Analysis and Forecast of Influencing Factors of House Price in Boston

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Keywords: Correlation Analysis, Multiple Linear Regression, All-Subsets Regression, Random Forest Regression,

Variable Importance Plot.

Abstract: The cost of housing is a significant indicator of economic activity. Both buyers and developers of real estate

closely monitor fluctuations in home prices. As a result, developing an accurate housing price forecasting model is critical for the financial market and people's livelihood. Boston dataset is used for data analysis in this paper. Firstly, this paper conducted a correlation study using Pearson's r and created scatter plots between each variable and the house price. In the multiple linear regression model, this study employed techniques including outlier removal, collinearity, and variable transformation to optimize the model for a more accurate prediction, finally getting an R-squared of 0.7927. In random forest model, this research finds the optimal parameter value and then compares the importance of predictors using the indicators %IncMSE and %IncNodePurity. Finally, this study determines that the random forest regression model is superior through the investigation of the factors that influence home prices, and this paper also analyze the elements

that influence house prices and consider the shortcomings of this paper.

1 INTRODUCTION

Due to economic growth as well as non-economic considerations including the caliber of schools, air pollution levels, and distance from commercial hubs, housing costs in major Chinese cities have been continuously rising (Chen, 2023). To completely comprehend the variables influencing house prices, a multidimensional analysis is required. This study examines the determining elements of housing costs, including non-economic ones, utilizing reliable data from 506 families in different Boston neighborhoods (Harrison and Rubinfeld, 1978). The data is analyzed using two different regression models, which can be used as a guide for future studies on the factors influencing housing costs in different Chinese cities. Numerous specialists domestically internationally have conducted statistical analyses on the factors influencing the cost of housing. For instance, Chen Zekun and Cheng Xiaorong used the gradient descent approach to perform regression analysis on the data set, train the model to achieve the fitting function, and then develop a housing price prediction model (Chen and Cheng, 2020). Yin Wenwen stated the coefficient function using a verification methodology, and then used Boston housing data to establish a variable coefficient model measurement errors, which adequately

explained the fluctuation trend of the median house price. The verification approach can handle complex error structure models while saving a significant amount of money (Yin, 2018). Keren Horn, and Mark Merante investigated the influence of Airbnb on rental pricing, attempting to determine whether changes in rental prices are related to a drop in housing availability (Horn and Merante, 2017).

2 RESEARCH METHOD

2.1 Pearson's Correlation Coefficient

It is a commonly used linear correlation metric. The covariance between two vectors, normalized by the product of their standard deviations, is how it is defined (Makowski, et al, 2020). The coefficient measures the tendency of two vector pairs to change collectively above or below their mean, indicating that the measurements for each pair are more likely to fall on one side or the other of the average for that pair.

2.2 Model Selection

A linear regression model with numerous explanatory

variables is known as a multiple linear regression model (Kumari and Yadav, 2018). This paper can take into consideration the following linear relationship:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_{13} x_{13} + \varepsilon$$
 (1)

y=MEDV(the median value of owner-occupied homes), $x_1, x_2, ..., x_{13}$ =factors affecting housing prices. β_0 and $\beta_1, ..., \beta_{13}$ are intercept term and the regression coefficient respectively, and ε is the error term representing random errors that the model cannot explain (Burton, 2021).

Random forest regression is an ensemble learning-based algorithm that performs regression tasks by building multiple decision trees and integrating their predictions. A random forest effectively lowers the danger of overfitting because each decision tree is independent and trained on randomly chosen subsamples. To obtain the final regression result, the random forest weights or averages the predictions of various decision trees 0 Qin and Song, 2021).

3 DATA ANALYSIS

Each of the 506 entries in the Boston housing data provides aggregated information about 14 attributes of dwellings in different Boston areas that were collected in 1978 (Harrison and Rubinfeld, 1978). The explanation of each variable is given in following table 1:

Table 1: Factors affecting Boston house prices.

Variable name	Variable interpretation			
CRIM x_1	Criminal rate per capita by town.			
$ZN x_2$	Residential use is permitted on lots Larger than 25,000 square feet.			
INDUS x_3	Each town's percentage of non-retail businesses.			
CHAS x_4	The dummy variable for Charles River (= 1 is a river; =0 is not a river).			
NOX x_5	Nitrogen oxides concentration (parts per 10 million).			
$RM x_6$	Rooms available per home.			
AGE x ₇	Proportion of pre-1940 owner-occupied housing units.			
DIS x_8	Distance from five Boston job centers, weighted.			
RAD x_9	Accessibility score for radial highways.			
TAX <i>x</i> ₁₀	Property tax rate per \$10,000 of full value.			
PIRATIO	Town-specific student-teacher ratio.			

<i>x</i> ₁₁			
B <i>x</i> ₁₂	$1000(Bk - 0.63)^2$ Where Bk is the		
	percentage of blacks.		
LSTAT x_{13}	Lower status of the population (percent).		
MEDV y	The average cost of an owner-occupied residence is \$1000.		
	residence is \$1000.		

To examine the factors influencing the level of house prices, this paper selects MEDV as my response variable and the other 13 variables as explanatory variables out of these 14.

3.1 Descriptive and Correlated Analysis of Data

This paper uses R software for analysis. For each variable, Table 2 displays the descriptive statistics.

Table 2: Descriptive Analysis of Explanatory Variables

	CRIM	ZN	INDUS
n	506	506	506
NA	0	0	0
mean	3.61	11.4	11.1

Based on initial analysis, this study concludes that the data do not contain any missing values. Intuitively, these factors will affect the level of housing prices in Boston in 1987. In order to further build the model, this research then assesses the link between the explanatory variable and the corresponding variable using the scatter plot and correlation coefficient plot.

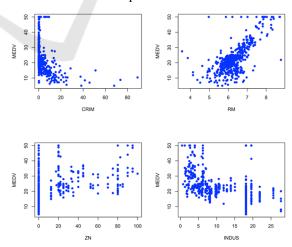


Figure 1: Scatter plot of each feature with MEDV (Picture credit: Original).

Figure 1 above shows a positive correlation between RM (the average number of rooms

available) and MEDV, implying that the price of real estate in the area will rise as the average number of rooms per residence rises. Similarly, there is a negative correlation between CRIM (for each person criminal activity by town) and MEDV, which means if the town's per capita crime rate rises, home values will fall.

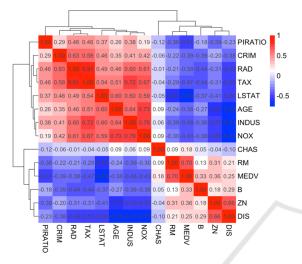


Figure 2: Plot of the correlation coefficient between variables (Picture credit: Original).

As can be observed, there is a significant association between LSTAT (the residents with a lower socioeconomic status.), RM, PIRATIO (the student-teacher ratios vary by town.), and MEDV, with correlation values of -0.74, 0.70, and -0.51 accordingly. Moreover, the correlation coefficient between the variable CHAS (the Charles River imaginary variable) and the other factors is often in the range of 0.1, demonstrating a minimal influence. Because of this, this paper will ignore the variable CHAS variable in further analysis.

3.2 Multiple Linear Regression Model -- First Regression Analysis

The Boston data was manually divided into two sets: the training set, which had 70% of the data, and the test set, which contained 30% of the data. The first fit to MEDV was performed using the remaining variables based on training data. The regression model is:

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\begin{array}{c} y = 34.120254 \, - \, 0.09745X1 \, + \\ 0.031720X2 \, - \, 0.019334X3 \, - \, 15.542128X5 \, + \\ 4.001339X6 \, + \, 0.002983X7 \, - \, 1.283889X8 \, + \\ 0.300291X9 \, - \, 0.012381X10 \, - \\ 0.973398X11 \, + \, 0.009031X12 \, - \, 0.498439X13 \end{array}
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The results show that p-value < 0.05, so the author can reject the null hypothesis, which means the regression equation is significant. A mediocre fitting effect is shown by the fitting coefficient R-squared of 0.742 and the adjusted R-squared of 0.733. The author then creates a fitting curve to compare the true value of housing prices with the expected value to assess the impact of the model of multiple linear regression. As shown in figure 3:

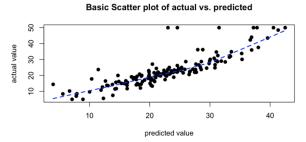


Figure 3: The expected and true value fitting curve (Picture credit: Original).

The extent of variation that can be explained by the regression equation is indicated by the values of R-squared, which are 0.719 and adjusted R-squared, respectively, of 0.717. Since most of the sample data points are grouped together around the regression line, the model has a high goodness of fit, as can be shown.

3.3 Model Optimization

First, this study uses the influence plot and outliers test function in the car package to remove the outliers, leverage values, and strong influence points such as 366, 369,419 and so on (Kabacoff, 2011). Next, this paper attempt to use the vif(fit) to identify any data collinearity (Shrestha, 2020). Although the VIF (variance inflation factor) of each variable is very minor, multicollinearity is present because the VIF values of NOX (the level of nitrogen oxides), RAD(the use of circular highways), and TAX(the full-value property-tax rate per\$10000) are all higher than 4. This paper can use the all-subsets regression approach to screen out multicollinearity by filtering variables on the boston data.

Figure 4 shows that CRIM, ZN (proportion of residential land zoned), INDUS (The fraction of acres used for non-retail businesses per village), and RAD have little impact; hence, this study eliminate these four variables. Finally, the scatter plot and kernel density estimation curve are utilized to assess the correlation between the variables and the MEDV median house price (Zhang, 2018). Additionally,

variable transformation is implemented to further enhance the efficiency of the model.

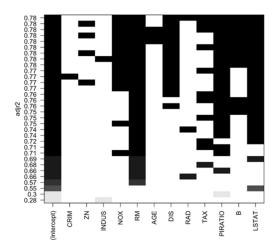


Figure 4: Full subset regression (Picture credit: Original).

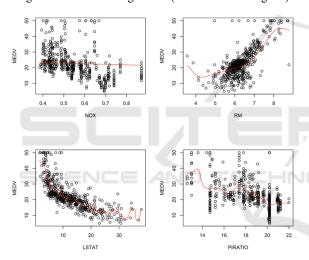


Figure 5: Scatter plot and kernel density estimation curve (Picture credit: Original).

According to Figure 5, there may be a quadratic correlation between RM and MEDV, a reciprocal correlation between LSTAT and MEDV. It's not immediately clear how much other factors and MEDV are correlated. Therefore, this paper rebuilts the regression model and added the two terms RM² and $\frac{1}{LSTAT}$ to the original model.

3.4 Multiple Linear Regression Model -- Final Regression Analysis

Below this paper carries out the fitting based on training data (Table 3), only three random lines of data are presented here:

Table 3: Coefficient of the regression Equation.

fitting	Estimate	Std.Error	t value	Pr(> t)
(Intercept)	100.23249	10.02214	10.00	< 2e-16
I(RM^2)	1.93103	0.21987	8.78	< 2e-16
I(1/LSTAT)	31.23698	5.57000	5.61	4.2e-08

The final regression model is obtained as:

$$\begin{array}{c} \mathbf{y} = 100.23249 - 13.58667x_5 - 21.14899x_6 - 0.001 \\ 49x_7 - 0.83616x_8 - 0.00261x_{10} - 0.66777x_{11} + 0.00 \\ 739x_{12} - 0.33427x_{13} + 1.93103x_6^2 + 31.23698\frac{1}{x_{13}} \end{array}$$

Because of the test's p-value of 2e-16, the results indicate that the null hypothesis can be rejected. Fitting coefficient R-squared is 0.815 and adjusted R-squared is 0.813. Similarly, this paper draws the fitting curve of the genuine house price and the forested house price to check the effect of model optimization:

Basic Scatter plot of actual vs. predicted

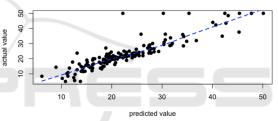


Figure 6: The expected and true value fitting curve (Picture credit: Original).

Residual standard error is 4.46. R-squared is 0.787 and adjusted R-squared is 0.786. therefore, compared to the initial regression model, the final multiple linear regression model reflects an improved goodness of fit for the clustered black points.

4 RANDOM FOREST REGRESSION MODEL

4.1 Determine the Parameters of mtry and ntree

This paper calls the random Forest function. First, this study choose the best mtry for the model (He, Fu and Liao, 2023). This paper adjusted the mtry parameter value to 6 during this modeling procedure since the traversal print result indicates that the optimal value for % Var explained=0.8583 for mtry=6. Next, this paper will find the model's ntree value, which indicates how many decision trees were

present at the time of modeling 0. To determine the value of the ntree parameter, this paper created a diagram below to illustrate the relationships between the quantity of decision trees and the rate of model inaccuracy.

Model error and the number of decision trees

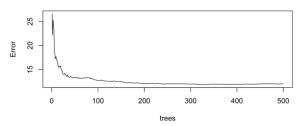


Figure 7: Relationship between model error rate and how many decision trees (Picture credit: Original).

As shown in figure7, the error rate decreases if the ntree value is 500. Therefore, this paper used mtry=6 and ntree=500 as the model parameters, and then the author trained the random forest model based on the training set. The R-squared is 0.855 and a mean squared residual of 11.76, indicating that the regression result using the random forest model is satisfactory. Next, this research will use the metrics %IncMSE(the rise in Mean squared Error of predictions) and %IncNodePurity(the mean decrease accuracy) to assess the variable's significance.

4.2 Check the Importance of Predictor Factors

Each predictor is given a random assignment, and the increase in the model's prediction error is then noted to determine the %IncMS (Liu et al, 2020). By computing the summation of the residuals' squares to compare the relative weights of the 13 variables, IncNodePurity assesses the impact of each variable on the heterogeneity of the observed values on the classification tree's nodes. As seen in figure 8:

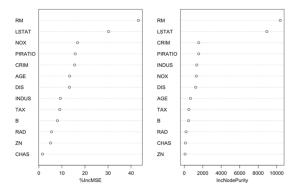


Figure 8: Variable Importance plot (Picture credit: Original).

It can see from the two figures above that the value of housing expenses are greatly influenced by RM, indicating that the amount of rooms is the primary factor determining the price of housing. The partial dependence plot can be used to determine whether there is a linear, monotonous, or complex relationship between the predictor and the house price. The following are examples of the partial dependent plot for the association between RM, LSTAT, and MEDV:

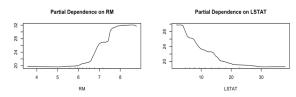


Figure 9: Partial dependence plot (Picture credit: Original).

As shown in figure 9, there is a strong linear relationship between housing price and RM, while LSTAT is negatively correlated with housing price, which is the same as the above conclusion.

4.3 The Results of the Training Set Prediction

This study tested the effect of the model by fitting the regression model. Figure 10 depicts the training set's prediction outcomes.

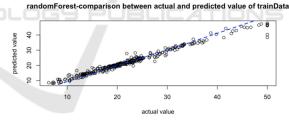


Figure 10: The results of the training set prediction (Picture credit: Original).

Through fitting regression analysis of MEDV and the predicted value of the training set, this study has the value of R-squared multiple is 0.9765 and R-squared adjusted is 0.9764, demonstrating the excellent data training effect of the model of random forests.

4.4 The Anticipated Scores of the Testing Set

This paper employs a trained random forest regression model to produce predictions for the test samples. The outcomes are as figure 11:

randomForest-comparison between actual and predicted value of testData

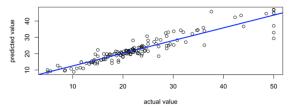


Figure 11: Testing set prediction outcomes (Picture credit: Original).

This paper performed fitting regression analysis on the median home price MEDV and the estimated value of the test data, and the results showed that the residual standard error is 3.033 and adjusted R-squared value is 0.859. Many of the points are clustered around the blue fitting line in the above image, even though a tiny portion of the points are dispersed across the fitting line, and the random-forests model has a usually accurate prediction consequence.

5 DISCUSSION

First, there are additional factors that affect the cost of housing, such as size, kind, height, and condition. Second, I eliminated variables and outliers that had an impact on the model's fit during the data analysis stage (Wang, Bah and Hammad, 2019).

6 CONCLUSION

6.1 Comparison between Different Models

Residual standard error of multiple linear regression prediction is 4.402 and the R-squared, a measure of determination, is 0.7927. R-squared coefficient of determination is 0.86, and the residual standard error of the prediction from random forest regression is 3.033. The information demonstrates that the random forest model is not only better than the linear regression model in data fitting optimization, but also has higher prediction accuracy than the linear regression model.

6.2 Different Factors' Effects on Home Prices

The regression coefficient and scatter plot of the model show that the percent of people with fewer socioeconomic status (LSTAT) and the sheer number of rooms in the house (RM) have the biggest effects on housing costs. In other words, the price of an area increases exponentially as the count of rooms increases. Likewise, when the population's share of the lower class rises, average disposable income falls, which in turn causes a decline in home values. The price of a home decreases with increasing weighted distance (DIS: locations of five Boston employment centers.) from Boston's five major neighborhoods, but prices increase in areas with low nitric oxide concentration (NOX), where there is greater housing dispersal. The price of housing decreases when the teacher-to-student ratio (PIRATIO) increases. High property taxes have a negative effect on home prices, but this effect is less pronounced in certain places.

6.3 Outlook

The above conclusions are from a macro point of view, the conclusion is only general. If researchers want to be specific to a particular house, they need to analyze according to the actual local situation. At the same time, due to time constraints, this study only built and trained linear regression and random forest models. It is hoped that more models will be added for analysis and comparison in future studies, and the optimal model will be selected for better prediction.

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