A Model to Determine Priority in AHP Using Coefficient Correlation

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Abstract: Analytic hierarchy process (AHP) is an effective decision-making method for making decisions on complex issues. Through AHP unstructured complex issues are simplified into structured parts in a hierarchy.Various methods in AHP are used to determine the priority of multi-criteria issues, but in this thesis a priority determination model is proposed through the correlation coefficient or Correlation Coefficient Maximization Approach (CCMA). Then compared with the addition normalization method or Additive normalization (AN). As an application of this method, it is about the election of a school principal by a foundation. Of the three alternatives specified, there are four criteria that must be possessed, namely knowledge, quality of work, responsibility, and work discipline. The results obtained show that the order of priority through the calculation of CCMA and AN is the same.

1 INTRODUCTION

Basically every individual is a decision maker. Everything that is done consciously or not is the result of a decision. The information obtained helps to understand events. In order to make the right and good decisions, clear and accurate information is needed. Not all information is useful for increasing understanding and consideration, if you only make decisions intuitively then you tend to believe that all kinds of information are useful and the better.

In making a decision, it is necessary to know the problem, needs and objectives of the decision, the decision criteria, the sub-criteria and the groups affected and the alternative actions to be taken. Then trying to determine the best alternative, for example in the case of resource allocation, it takes priority for alternatives to allocate the right resources.

Decision making, which gathers most of the information, has become a science of mathematics today. Decision making involving many criteria and many sub-criteria, is used to rank the alternatives for a decision.

The Analytic Hierarchy Process (AHP) was developed in the early 1970s by Thomas L. Saaty, is an effective decision-making method by using factors of logic, intuition, experience, knowledge, emotion and feeling to be optimized in a systematic process. (Saaty, 1990), argued that AHP has been accepted as the most superior multi-criteria decision model, both among academics and among practitioners for making decisions on complex issues. The rules in AHP relate to the workings of the human mind, because AHP simply relies on intuition as its main input, but intuition must come from a decision maker who is well informed and understands the decision problem.

The preparation of the decision structure in prioritizing a problem is carried out by decomposing, namely breaking the whole problem into elements of the problem, so that the influencing factors and alternative decisions will be described which will be determined in the form of a hierarchy of all elements.

In setting the priority of the elements in a decision problem is to make a pairwise comparison of a specified criterion, so that a very influential scale is obtained to compare the two elements. The results of this assessment are presented in the form of a matrix called the Pairwise Comparison Matrix. The preparation of the pairwise comparison matrix is to determine the importance value of each element in the decision structure. Pairwise comparison matrices are made based on the levels of each factor.

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2 METHOD

Priority estimation from the pairwise comparison matrix is a major part of AHP. Through the pairwise comparison matrix, the priority level of each element will be determined. By developing priority vectors for all matrices in the hierarchies formed for a particular decision problem, it is possible to perform aggregation and obtain the final priority vector (total priority). There are different techniques for determining priority vectors from comparison matrices and much research effort has been directed towards finding the best estimation method. The eigenvector (EV) method, which was proposed for the first time by (Saaty, 1977), proved that the eigenvector principle of the comparison matrix can be used as the required priority vector, for consistent or non-consistent judgments from decision makers. The standard procedure for determining priority vectors with the EV method is based on a square comparison matrix and normalizing the number of rows. Saaty also proposed several simple approximation methods to obtain the required vectors.

The simplest method is the addition normalization method or the additive normalization (AN) method. This method generates priority by taking the sum of the columns in the comparison matrix and by averaging the values obtained in the rows. Although AN is not widely accepted in the scientific community which prefers more sophisticated methods, it is still widely used because of its simplicity. The results of the analysis show that this method is competitive with other methods.

An interesting modification of the EV method proposed by (Cogger and Yu, 1985), is based on the premise that the overall preference intensity information is contained in the upper triangular matrix of the comparison matrix. The calculation procedure is recursive and simple, but the study by (Golany and Kress, 1993), shows that this method is not effective and can be excluded.

Used the EV method to investigate prioritization with the addition of alternatives to the Analytic Hierarchy Process. With the addition of alternatives to the Analytic Hierarchy Process for certain cases, the priority order of the previous alternatives can be changed. But by modifying the way to normalize the Eigen Vector (EV) it produces a procedure that is able to maintain the order of priority (Schmidt et al., 2015).

Most of the other methods of obtaining priority from a comparison matrix are considered extreme because they are based on an optimization approach. The prioritization problem is expressed as minimizing a certain objective function that measures the deviation between the ideal solution and the actual solution, subject to some additional constraints. As stated by (Mikhailov and Singh, 1999), priority assessment can be formulated as a non-linear optimization problem with constraints and solved by the direct least-squares (DLS) method (Chu et al., 1979), stated that although this method minimizes the Euclidean distance between the ideal solution and the actual solution, this method generally results in multiple solutions which can be considered as a drawback from a practical point of view. To eliminate it, several optimization methods such as the weighted least-squares method or the weighted least-squares (WLS) method are proposed, using a modified Euclidean norm as the objective function.

(Crawford and Williams, 1985) suggested the logarithmic least squares (LLS) method provides an explicit solution through an optimization procedure that minimizes the logarithm of the objective function by fulfilling the multiplication constraints. (Cook and Kress, 1988), put forward the logarithmic least absolute values (LLAV) method, a median-relation method that is not biased toward determining extreme values. (Wang et al., 2007), put forward an estimation of priority in AHP through a correlation coefficient, known as the Correlation Coefficient Maximization Approach (CCMA). CCMA is able to maximize the correlation coefficient between its own priorities and each column of the pairwise comparison matrix. Of the various methods in determining priorities, the authors put forward a model that will be discussed in setting priorities, namely through the correlation coefficient or the Correlation Coefficient Maximization Approach (CCMA).

3 RESULTS AND DISCUSSION

Correlation Coefficient Maximization Approach (CCMA) as an approach to maximizing the correlation coefficient used in determining the priority of the Pairwise Comparison Judgment Matrices (PCJM). According to (Wang et al., 2007), prioritization by maximizing the correlation coefficient can maximize the correlation coefficient between priorities and each column of the pairwise comparison matrix. Suppose $A = (a_{ij})_{nxn}$ is a pairwise comparison matrix $a_{ij} = 1/a_{ij}, a_{ii} = 1$ and $a_{ij} > 0$ for $i, j = 1,..,n$ and $W = (w_1,...,w_n)^T$ as a priority vector with $\sum_{i=1}^{n} w_i = 1$ and $w_i \geq 0$, for $i = 1,...,n$. According to (Saaty, 1988), if $a_{ij} = a_{ik}a_{kj}$ for $k = 1,...,n$ then $A = (a_{ij})_{nxn}$ it is called a perfectly consistent pairwise comparison matrix. For a perfectly consistent comparison matrix $A = (a_{ij})_{nxn}$, it can be characterized exactly by a priority vector $W = (w_1, ..., w_n)^T$ as follows:

$$a_{ij} = w_i/w_j, withi, j = 1, ..., n \quad (1)$$

From equation (1) obtained:

$$w_j = \frac{\sum_{j=1}^{n} w_i}{\sum_{j=1}^{n} a_{ij}} = \frac{1}{\sum_{j=1}^{n} a_{ij}}, j = 1, ..., n \quad (2)$$

Based on equation 1, a consistent comparison matrix $A = (a_{ij})_{nxn})$ can be presented as an n column vector as follows:

$$\frac{1}{w_1}\begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_n \end{bmatrix}, \frac{1}{w_2}\begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_n \end{bmatrix}, ..., \frac{1}{w_n}\begin{bmatrix} w_1 \\ w_2 \\ \cdots \\ w_n \end{bmatrix} \quad (3)$$

This shows that the n column vector is perfectly correlated with the priority vector $W = (w_1, ..., w_n)^T$.

For example, R_j is the correlation coefficient between the priority vector W and the j-column vector of the pairwise comparison matrix $A = (a_{ij})_{nxn}$, then it is obtained

$$R_j = \frac{\sum_{i=1}^{n}(a_{ij} - \bar{a}_j)(w_i - \bar{w})}{(\sqrt{\sum_{i=1}^{n}(a_{ij} - \bar{a}_j)^2})(\sqrt{\sum_{i=1}^{n}(w_i - \bar{w})^2})} \quad (4)$$

with $j = 1, ...n, \bar{a}_j = \frac{1}{n}\sum_{i=1}^{n} a_{ij}$ and $\bar{w} = \frac{1}{n}\sum_{i=1}^{n} w_i = \frac{1}{n}$

For a comparison matrix $A = (a_{ij})_{nxn}, asa_{ij} = w_1/w_j$, with $(i, j = 1, 2, ..., n)$ and $\bar{a}_j = \bar{w}/w_j$ for $(j = 1, ..., n)$, Equation 4 can be written as :

$$R_j = \frac{\frac{1}{w_j}\sum_{i=1}^{n}(w_i - \bar{w})(w_i - \bar{w})}{(\sqrt{\frac{1}{w_j^2}\sum_{i=1}^{n}(w_i - \bar{w})^2})(\sqrt{\sum_{i=1}^{n}(w_i - \bar{w})^2})} \quad (5)$$

$$= 1, for\ j = 1, ..., n$$

Based on the expression above, the optimization model can be formulatee as follows.

$$MaximizeR = \sum_{j=1}^{n} R_j \sum_{j=1}^{n}\sum_{i=1}^{n} \frac{(a_{ij} - \bar{a})}{\sqrt{\sum_{i=1}^{n}(a_{ij} - \bar{a}_j)^2}} \cdot \frac{(w_i - \bar{w})}{\sqrt{\sum_{i=1}^{n}(w_i - \bar{w})^2}} \quad (6)$$

Consider that $\sum_{i=1}^{n} w_i = 1$, and $w_i \geq 0, fori = 1, ..., n$, and let

$$\hat{w}_i^* = \frac{(w_i - \bar{w})}{\sqrt{\sum_{i=1}^{n}(w_i - \bar{w})^2}} \quad (7)$$

$$b_{ij} = \frac{(a_{ij} - \bar{a}_j)}{\sqrt{\sum_{i=1}^{n}(a_{ij} - \bar{a}_j)^2}}, i, j = 1, ..., n \quad (8)$$

We obtain:
$\sum_{i=1}^{n} \hat{w}_i^2 = 1$ dan $\sum_{i=1}^{n} b_{ij}^2 = 1, (i, j = 1, ..., n)$

Due to the Equations 5 - 7, the optimization model can be transformed into the following expression.

$$MaximizeR = \sum_{j=1}^{n}\sum_{i=1}^{n} b_{ij}\hat{w}_i = \sum_{i=1}^{n}(\sum_{j=1}^{n} b_{ij})\bar{w})_1 \quad (9)$$

We can derive Theorems from that optimization model.

Theorem 1.
Let $\hat{w}_i^*$ be the solution of optimization model and R^* is the value of objective function, then:

$$\hat{w}_i^* = \frac{\sum_{j=1}^{n} b_{ij}}{\sqrt{\sum_{i=1}^{n}(\sum_{j=1}^{n} b_{ij})^2}} \quad (10)$$

$$R^* = \sqrt{\sum_{i=1}^{n}(\sum_{j=1}^{n} b_{ij})^2} \quad (11)$$

From Equation 6, we get

$$w_i = \bar{w} + \sqrt{\sum_{i=1}^{n}(w_i - \bar{w})^2}.w_i^*, (i \hat{=} 1, ..., n) \quad (12)$$

Let $\beta = \sqrt{\sum_{i=1}^{n}(w_i - \bar{w}))^2} \geq 0$, Equation 11 becomes:

$$w_i = \bar{w} + \beta\hat{w}_i = \frac{1}{n} + \beta\hat{w}_i, (i = 1, ..., n) \quad (13)$$

(The differences of value will give different priority vector).

To get the value of parameter, we could solve two optimization model, as follows:

$$J = \sum_{i=1}^{n}(\sum_{j=1}^{n}(w_i - a_{ij}w_j)^2 = \sum_{i=1}^{n}(\sum_{j=1}^{n}\left[\beta(\hat{w}_i - a_{ij}\hat{w}_j^*) - \frac{1}{n}(a_{i,j} - 1)\right]^2, for\beta \geq 0 \quad (14)$$

$$J = \sum_{i=1}^{n}(\sum_{j=1}^{n}(a_{ij} - \frac{w_i}{w_j})^2) = \sum_{i=1}^{n}(\sum_{j=1}^{n}(a_{ij} - \frac{1/n + \beta\hat{w}_i^*}{1/n + \beta\hat{w}_j^*}, for\beta \geq 0 \quad (15)$$

Teorema 2.

$$\beta^* = \frac{\sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij}-1)(\hat{w}_i^* - a_{ij}\hat{w}_j^*)}{n\sum_{i=1}^{n}\sum_{j=1}^{n}(\hat{w}_i^* - a_{ij}\hat{w}_j^*)} \tag{16}$$

To determine the priority of a consistent pair comparison matrix, the CCMA determines the following steps:

- Step 1. Normalize the pairwise comparison matrix using Equation 8.
- Step 2. Calculate the weight of the transformation using Equation 10 and maximizing the sum on the correlation coefficient with Equation 11,
- Step 3. Determine the value of the coefficient with Equation 16.
- Step 4. Calculate the final priority $(i = 1,2,...,n)$, with the Equation 13.

4 CONCLUSIONS

This study proposes a model for determining priorities in decision-making using the Analytic Hierarchy Process (AHP) and the Correlation Coefficient Maximization Approach (CCMA) to be applied to the selection of school principals by a foundation, where three alternatives and four criteria must be determined, namely knowledge, quality of work, responsibility and work discipline. The results show that the priority order through CCMA and Additive Normalization (AN) calculations is the same. The proposed model provides a competitive method for ranking alternatives in decision-making and can be used to determine priorities in complex decision scenarios.

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