work.

# 2 RELATED WORK

## 2.1 Related Works on the QAP

A wide variety of heuristics has been applied to the QAP, which differ in their algorithmic design philosophies, search strategies, operators, and parameters (Dréo et al., 2006). In this work here, we investigate whether the new paradigm FFA is applicable to the QAP. Beating the state of the art is not our goal. Nevertheless, it is important to at least provide a brief overview of some of the diverse historical heuristic solution ideas for the QAP.

(Wilhelm and Ward, 1987) studied the application of simulated annealing to the QAP. They showed that the simulated annealing algorithm produces good results but is sensitive to the setting of parameters and tested the effect of several parameters on the performance of the algorithm and CPU usage time.

(Taillard, 1991) developed a robust tabu search algorithm for the QAP, which today still is considered as competitive. It explores the neighborhood of the current solution by pairwise exchanges. The aspiration criterion allows forbidden moves if they produce a solution better than the best so far one. A subset of the QAPLIB instances with scales from 5 to 100 were used to investigate the algorithm performance.

Soon thereafter, (Fleurent and Ferland, 1993) presented a hybrid genetic algorithm, which combines the population-based evolutionary heuristic with local search. In traditional genetic algorithms, the quality of individuals can only be improved by crossover, mutation, and other operators. However, hybrid genetic algorithms can improve the solution also by local search or even tabu search. In experiments on the sko-class of instances (Skorin-Kapov, 1990) with scales up to 100, the hybrid algorithm outperformed its component algorithms. (Merz and Freisleben, 1999) introduced a memetic algorithm (MA), which, basically, is another hybrid evolutionary algorithm. The experiment was based on another subset of the QAPLIB instances and the MA outperformed several other heuristics on all instances of practical significance (i.e., except for the randomly generated ones).

In the same year, (Gambardella et al., 1999) presented an ant colony system hybridized with a local search. A comprehensive comparison experiment on several QAPLIB instances with scales $n$ between 19 and 90 showed that this algorithm performs especially well on irregular problems (that is, instances whose distance and/or flow matrix contain disparate values) and representative real-world instances.

(Horng et al., 2000) applied an evolutionary strategy (ES) to the QAP. In order to prevent premature convergence to local optima, this method adds the concept of clustering and family competition to the population handling. The resulting higher diversity leads to good performance on instances with $n \in 19 \dots 90$. In this work we take the alternative approach of FFA, which – different from the clustering-based idea of that work – does not require any population. Also, diversity is often considered from the search space perspective, whereas FFA tries to create diversity in the objective space in the hope that this induces diversity also in the genotypic representation of the solutions.

As maybe the last of these historical research directions to approach the QAP, (Hafiz and Abdennour, 2016) proposed a discretization framework for particle swarm optimization. This continuous optimization technique, too, can produce good results on the QAP.

Some of the above algorithms, like tabu search or the ES, introduce methods to increase the diversity of the solutions under investigation. Thus, they have components that try to prevent the algorithms from converging to local optima. However, all of them prefer better solutions over worse ones. In the following section, we therefore discuss why FFA is a uniquely different approach to diversity and optimization and why investigating its performance on the QAP is necessary.

It should be noted that in (Thomson et al., 2024), we applied fitness landscape analysis to FFA on the taie27 set of 20 QAP instances of the same scale $n = 27$, which are not part of QAPLIB. In that paper, our goal was to explain why and how FFA-based search works. We presented visualizations of metrics for algorithm trajectories which substantiate the good exploration ability of FFA-based algorithms. The question of whether FFA is a suitable technique for more general QAPs, however, was explicitly left unanswered. We answer it now, by using many more and entirely different instances. We also complement the analysis with several new perspectives, such as an analysis of the last improvement step or which kind of instances FRLS can solve to optimality within a reasonable computational budget.

## 2.2 Related Works Against Convergence to Local Optima

The problem of premature convergence to local optima is well-known in many fields of soft computing. It occurs, for example, in $k$-means clustering (Shalev-Shwartz and Ben-David, 2014; Arthur and Vassilvitskii, 2007) and the training of ANNs (Shalev-Shwartz and Ben-David, 2014; Treadgold and Gedeon, 1998). In optimization, it has been researched for a long time (Weise et al., 2012; Weise et al., 2009).

Tabu Search (TS) (Glover and Taillard, 1993), one of the most prominent methods to prevent premature convergence, improves upon local search by declaring solutions (or solution traits) that have been visited as tabu, which prevents the algorithm from getting stuck. It has found application in the QAP in several different variants (Misevičius, 2008; Merz and Freisleben, 1999; Skorin-Kapov, 1990).

In the field of Evolutionary Algorithms, the old ideas of sharing, niching, and clearing (Mahfoud, 1997; Goldberg and Richardson, 1987; Deb and Goldberg, 1989; Pétrowski, 1996) as well as clustering (Weise et al., 2011) combine density information with the objective values into so-called fitness values to increase the diversity in the populations of candidate solutions. These methods only consider the present populations and do not consider the history of the search, whereas FFA incorporates and aggregates knowledge over the whole course of optimization.

Methods that try to balance between solution quality and (population) diversity are today grouped under the term Quality-Diversity (QD) algorithms (Cully and Demiris, 2018; Gravina et al., 2019). QD algorithms are mainly applied to games, maze solving, and shape or robotics behavior evolution, but rarely in the context of discrete or hard optimization tasks from operations research.[1]

Novelty Search (NS) (Lehman and Stanley, 2008; Lehman and Stanley, 2011a) is an early QD algorithm. NS is driven by a dynamic novelty metric $\rho$ measuring the mean behavior difference to the $k$-nearest neighbors in the set of past solution "behaviors." NS with Local Competition (NSLC) (Lehman and Stanley, 2011b) combines the search for diverse solutions with a local competition objective rewarding solutions that can outperform those most similar to them.

In the QD method Surprise Search (SS) (Gravina et al., 2016), a solution is rated by the difference between its observed behavior from the expected

[1] At least the comprehensive paper QD paper list by (Mouret and Cully, 2024) does not list a single work referring to the QAP or the TSP in its abstract.

Algorithm 1: RLS($f : \mathbb{S} \mapsto \mathbb{N}$).

```
sample s_c from S u.a.r.;  z_c ← f(s_c);
for 10^8 − 1 times do ▷ our termination criterion
    s_n ← swap 2 values in s_c u.a.r.;  z_n ← f(s_n);
    if z_n ≤ z_c then s_c ← s_n;  z_c ← z_n;
return s_c, z_c
```

Algorithm 2: FRLS($f : \mathbb{S} \mapsto \mathbb{N}$).

```
H ← (0,0,··· ,0);        ▷ H-table initially all 0s
sample s_c from S u.a.r.;  z_c ← f(s_c);
s_b ← s_c;  z_b ← z_c;   ▷ best may otherwise get lost
for 10^8 − 1 times do ▷ our termination criterion
    s_n ← swap 2 values in s_c u.a.r.;  z_n ← f(s_n);
    if z_n < z_b then s_b ← s_n;  z_b ← z_n;
    H[z_c] ← H[z_c] + 1;  H[z_n] ← H[z_n] + 1;
    if H[z_n] ≤ H[z_c] then s_c ← s_n;  z_c ← z_n;
return s_b, z_b               ▷ return preserved best
```

behavior. A history of discovered solution behaviors is maintained and used to predict the behavior of new solutions. (Gravina et al., 2019) combine SS and NSLC.

Finally, the MAP-Elites algorithm by (Mouret and Clune, 2015) combines a performance objective $f$ and a user-defined space of features that describe candidate solutions. MAP-Elites searches for the highest-performing solution in each cell of the discretized feature space.

Sharing techniques require a population and all the other methods discussed above were designed as optimization algorithms themselves. FFA, however, can be plugged into a wide range of optimization algorithms as long as their objective functions are discrete. Instead of using the objective values $z$ computed by the objective function $f(s) = z$ when comparing solutions $s$, FFA prescribes using their observed encounter frequencies $H[z]$. This makes FFA invariant under all injective transformations of the objective function value, a property further distinguishing it from all related techniques (Weise et al., 2021b; Weise et al., 2023).

## 3 OUR APPROACH

The pure randomized local search algorithm RLS is illustrated in Algorithm 1. This algorithm starts by sampling a solution $s_c$ from the set $\mathbb{S}$ of all permutations of the first $n$ natural numbers uniformly at random (u.a.r.). It evaluates the objective function $f$ and obtains the quality $z_c$ of $s_c$. In a loop, it then creates a copy $s_n$ of $s_c$ in which two values are swapped, u.a.r.. The quality $z_n = f(s_n)$ of $s_n$ is computed. If $s_n$ is better than or equally good as $s_c$, it will replace $s_c$. The loop is repeated until the termination criterion is met, which, in our case, is the consumption of a total of $10^8$ objective function evaluations (FEs, including the evaluation of the random initial solution).

We plug FFA into this algorithm and obtain the FRLS in Algorithm 2. While RLS accepts the new solution $s_n$ if its objective value $z_n$ is not worse than the objective value $z_c$ of the current solution $s_c$, FRLS accepts $s_n$ if the encounter frequency $H[z_n]$ of $z_n$ in the selection decision is not higher than the encounter frequency $H[z_c]$ of $z_c$. For this purpose, it begins by filling the frequency table $H$ with zeros at the beginning of the algorithm. In each iteration, $H[z_n]$ and $H[z_c]$ are both incremented by one and then replace $z_n$ and $z_c$ in the selection decision. This means that FRLS is not biased towards better solutions and will replace $s_c$ with a worse $s_n$ if its objective value $z_n$ is encountered less than or equally often as $z_c$. Therefore, instead of returning $s_c$ and $z_c$ at the end, FRLS must remember the best-encountered solution and objective value in additional variables $s_b$ and $z_b$, respectively.

## 4 EXPERIMENTS AND RESULTS

The QAPLIB by (Burkard et al., 1997) is a commonly used and continuously updated database of QAP benchmark instances and their solutions. It contains both real-life instances and randomly generated instances. In our experiments, we use all 134 instances of the latest version of the QAPLIB at the time of this writing, which is maintained by (Hahn and Anjos, 2018) and was last updated in 2018. From this resource, we also take the lower bounds $lb$ of the objective functions $f$. For each instance, we perform 3 independent runs which, together with the many instances, are already sufficient to observe very clear differences in performance. The instances have the following properties:

- bur$n$* (Burkard and Offermann, 1977), 8 instances, $n = 26$, all optima known
- chr$n$* (Christofides and Benavent, 1989), 14 instances, $n \in \{12, 15, 18, 20, 22, 25\}$, all optima known
- els19 (Elshafei, 1977), 1 instance, $n = 19$, optimum known
- esc$n$* (Eschermann and Wunderlich, 1990), 19 instances, $n \in \{16, 32, 64, 128\}$, all optima known
- had$n$ (Hadley et al., 1992), 5 instances, $n \in \{12, 14, 16, 18, 20\}$, all optima known

- kra$n$* (Krarup and Pruzan, 1978), 3 instances, $n \in \{30, 32\}$, all optima known
- lipa$n$* (Li and Pardalos, 1992), 16 instances, $n \in \{20, 30, 40, 50, 60, 70, 80, 90\}$, all optima known
- nug$n$* (Nugent et al., 1968), 15 instances, $n \in \{12, 14, 15, 16, 17, 18, 20, 21, 22, 24, 25, 27, 28, 30\}$, all optima known
- rou$n$ (Roucairol, 1987), 3 instances, $n \in \{12, 15, 20\}$, all optima known
- scr$n$ (Scriabin and Vergin, 1975), 3 instances, $n \in \{12, 15, 20\}$, all optima known
- sko$n$* (Skorin-Kapov, 1990), 13 instances, $n\{42, 49, 56, 64, 72, 81, 90, 100\}$, *all optima unknown*
- ste36* (Steinberg, 1961), 3 instances, $n = 36$, all optima known
- tai$n$* (Taillard, 1991; Taillard, 1995), 26 instances, $n \in \{12, 15, 17, 20, 25, 30, 35, 40, 50, 60, 64, 80, 100, 150, 256\}$, *optima of 16 instances unknown*
- tho$n$ (Thonemann and Bölte, 1994), 3 instances, $n \in \{30, 40, 150\}$, *only optimum of tho30 known*
- wil$n$ (Wilhelm and Ward, 1987), 2 instances, $n \in \{50, 100\}$, *no optimum known*

We implement our algorithms using the moptipy (Weise and Wu, 2023) framework and run the experiments on a Windows 10 machine using Python 3.10 and the numba JIT.

Table 1 shows the arithmetic mean of the best objective values achieved by RLS and FRLS over the 3 runs per QAPLIB instance. The last row, **# best**, tells us that FRLS achieved the best average result 113 times, while RLS did this only 35 times. The average result of FRLS hits the lower bound $lb$, i.e., is optimal 73 times. Its best-of-3-runs results (not tabulated) reach it 78 times. RLS achieves this feat only 14 respectively 20 times. In other words, not only does FRLS outperform RLS on 74% of the QAPLIB instances in terms of its average result, it also solves 58% of them to optimality.

In (Liang et al., 2022), it was found that the performance of FRLS may strongly depend on the number $M$ of different objective values that an optimization problem exhibits. The good performance of FRLS on the esc$n$ problems may be caused by the many zeros in their flow matrix resulting in few different possible object values.

Exactly determining $M$ for the QAPLIB instances would be another $\mathcal{NP}$-hard problem in itself. Therefore, we do not have the exact values of this measure available. However, we can approximate it using the estimate, or better, a lower bound $m$: Each run of FRLS maintains its own frequency table $H$ and we collect these tables in our log files. We also log all improving moves that any algorithm makes, so we additionally have at least the strictly monotonous sequence of visited $f$-values for RLS. Finally, the website of the QAPLIB offers the best-known or even optimal solutions for all instances, which are better than our results on 42% of the instances. Therefore, by setting $m$ to be the size of the joint set of all of these values of all runs, we can get a lower bound for $M$. When $m$ is much smaller than our total computational budget over all runs of FRLS (for which we collect the complete $H$-tables), i.e., where $m \ll 3 * 10^8$, it should be a reasonable estimate of $M$. Otherwise, at least it informs us whether $M$ is probably small or large. We therefore also include it in the tables.

Revisiting the results of both algorithms in Table 1 and considering them from the perspective of $m$ confirms the findings by (Liang et al., 2022). If $m$ of an instance is small, FRLS tends to solve the instance to optimality (and hit the lower bound $lb$), even if the scale $n$ is not small (e.g., at lipa50a). Vice versa, the tables also show that FRLS is outperformed by RLS even on small problems if their $m$ is large, see, e.g., tai15b. The comparatively good performance of FRLS on the tai$n$a instances versus the tai$n$b instances is also interesting because the former are usually considered as harder (Ochoa and Herrmann, 2018).

A remarkable piece of evidence of the exploration power of FRLS, which discovers most of the encountered objective values, are the high $m$-values for many instances. FRLS contributed 215 196 721 values to the estimation $m = 215\,196\,971$ for tai30b. Since we conducted only 3 runs at $10^8$ FEs each, this means that 71% of *all* the solutions that these FRLS runs have sampled had *unique* objective values. If all solutions on a problem instance would have unique objective values, then FRLS would always accept the new solution $s_n$ and hence become a random walk. But this does not seem to be the case: On tai20b, FRLS encountered 173 058 828 different objective values – and outperformed RLS by a margin of over 10%.

The strong ability to explore and keep improving of FRLS is further illustrated in Figure 1. Here, we plot the average *life* index of the objective function evaluation (FE) where the last improving move was made over the problem scale $n$. In other words: Each run of an algorithm on a given problem instance eventually stops improving its best-so-far solution. It may or may not have discovered the optimal solution by then, but after that, no more improvement is made (within the provided computational budget, at least). The index of the algorithm step when, for the last time in a run, a new (better) best-so-far solution is discov-

Table 1: The average result over 3 runs of the RLS and the FRLS on the 134 QAPLIB instances, in comparison with the lower bound *lb* of $f$ and the number $m$ of observed and known objective values as a lower bound for the number of possible different objective values. The best result is marked in **boldface**.

| instance | *lb* | *m* | RLS | FRLS | instance | *lb* | *m* | RLS | FRLS |
|---|---|---|---|---|---|---|---|---|---|
| bur26a | 5 426 670 | 1 480 802 | 5 442 929 | **5 434 256** | lipa60a | 107 218 | 4 915 | 108 368 | **107 461** |
| bur26b | 3 817 852 | 1 021 194 | 3 838 077 | **3 818 291** | lipa60b | 2 520 135 | 456 660 | 3 016 957 | **3 005 080** |
| bur26c | 5 426 795 | 1 384 071 | 5 440 307 | **5 428 857** | lipa70a | 169 755 | 6 880 | 171 358 | **170 429** |
| bur26d | 3 821 225 | 945 677 | 3 833 028 | **3 821 540** | lipa70b | 4 603 200 | 651 696 | **5 569 556** | 5 642 958 |
| bur26e | 5 386 879 | 1 579 830 | 5 405 301 | **5 389 526** | lipa80a | 253 195 | 7 772 | 255 351 | **254 606** |
| bur26f | 3 782 044 | 1 111 729 | 3 793 182 | **3 782 454** | lipa80b | 7 763 962 | 940 457 | **9 423 095** | 9 650 856 |
| bur26g | 10 117 172 | 2 672 208 | 10 145 555 | **10 127 889** | lipa90a | 360 630 | 9 976 | 363 412 | **362 571** |
| bur26h | 7 098 658 | 1 876 059 | 7 141 228 | **7 101 399** | lipa90b | 12 490 441 | 1 277 577 | **15 173 637** | 15 617 417 |
| chr12a | 9 552 | 33 801 | 14 899 | **9 552** | nug12 | 578 | 232 | 606 | **578** |
| chr12b | 9 742 | 33 627 | 14 589 | **9 742** | nug14 | 1 014 | 366 | 1 037 | **1 014** |
| chr12c | 11 156 | 33 377 | 14 939 | **11 156** | nug15 | 1 150 | 432 | 1 182 | **1 150** |
| chr15a | 9 896 | 52 353 | 16 015 | **9 896** | nug16a | 1 610 | 532 | 1 673 | **1 610** |
| chr15b | 7 990 | 53 657 | 11 952 | **7 990** | nug16b | 1 240 | 483 | 1 297 | **1 240** |
| chr15c | 9 504 | 50 900 | 14 913 | **9 504** | nug17 | 1 732 | 608 | 1 813 | **1 732** |
| chr18a | 11 098 | 66 156 | 18 142 | **11 098** | nug18 | 1 930 | 650 | 1 978 | **1 930** |
| chr18b | 1 534 | 3 083 | 1 648 | **1 534** | nug20 | 2 570 | 826 | 2 681 | **2 570** |
| chr20a | 2 192 | 8 429 | 3 325 | **2 192** | nug21 | 2 438 | 970 | 2 510 | **2 438** |
| chr20b | 2 298 | 8 307 | 3 556 | **2 335** | nug22 | 3 596 | 1 531 | 3 759 | **3 596** |
| chr20c | 14 142 | 91 709 | 31 659 | **14 142** | nug24 | 3 488 | 1 254 | 3 608 | **3 488** |
| chr22a | 6 156 | 16 932 | 6 824 | **6 156** | nug25 | 3 744 | 1 277 | 3 950 | **3 744** |
| chr22b | 6 194 | 16 846 | 6 861 | **6 215** | nug27 | 5 234 | 1 860 | 5 470 | **5 234** |
| chr25a | 3 796 | 21 052 | 6 509 | **3 796** | nug28 | 5 166 | 1 728 | 5 417 | **5 166** |
| els19 | 17 212 548 | 30 545 903 | 25 266 593 | **18 821 866** | nug30 | 6 124 | 2 018 | 6 439 | **6 124** |
| esc16a | 68 | 34 | **68** | **68** | rou12 | 235 528 | 58 475 | 248 938 | **235 528** |
| esc16b | 292 | 22 | **292** | **292** | rou15 | 354 210 | 97 118 | 378 899 | **354 210** |
| esc16c | 160 | 73 | **160** | **160** | rou20 | 725 522 | 175 690 | 759 802 | **725 522** |
| esc16d | 16 | 36 | **16** | **16** | scr12 | 31 410 | 28 833 | 33 079 | **31 410** |
| esc16e | 28 | 29 | **28** | **28** | scr15 | 51 140 | 53 073 | 56 646 | **51 140** |
| esc16f | 0 | 1 | **0** | **0** | scr20 | 110 030 | 120 453 | 126 571 | **110 030** |
| esc16g | 26 | 35 | **26** | **26** | sko42 | 15 332 | 4 201 | 16 351 | **15 812** |
| esc16h | 996 | 272 | **996** | **996** | sko49 | 22 650 | 5 802 | 23 909 | **23 403** |
| esc16i | 14 | 37 | **14** | **14** | sko56 | 33 385 | 8 202 | 35 337 | **34 467** |
| esc16j | 8 | 20 | **8** | **8** | sko64 | 47 017 | 10 379 | 49 509 | **48 524** |
| esc32a | 130 | 253 | 151 | **130** | sko72 | 64 455 | 13 439 | 67 707 | **66 378** |
| esc32b | 168 | 124 | 183 | **168** | sko81 | 88 359 | 17 353 | 92 575 | **91 107** |
| esc32c | 642 | 194 | **642** | **642** | sko90 | 112 423 | 20 859 | 117 639 | **115 853** |
| esc32d | 200 | 117 | 205 | **200** | sko100a | 143 846 | 25 618 | 153 965 | **152 557** |
| esc32e | 2 | 50 | **2** | **2** | sko100b | 145 522 | 26 389 | 156 111 | **154 557** |
| esc32g | 6 | 37 | **6** | **6** | sko100c | 139 881 | 25 903 | 151 014 | **148 430** |
| esc32h | 438 | 175 | 467 | **438** | sko100d | 141 289 | 25 616 | 151 863 | **150 203** |
| esc64a | 116 | 124 | **116** | **116** | sko100e | 140 893 | 26 623 | 151 569 | **149 795** |
| esc128 | 64 | 192 | 65 | **64** | sko100f | 140 691 | 25 266 | 151 695 | **149 570** |
| had12 | 1 652 | 228 | 1 665 | **1 652** | ste36a | 9 526 | 14 213 | 10 213 | **9 526** |
| had14 | 2 724 | 394 | 2 753 | **2 724** | ste36b | 15 852 | 96 128 | 17 766 | **15 852** |
| had16 | 3 720 | 478 | 3 815 | **3 720** | ste36c | 8 239 110 | 7 804 921 | **8 970 338** | 10 698 219 |
| had18 | 5 358 | 622 | 5 413 | **5 358** | tai12a | 224 416 | 64 051 | 240 311 | **224 416** |
| had20 | 6 922 | 856 | 6 969 | **6 922** | tai12b | 39 464 925 | 60 287 923 | 45 156 248 | **39 492 474** |
| kra30a | 88 900 | 8 200 | 94 843 | **88 900** | tai15a | 388 214 | 94 668 | 400 575 | **388 214** |
| kra30b | 91 420 | 8 581 | 95 967 | **91 420** | tai15b | 51 765 268 | 35 623 423 | **51 943 701** | 52 001 756 |
| kra32 | 88 700 | 8 828 | 93 730 | **88 700** | tai17a | 491 812 | 122 615 | 523 634 | **491 812** |
| lipa20a | 3 683 | 435 | 3 795 | **3 683** | tai20a | 703 482 | 178 309 | 751 881 | **704 195** |
| lipa20b | 27 076 | 10 778 | 31 241 | **27 076** | tai20b | 122 455 319 | 173 058 953 | 143 287 002 | **129 766 839** |
| lipa30a | 13 178 | 1 088 | 13 442 | **13 178** | tai25a | 1 167 256 | 252 308 | 1 231 845 | **1 174 603** |
| lipa30b | 151 426 | 57 162 | 178 015 | **151 426** | tai25b | 344 355 646 | 202 832 378 | **384 043 042** | 395 447 601 |
| lipa40a | 31 538 | 1 976 | 32 042 | **31 538** | tai30a | 1 706 855 | 319 665 | 1 918 997 | **1 853 616** |
| lipa40b | 476 581 | 184 489 | 563 999 | **476 581** | tai30b | 637 117 113 | 215 196 971 | **710 795 743** | 721 008 038 |
| lipa50a | 62 093 | 3 296 | 62 902 | **62 093** | tai35a | 2 216 627 | 397 009 | 2 559 439 | **2 509 553** |
| lipa50b | 1 210 244 | 348 151 | 1 438 601 | **1 308 415** | tai35b | 269 532 400 | 125 920 739 | **317 695 376** | 334 904 454 |
| tai40a | 2 843 274 | 468 546 | 3 307 957 | **3 281 287** | tai100b | 1 151 591 000 | 152 455 325 | **1 240 769 163** | 1 543 004 655 |
| tai40b | 608 808 400 | 165 765 853 | **693 265 760** | 806 047 127 | tai150b | 441 786 736 | 43 282 106 | **509 821 471** | 612 165 283 |
| tai50a | 4 390 920 | 612 489 | **5 152 389** | 5 245 447 | tai256c | 44 095 032 | 5 758 252 | **44 940 419** | 48 194 560 |
| tai50b | 431 090 700 | 129 307 016 | **504 050 091** | 590 061 093 | tho30 | 149 936 | 63 292 | 157 237 | **149 936** |
| tai60a | 6 325 978 | 752 294 | **7 553 963** | 7 717 479 | tho40 | 226 490 | 102 223 | 251 221 | **240 708** |
| tai60b | 592 371 800 | 137 418 134 | **643 368 525** | 791 205 717 | tho150 | 7 620 628 | 918 879 | **8 319 988** | 8 855 816 |
| tai64c | 1 855 928 | 1 691 310 | **1 860 059** | 1 861 098 | wil50 | 48 121 | 6 760 | 49 465 | **48 835** |
| tai80a | 11 657 010 | 1 019 112 | **14 030 598** | 14 523 961 | wil100 | 268 955 | 26 012 | 275 203 | **273 622** |
| tai80b | 786 298 800 | 120 691 529 | **873 374 711** | 1 075 394 622 | | | **# best** | 35 | **113** |
| tai100a | 17 853 840 | 1 268 760 | **21 828 809** | 22 720 707 | | | | | |

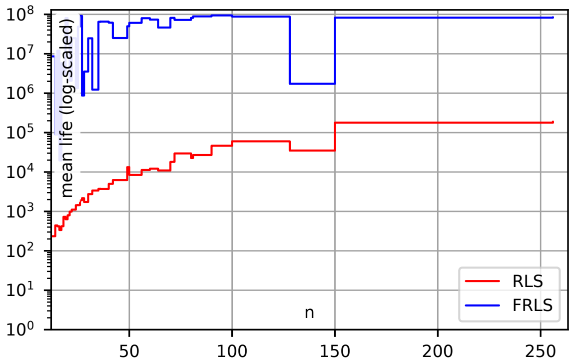


Figure 1: The average *life* index of the objective function evaluation (FE) where the last improving move was made, plotted in log-scale over the problem scale $n$.

ered, averaged over the runs, is presented as *life*.

We find that the time during which the RLS can keep improving increases slightly with $n$. However, it remains roughly in the range of at most a few 100 000 FEs. Over almost all problem scales, FRLS can keep improving for, basically, the complete available budget of $10^8$ FEs. This strongly indicates that if we had allocated not $10^8$ FEs but $10^{10}$, as it was done in (Weise et al., 2021b; Weise et al., 2023; Liang et al., 2022; Liang et al., 2024), we very likely would have seen several more instances solved to optimality. The single downward rectangular slot in both curves in the diagram is caused by esc128, at which both algorithms converge earlier (FRLS to the optimum, after which no further improvement is possible). The next larger instances are at $n = 150$ where the trend resumes.

We now plot the progress of the two algorithms in terms of the best-so-far objective value divided by the lower bound $lb$ of the objective function $f$ over time measured in FEs and averaged over all the runs and instances in each of the 15 groups of QAPLIB. Instance esc16f with $lb = 0$ is omitted. From Figure 2, it is visible that FRLS finds better average end result qualities on all groups except ste$n$ and tai$n$. Even on these groups, it would have probably overtaken RLS if we had given more runtime. In most of the diagrams, RLS is initially faster and then stagnates, while FRLS steadily and continuously keeps improving.

## 5 CONCLUSIONS

In the past, Frequency Fitness Assignment (FFA) has led to surprisingly good results on several $\mathcal{NP}$-hard optimization problems, including Max-Sat (Weise et al., 2021b; Weise et al., 2023), the JSSP (Weise et al., 2021a; de Bruin et al., 2023), and the TSP (Liang et al., 2022; Liang et al., 2024). In this work, we conclusively showed that FFA can achieve this on one more of these classical hard tasks from operations research: the Quadratic Assignment Problem (QAP).

We find that the FFA-based randomized local search FRLS does not just find better solutions than the objective-guided RLS algorithm on the vast majority of the QAPLIB instances, it also keeps improving its current best solution for the complete computational budget of $10^8$ FEs that we assigned to the runs. With this budget, it can discover the optimal solutions of over 58% of the QAPLIB instances. Had we assigned a larger budget – (Liang et al., 2022; Liang et al., 2024; Weise et al., 2021b; Weise et al., 2023) use $10^{10}$ FEs – we would likely have seen even more instances solved.

We furthermore confirm the remarkable ability of FFA to discover very diverse solutions (at least from the perspective of the objective function). It is known that on the QAP, many solutions tend to have the same objective values (Tayarani-N. and Prügel-Bennett, 2015). Yet, on some of the instances, more than half of the objective values discovered by FRLS were unique.

The QAP is strongly related to the TSP (Dréo et al., 2006). (Liang et al., 2022; Liang et al., 2024) found that the FFA performance strongly depends on the number $M$ of possible different objective values. We are the first to report a lower bound and estimate $m$ of $M$ for each of the QAPLIB instances. We confirm that, indeed, if $m$ is high, then the performance of the FRLS declines in comparison to the objective-guided RLS, adding to our understanding of the performance of this algorithm.

(Liang et al., 2022; Liang et al., 2024) showed that the performance of the FRLS can significantly be improved if it is hybridized with RLS sharing the budget in a round-robin fashion and if simulated annealing (SA) is used as a basic algorithm. Investigating plugging FFA in other algorithms on the QAP, such as the SA by (Wilhelm and Ward, 1987), the tabu search by (Taillard, 1991), the hybrid evolutionary algorithms by (Fleurent and Ferland, 1993; Merz and Freisleben, 1999), or the ant colony optimization method by (Gambardella et al., 1999), is therefore an important branch of our future work.

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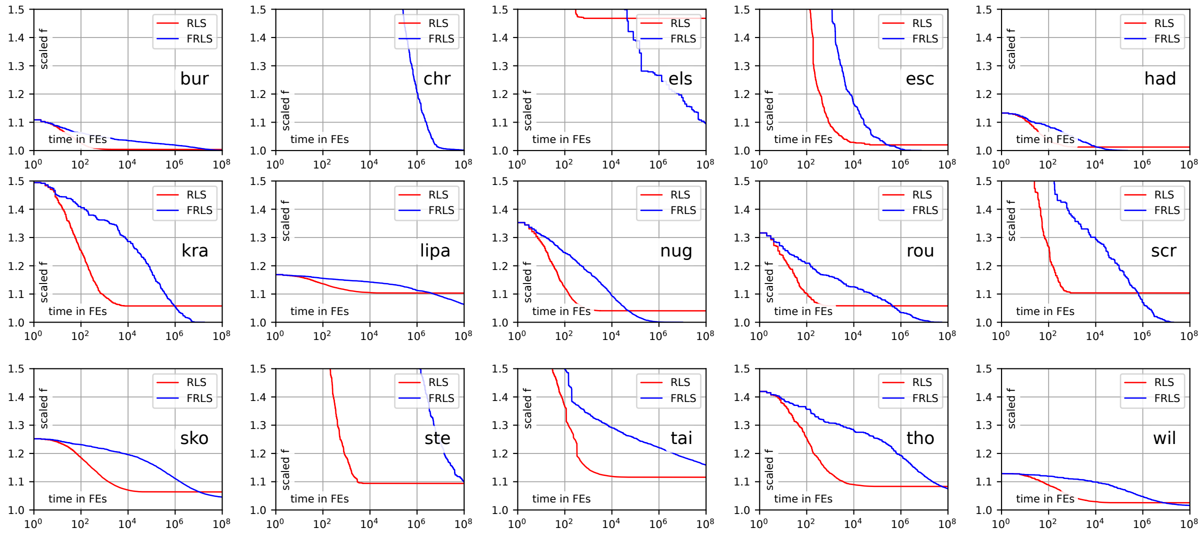


Figure 2: The progress in terms of the best-so-far objective value divided by the lower bound *lb* of *f* averaged over all runs and instances of an instance group and plotted over the time measured in FEs (log-scaled). Instance esc16f is omitted from this statistic (the esc group) due to having a lower bound of 0. On the chr*n* instances, RLS is off the scale.

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