General Studies on Electromagnetic Wave Transmission Behaviors with the High-Speed Dielectric Interface

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Abstract: The propagation of electromagnetic wave with high-speed moving medium is very complex and one of the important research contents of electromagnetic wave theory. It can not only reveal the characteristics of electromagnetic waves themselves and the properties of the medium, but also has important application value. In this paper, the laws of reflection and refraction of electromagnetic waves at the interface of high-speed moving medium are derived by using the Special Relativity transformation relations of electromagnetic field quantities and four-dimensional wave vectors between different inertial systems. Meanwhile the phenomena of total reflection, Brewster's angle and half-wave loss, etc. are discussed in detail, and some new useful results are obtained.

1 INTRODUCTION

The problem of the reflection and refraction of electromagnetic waves (the following are referred to as waves for simplicity) at the motion interface is fundamental to the electrodynamics transmission and has been valued by people for a long time, which is widely used in optics, radar engineering, radio astronomy and other aspects (Jean Van Bladel, 1984), and is also one of the core theories of remote sensing technology (Li Jindong, 2018). However, due to the excessive complexity, the problem mentioned above are rarely involved in classical electrodynamics textbooks (Guo Shuohong, 2008) in which the reflection and refraction laws with static interface have been discussed in more detail. Although it has been studied in some literatures, it is only discussed from some special angles that are easy to solve, such as the wave is incident from a vacuum to a medium moving parallel or perpendicular to the interface (Wen Shengle et al., 2001; Wei Renhuai et al., 2009), etc, but does not give universal solutions, especially the case that the medium interface moves along any direction.

The studies in this paper are mainly based on Lorentz transformation in Special Relativity and Maxwell's electromagnetic theory, taking plane wave as the research object for which has many advantages in theoretical research. Firstly, the case of incidence from vacuum to medium is discussed detailedly, and the reflection and refraction laws of waves at high-speed moving interface with arbitrary direction are obtained. Then it is extended to medium to medium. Finally the half-wave loss, total reflection and Brewster's angle, etc. are discussed in detail.

2 SOME USEFUL RELATIVISTIC TRANSFORMATIONS

Here we take laboratory system as Σ and the system at rest relative to the motion interface as Σ'. The incidence of plane wave from vacuum to medium is considered. In Σ, the electric field strength and wave vector of waves, normal direction vector and motion velocity of the interface, and relative refractive index of medium are $\vec{E}_i$, $\vec{k}_i$, $\vec{n}$, $\vec{v}$ and n_f respectively. The Lorentz transformations from Σ to Σ' of the electric field strength, wave vector and frequency of waves are as Eq.(1).

$$\vec{E}'=\gamma\left[\vec{E}+\vec{v}\times\vec{B}-\frac{\gamma}{\gamma+1}\frac{\vec{v}(\vec{v}\cdot\vec{E})}{c^2}\right],\quad \vec{k}'=\vec{k}+(\gamma-1)\frac{(\vec{k}\cdot\vec{v})\vec{v}}{|\vec{v}|^2}-\frac{\gamma\omega}{c^2}\vec{v},\quad \omega'=\gamma(|\vec{k}_i|c-\vec{v}\cdot\vec{k}_i) \quad (1)$$

For $\vec{n}'$ in Σ', $n_{//}r_{//}+n_{\perp}r_{\perp}+c=0$ and $n'_{//}r'_{//}+n'_{\perp}r'_{\perp}+c'=0$ are obtained by the decomposition of plane equations $\vec{n}\cdot\vec{r}+c=0$ in Σ and $\vec{n}'\cdot\vec{r}'+c'=0$ in Σ' into parallel and perpendicular to the velocity direction. By the length contraction, the Lorentz transformation of $\vec{n}'$ can be written as Eq.(2).

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$$\vec{n}'=\vec{n}+\left(\frac{1}{\gamma}-1\right)\frac{(\vec{n}\cdot\vec{v})\vec{v}}{|\vec{v}|^2} \qquad (2)$$

3 ELECTROMAGNETIC WAVE IN Σ'

In this section, the electric field strength, magnetic induction strength and wave vector of reflected and refracted waves in Σ′ will be obtained.

3.1 Wave Vector in Σ'

The wave vectors $\vec{k}_i', \vec{k}_R'$ and $\vec{k}_T'$ are taken to analyse in the direction parallel and perpendicular to $\vec{n}'$. Decomposition of $\vec{k}_i'$ into parallel and perpendicular to $\vec{n}'$ gets $\vec{k}_i'_{//}=(\vec{k}_i'\cdot\vec{n}')\vec{n}'/|\vec{n}'|^2$ and $\vec{k}_i'_{\perp}=\vec{k}_i'-\vec{k}_i'_{//}$. For $\vec{k}_R'$, $\vec{k}_R'_{//}=-\vec{k}_i'_{//}$ and $\vec{k}_R'_{\perp}=\vec{k}_i'_{\perp}$ can be obtained from the reflection law in static system (Guo Shuohong, 2008). For $\vec{k}_T'$, combining $\omega=|\vec{k}_i|c$ and the equal magnitude of the frequency of wave in different medium, $|\vec{k}_T'_{//}|^2+|\vec{k}_T'_{\perp}|^2=|\vec{k}_T'|^2=n_f^2\omega'^2/c^2=n_f^2|\vec{k}_i'|^2=n_f^2\left(|\vec{k}_i'_{//}|^2+|\vec{k}_i'_{\perp}|^2\right)$ can be obtained. The Snell's law in Σ′ is $n_f=\sin\theta'/\cos\alpha'$, where θ' is the incident angle and α' is the refraction angle. Combining above equations we can get $|\vec{k}_T'_{\perp}|=|\vec{k}_i'_{\perp}|$. Since $\vec{k}_T'_{\perp}$ and $\vec{k}_i'_{\perp}$ are both in the incident plane and perpendicular to $\vec{n}'$, they are parallel. Then Eq.(3) about $\vec{k}_R'$ and $\vec{k}_T'$ are as the following.

$$\vec{k}_R'=\vec{k}_i'-2\frac{(\vec{k}_i'\cdot\vec{n}')\vec{n}'}{|\vec{n}'|^2},\quad \vec{k}_T'=\vec{k}_i'+\left[\sqrt{(n_f^2-1)|\vec{n}'|^2|\vec{k}_i'|^2+(\vec{k}_i'\cdot\vec{n}')^2}-(\vec{k}_i'\cdot\vec{n}')\right]\frac{\vec{n}'}{|\vec{n}'|^2} \qquad (3)$$

According to the reflection law $\theta'=\varphi'$, the trigonometric functions of the incident angle θ', reflection angle φ' and refraction angle α' can be written as Eq.(4).

$$\begin{cases}\sin\theta'=\sin\varphi'=\dfrac{|\vec{k}_i'_{\perp}|}{|\vec{k}_i'|}=\dfrac{\sqrt{|\vec{n}'|^2|\vec{k}_i'|^2-(\vec{k}_i'\cdot\vec{n}')^2}}{|\vec{n}'||\vec{k}_i'|},\quad \sin\alpha'=\dfrac{|\vec{k}_T'_{\perp}|}{|\vec{k}_T'|}=\dfrac{|\vec{k}_i'_{\perp}|}{n_f|\vec{k}_i'|}=\dfrac{\sqrt{|\vec{n}'|^2|\vec{k}_i'|^2-(\vec{k}_i'\cdot\vec{n}')^2}}{n_f|\vec{n}'||\vec{k}_i'|}\\ \cos\theta'=\cos\varphi'=\dfrac{|\vec{k}_i'_{//}|}{|\vec{k}_i'|}=\dfrac{|\vec{k}_i'\cdot\vec{n}'|}{|\vec{n}'||\vec{k}_i'|},\quad \cos\alpha'=\dfrac{|\vec{k}_T'_{//}|}{|\vec{k}_T'|}=\dfrac{\sqrt{(n_f^2-1)|\vec{n}'|^2|\vec{k}_i'|^2+(\vec{k}_i'\cdot\vec{n}')^2}}{n_f|\vec{n}'||\vec{k}_i'|}\\ \tan\theta'=\tan\varphi'=\dfrac{\sin\theta'}{\cos\theta'}=\dfrac{\sqrt{|\vec{n}'|^2|\vec{k}_i'|^2-(\vec{k}_i'\cdot\vec{n}')^2}}{|\vec{k}_i'\cdot\vec{n}'|},\quad \tan\alpha'=\dfrac{\sin\alpha'}{\cos\alpha'}=\sqrt{\dfrac{|\vec{n}'|^2|\vec{k}_i'|^2-(\vec{k}_i'\cdot\vec{n}')^2}{(n_f^2-1)|\vec{n}'|^2|\vec{k}_i'|^2+(\vec{k}_i'\cdot\vec{n}')^2}}\end{cases} \qquad (4)$$

3.2 Electric Field Strength in Σ'

The normal direction of the incident plane in Σ′ is denoted as $\vec{m}'=\vec{k}_i'\times\vec{n}'/|\vec{k}_i'\times\vec{n}'|$. Because Fresnel's law (Guo Shuohong, 2008) studies the reflection and refraction coefficients relevant to the *s* and *p* components of waves, we need to know the magnitude of the *s* and *p* components $E_i'_{\perp}$ and $E_i'_{//}$ of the incident waves, where the direction of $\vec{E}_i'_{\perp}$ is along the normal direction $\vec{m}'$. Then $E_i'_{\perp}$ and $E_i'_{//}$ can be written as Eq.(5) by using $\vec{B}=\vec{k}\times\vec{E}/\omega$.

$$E_i'_{\perp}=\vec{E}_i'\cdot\vec{m}'=\vec{E}_i'\cdot\frac{\vec{k}_i'\times\vec{n}'}{|\vec{k}_i'\times\vec{n}'|},\quad E_i'_{//}=\vec{E}_i'\cdot\left(\vec{m}'\times\frac{\vec{k}_i'}{|\vec{k}_i'|}\right)=\vec{E}_i'\cdot\frac{\vec{k}_i'\times\vec{n}'\times\vec{k}_i'}{|\vec{k}_i'\times\vec{n}'\times\vec{k}_i'|} \qquad (5)$$

Fresnel's law in Σ′ is as Eq.(6).

$$\begin{cases}\dfrac{E_R'_{\perp}}{E_i'_{\perp}}=-\dfrac{\sin\theta'\cos\alpha'-\cos\theta'\sin\alpha'}{\sin\theta'\cos\alpha'+\cos\theta'\sin\alpha'},\quad \dfrac{E_R'_{//}}{E_i'_{//}}=\dfrac{(1-\tan\theta'\tan\alpha')(\tan\theta'-\tan\alpha')}{(1+\tan\theta'\tan\alpha')(\tan\theta'+\tan\alpha')}\\ \dfrac{E_T'_{\perp}}{E_i'_{\perp}}=\dfrac{2\cos\theta'\sin\alpha'}{\sin\theta'\cos\alpha'+\cos\theta'\sin\alpha'},\quad \dfrac{E_T'_{//}}{E_i'_{//}}=\dfrac{2\cos\theta'\sin\alpha'}{(\sin\theta'\cos\alpha'+\cos\theta'\sin\alpha')(\cos\theta'\cos\alpha'+\sin\theta'\sin\alpha')}\end{cases} \qquad (6)$$

Substituting the Eq.(4) into (6), $E_R'_{\perp}$, $E_T'_{\perp}$, $E_R'_{//}$ and $E_T'_{//}$ can be obtained as Eq.(7),

$$\begin{cases}E_R'_{\perp}=-\dfrac{a_-}{a_+}E_i'_{\perp},\quad E_R'_{//}=\dfrac{b_-}{b_+}\dfrac{a_-}{a_+}E_i'_{//}\\ E_T'_{\perp}=\dfrac{a_+-a_-}{a_+}E_i'_{\perp},\quad E_T'_{//}=n_f\dfrac{2(b_+-b_-)+(a_+-a_-)^2}{4b_+}\dfrac{a_+-a_-}{a_+}E_i'_{//}\end{cases} \qquad (7)$$

Where $a_{\pm}=\left[(n_f^2-1)|\vec{n}'|^2|\vec{k}_i'|^2+(\vec{k}_i'\cdot\vec{n}')^2\right]^{1/2}\pm|\vec{k}_i'\cdot\vec{n}'|$, $b_{\pm}=|\vec{k}_i'\cdot\vec{n}'|\left[(n_f^2-1)|\vec{n}'|^2|\vec{k}_i'|^2+(\vec{k}_i'\cdot\vec{n}')^2\right]^{1/2}\pm\left[|\vec{n}'|^2|\vec{k}_i'|^2-(\vec{k}_i'\cdot\vec{n}')^2\right]$.

Finally, from Eq.(5)-(7), electric field strengths in Σ′ are written as Eq.(8). And the magnetic induction strength in Σ′ can be written by using $\vec{B}=\vec{k}\times\vec{E}/\omega$ as Eq.(9).

$$\vec{E}_R'=E_R'_{//}\frac{\vec{k}_i'\times\vec{n}'\times\vec{k}_R'}{|\vec{k}_i'\times\vec{n}'\times\vec{k}_R'|}+E_R'_{\perp}\frac{\vec{k}_i'\times\vec{n}'}{|\vec{k}_i'\times\vec{n}'|},\quad \vec{E}_T'=E_T'_{//}\frac{\vec{k}_i'\times\vec{n}'\times\vec{k}_T'}{|\vec{k}_i'\times\vec{n}'\times\vec{k}_T'|}+E_T'_{\perp}\frac{\vec{k}_i'\times\vec{n}'}{|\vec{k}_i'\times\vec{n}'|} \qquad (8)$$

$$\vec{B}_R'=\frac{1}{\omega'}\left[E_R'_{//}\frac{|\vec{k}_i'|^2\,\vec{k}_i'\times\vec{n}'}{|\vec{k}_i'\times\vec{n}'\times\vec{k}_R'|}-E_R'_{\perp}\frac{\vec{k}_i'\times\vec{n}'\times\vec{k}_R'}{|\vec{k}_i'\times\vec{n}'|}\right],$$

$$\vec{B}_T'=\frac{1}{\omega'}\left[E_T'_{//}\frac{n_f^2|\vec{k}_i'|^2\,\vec{k}_i'\times\vec{n}'}{|\vec{k}_i'\times\vec{n}'\times\vec{k}_T'|}-E_T'_{\perp}\frac{\vec{k}_i'\times\vec{n}'\times\vec{k}_T'}{|\vec{k}_i'\times\vec{n}'|}\right] \qquad (9)$$

4 ELECTROMAGNETIC WAVE IN Σ

In this section, we will give the electric field strength and wave vector of the reflected and refracted waves in Σ based on the results in section III, and then the reflection coefficient, refraction coefficient, half-wave loss, total reflection law and Brewster's law will be analyzed in detail.

4.1 Electric Field Strength and Wave Vector in Σ

Because both incident and reflected waves are in vacuum, the Lorentz transformations of the electric field strength and wave vector of reflected wave in Eq.(10) are familiar as the incident wave.

$$\vec{E}_R=\gamma\left[\vec{E}_R'-\vec{v}\times\vec{B}_R'-\frac{\gamma}{\gamma+1}\frac{\vec{v}(\vec{v}\cdot\vec{E}_R')}{c^2}\right],\quad \vec{k}_R=\vec{k}_R'+(\gamma-1)\frac{(\vec{k}_R'\cdot\vec{v})\vec{v}}{|\vec{v}|^2}+\frac{\gamma\omega'}{c^2}\vec{v} \qquad (10)$$

The Lorentz transformation of wave vector of the refracted wave is the same as the incident wave for it is independent of the medium. However, when there is a medium, the Lorentz transformation of electric field strength $\vec{E}$ should be replaced by the Lorentz transformation of electric displacement vector $\vec{D}$. The Lorentz transformation of $\vec{D}$ along the X-axis direction has been given in reference (Liu Liao et al., 2008). Analogously to the Lorentz transformation of $\vec{E}$, the Lorentz transformation of $\vec{D}$ along any direction is written as $\vec{D}'=\gamma\left[\vec{D}+\vec{v}\times\vec{H}/c^2-\gamma\vec{v}\left(\vec{v}\cdot\vec{D}\right)/(\gamma+1)/c^2\right]$. Where $\vec{D}=\varepsilon\vec{E}$, $\vec{H}=\vec{B}/\mu$, ε and μ are permittivity and magnetic permeability in the relative static medium respectively, and $\mu\approx\mu_0$ for the ordinary linear medium. By wave velocity $v=(\mu\varepsilon)^{-1/2}$, vacuum light velocity $c=(\mu_0\varepsilon_0)^{-1/2}$ and $n_f=c/v$, we can get $\vec{E}_T$ and $\vec{k}_T$ as in Eq.(11).

$$\vec{k}_T=\vec{k}_T'+(\gamma-1)\frac{(\vec{k}_T'\cdot\vec{v})\vec{v}}{|\vec{v}|^2}+\frac{\gamma\omega'}{c^2}\vec{v},\quad \vec{E}_T=\frac{\vec{D}_T}{\varepsilon}=\frac{\vec{D}_T}{\varepsilon_0 n_f^2}=\gamma\left[\vec{E}_T'-\frac{1}{n_f^2}\vec{v}\times\vec{B}_T'-\frac{\gamma}{\gamma+1}\frac{\vec{v}(\vec{v}\cdot\vec{E}_T')}{c^2}\right] \quad (11)$$

The above results can be demonstrated visually in Fig.1. Fig.1 (a) vs. (c) and (d) show the refractive index of medium and the speed of the interface have the significant influence on the reflected and refracted waves respectively. While it can be seen from Fig.1 (a) vs. (b) that frequency ω has no effect on the reflected and refracted waves.

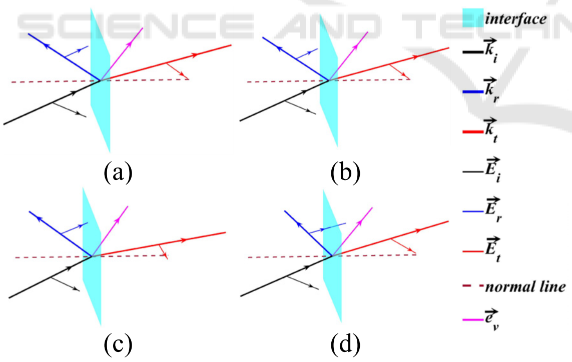
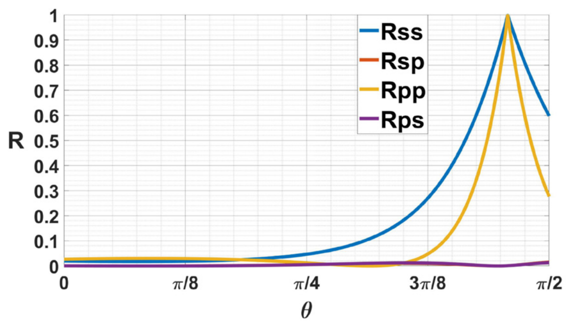


Figure 1. Demonstration of incident, reflected and refracted waves from at the interface between vacuum and the moving medium. The subscript i, R and T represent incident, reflected and refracted waves respectively, and the relative length represents the absolute value of corresponding vector. $\vec{n}$ and $\vec{e}_v$ represent the normal direction and motion direction of the interface respectively. The conditions of (a) are $\omega=2\pi\cdot 50$kHz, $\vec{E}_i=\vec{e}_x+\vec{e}_y-2\vec{e}_z$ (V/m), $n_f=1.5$, $\vec{e}_{ki}=\left(\vec{e}_x+\vec{e}_y+\vec{e}_z\right)/\sqrt{3}$, $\vec{v}=0.5c\left(\vec{e}_x+2\vec{e}_y+3\vec{e}_z\right)/\sqrt{14}$ (m/s) and $\vec{n}=\vec{e}_x$, while the conditions of (b, c, d) vary only in (b) $\omega=2\pi\cdot 500$kHz, (c) $n_f=2.5$ and (d) $\vec{v}=0.75c\left(\vec{e}_x+2\vec{e}_y+3\vec{e}_z\right)/\sqrt{14}$ (m/s) separately.

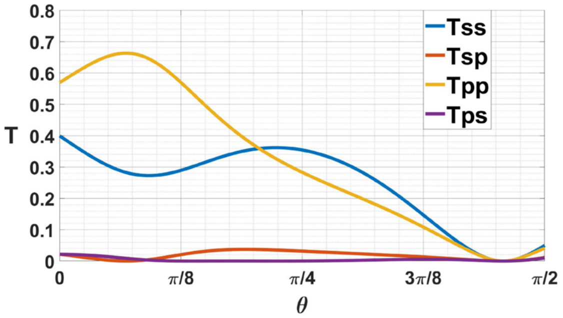
Combining $\omega=\left|\vec{k}_i\right|c$ and Eq.(1), $\vec{k}_i'=\omega[\vec{e}_{ki}+(\gamma-1)(\vec{e}_{ki}\cdot\vec{v})\vec{v}/|\vec{v}|^2-\gamma\vec{v}/c]/c$ is obtained. It is obvious that the direction of the incident wave in Σ' is independent of ω. Through the analysis of Eq.(4) and (6), it can be seen that $\left|\vec{k}_i'\right|$ has no effect on Eq.(6). Considering Eq.(8), it can be concluded that ω has no effect on electric field strength direction of the reflected and refracted waves. Then the frequency of wave has no effect on the electric field strength of the reflected and refracted waves.

4.2 Coefficients of Reflection and Transmission

In order to show more clearly the electric field strengths of the reflected and refracted waves, we make the curve of reflection coefficient R and refraction coefficient T with the change of incident angle θ as Fig.2. Among them, there are four cases of reflection coefficient and refraction coefficient respectively. According to $\omega=\left|\vec{k}_i\right|c$ and Eq.(1), when electric field strength and wave vector are transformed between Σ and Σ', the magnitude and direction of the transformed electric field strength and wave vector are affected by $\vec{v}$. Thus, reflected and refracted waves will exist p (s) wave component when the incident wave is just s (p) wave. Hence the reflected and refracted waves would have different polarization types from the incident wave, just like Faraday effect and Kerr effect which are caused by some characteristics of special media. Therefore, R and T can be divided into $R_{\alpha\beta}$ and $T_{\alpha\beta}$ respectively, where α, β=s, p. $R_{\alpha\beta}$ ($T_{\alpha\beta}$) refers to the ratio of the intensity of the β wave of reflected (refracted) wave to the intensity of the α wave of incident wave, where $R_{\alpha\beta}=E_{R\alpha}^2/E_{i\beta}^2$ and $T_{\alpha\beta}=E_{T\alpha}^2/E_{i\beta}^2$. As can be seen from the Fig.2, $R_{\alpha\beta}$ and $T_{\alpha\beta}$, $\alpha=\beta$ are significantly greater than $R_{\alpha\beta}$ and $T_{\alpha\beta}$, $\alpha\neq\beta$ in most angle ranges.



(a)



(b)

Figure 2. The relation of (a) reflection coefficient R and (b) refraction coefficient T with the incident angle θ, where the incident wave of $\omega=2\pi\cdot 50$kHz, $\vec{E}_i=\vec{e}_x+\vec{e}_y-2\vec{e}_z$ (V/m) and wave vector in the XOY plane are reflected and refracted at the interface of $\vec{n}=\vec{e}_x$, $n_f=1.5$ and $\vec{v}=0.5c\left(\vec{e}_x+2\vec{e}_y+3\vec{e}_z\right)/\sqrt{14}$ (m/s).

4.3 Half-Wave Loss, Total Reflection and Brewster's Law

Half-wave loss, total reflection and Brewster's law are the classical behaviors of waves at interface. The influence of high-speed moving interface on these phenomena are discussed in the following. According to the first formula of Eq.(7), the requirement for half-wave loss is $(n_f^2-1)|\vec{n}'|^2|\vec{k}_i'|^2>0$. It is obvious that when $n_f>1$ there is half-wave loss between the incident and the reflected waves. Therefore, the condition of half-wave loss at the moving interface is the same as stationary interface. According to Eq.(3), when $|\vec{k}_{T}'{}_{//}|^2\leq 0$, the wave vector in the medium along the normal direction is imaginary and the total reflection would happen. The critical angle condition for satisfying total reflection is $\sin\theta'\geq n_f$. The meaningful condition for the above is $n_f<1$, while here is no total reflection of incident from vacuum to medium. In other word, the incident wave should be from the optically denser medium to the thinner one. Total reflection critical angle θ' increases with n_f. The Brewster's law says that the component of electric field strength parallel to incident plane in reflected wave is vanishing, that is to say, $E_R'{}_{//}=0$ and $E_R'{}_{\perp}\neq 0$. The Brewster's angle condition of $\cos\theta'_b=[n_f^2/(n_f^2+1)]^{1/4}$ can be obtained by substituting $\theta'\neq\alpha'$, $E_R'{}_{//}=0$ and $E_R'{}_{\perp}\neq 0$ into Eq.(4), and $\cos\theta'$ (θ') increases (decreases) with n_f.

5 FURTHER DISCUSSION ON INCIDENCE FROM ONE MEDIUM TO ANOTHER

The incidence from vacuum to medium has been studied in former sections, and the case of one medium to another would be discussed based on the above. There are two points which should be considered here: firstly, the Lorentz transformation of the field strength of the incident and reflected waves should use $\vec{D}$ instead of $\vec{E}$; secondly, when the phenomenon of dispersion is considered, the relative refractive index has the relation with the interface velocity, and then it will affect the rule of half-wave loss, total reflection and Brewster's phenomenon.

5.1 Effect on Fresnel's Law

The Lorentz transformation of the field strength in medium is different from the one in vacuum. When wave spreads in medium, it is only need to rewrite the Lorentz transformations $\vec{E}_i$ to $\vec{E}_i'$ and $\vec{E}_R'$ to $\vec{E}_R$ by imitating the second formula in Eq.(11), and no other equations need to be changed. The Fresnel's law of wave incident from one medium to another and the interface moving at high speed can be obtained.

5.2 Effect on Half-Wave Loss, Total Reflection and Brewster's Law

The relative refractive index of medium to vacuum is greater than 1 surely, but the one between two media may have all kinds of possible values. In this paper, the reflection and refraction of waves at interface are analyzed from Σ'. The equations for reflection and refraction laws in Σ are derived by using the equations for reflection and refraction laws in Σ'. However, the reflected and refracted behaviors of wave at interface is related to the relative refractive index concerned with two media. And the n_f in the previous sections also should have relation with the wave frequency ω' in Σ', that is $n_f=n_f(\omega')$, hence the conditions of half-wave loss, total reflection and Brewster's law in section IV are also related to $n_f(\omega')$. The relation between the refractive index of medium and the wavelength in vacuum satisfies the Cauchy's dispersion formula $n_f(\lambda')=A+B/\lambda'^2+C/\lambda'^4$ [7], which is rewritten as $n_f(\omega')=A+B\omega'^2/(4\pi^2c^2)+C\omega'^4/(16\pi^4c^4)$ by using $\lambda'=2\pi c/\omega'$, where λ' and ω' are wavelength and frequency in Σ' and A, B and C are the coefficients determined by experiments. Eq.(1) shows that the ω' of the wave changes with $\vec{v}$, and the range of ω'

is $\gamma|\vec{k}_i|(c\text{-}v)\leq\omega'\leq\gamma|\vec{k}_i|(c+v)$, or $\gamma(1\text{-}\beta)\omega\leq\omega'\leq\gamma(1+\beta)\omega$. Suppose there exist two suitable media 1 and 2, and their refractive indexes satisfy the following conditions: the ω' is smaller but greater than $\gamma(1-\beta)\omega$, and the refractive indexes of the two media meet $n_{f1}(\omega')<n_{f2}(\omega')$. In other word, the relative refractive index $n_{f21}>1$. With the increase of ω', when ω' is greater but less than $\gamma(1+\beta)\omega$, the refractive indexes of the two media can meet $n_{f1}(\omega')>n_{f2}(\omega')$, that is, the relative refractive index $n_{f21}<1$. There is a special case of $n_{f21}=1$ between the two cases. When the media are determined, both $n_{f1}(\omega')$, $n_{f2}(\omega')$ and n_{f21} are uniquely determined by ω', while ω' is determined by ω and $\vec{v}$ according to Eq.(1). Hence $\vec{v}$ is bound to affect the laws of half-wave loss, critical angle of total reflection, and Brewster's angle. However, the relations of refractive indexes and wave frequencies are so complex in fact, and it is difficult to have a unified theoretical formula to describe for different media. This is another interesting subject valuable to investigate deeply in the next.

6 CONCLUSION

In this paper, the problem of electromagnetic waves transmission at high-speed interface moving in any direction has been studied extensively. Firstly, the case of incidence from vacuum to medium is discussed, and the reflection and refraction laws are obtained by Lorentz transformation, Snell's law and Fresnel's law, which are different vastly from the laws in static system. Besides the half-wave loss, total reflection and Brewster's law are analyzed. Furthermore, the case of incidence from one medium to another is also discussed. In the specific application, according to the method and theoretical formula in this paper, the final results needed can be obtained by numerical calculation by substituting concrete conditions like ω, n_f and $\vec{v}$.

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