

Multi-Coil Electromagnetic Field Calculation with Two Methods and Software Implementation

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Abstract: There are many methods to calculate multi-coil composite electromagnetic fields, but which one is optimal requires further research. The best method should take into account the calculation efficiency and calculation accuracy. We want the fastest operation time to obtain the ideal accuracy. This paper discusses the two methods of series expansion and elliptic integration to calculate the multi-coil composite electromagnetic field, finds that the elliptic integral method has better operation effect. Series expansion can be a supplemental option for certifying result. Lastly, A software program is written to realize data input, start calculation and final result display with a good human-machine interface.

1 INTRODUCTION

Lacking specialized high precision calculation tool for electromagnetic field strength, we set about developing a kind of magnetic field calculation program software. After contrast between series expansion method and elliptic integral method, we find the two methods are both effective on solving circle current electromagnetic field, but the former is much easier to control the resultant precision in computer operation. Therefore, besides the traditional elliptic integral method, series expansion method affords another newer effective method optional for us when we do high precision magnetic field calculation. They are both convenient for us to figure out a visualized magnetic cloud map meeting our majority usage case.

Calculation for kinds of shapes of coil group is badly needed with high frequency in High Magnetic Field Laboratory of Chinese Academy of Science (Ren Yong - He P.) and other institutional units. The demand of the calculation is normally high-precision (generally exceeding 0.01ppm). There is a consistent rigid demand to work out many types of coil groups' field strengths or homogeneity.

When we calculate the magnetic field strength, we normally use these formulas or methods: like as Biot-Savart Law (Binder P-M, 2016), Laplace Equations (Du Di, 2016), Maxwell Equations (Arbab A.I., Chovan Jaroslav), and so on. Those will further turn

out some complicated integral equations or other forms, often impossibly solved artificially.

On another hand, magnets are composed of multiple coils with different locations and various shape dimensions. A large amount of superconducting wires are enriched in variable currents and section shapes. All of these factors are hardly difficult to calculate manually.

So, developing a kind of high fitness calculation program based on high speed computer numerical calculation is the only way to face all of these hard cases (Zhilichev Y, Takahashi Keita).

We hope the program can be widely used in almost all kinds of magnet figuring. In this case, we choose Matlab as development platform, because its high performance of matrix arithmetic capability, visualized figure output character, and fine operate accuracy are attracted deeply for majority programmers (Ramana Reddy J.V.- Fang Xiaorong), also for us.

The software adopts plenty of functions and modularity program structures. That makes it have strong readability and good transplantable. We preserve some modular interfaces for latter improvements for specific conditions and functional extensions. It has a good graphic user interface for convenient and efficient inputting kinds of parameters. The number for coils countable is limitless expandable. So it has a widely usage in practical magnetic calculation fields.

Computational accuracy is sufficient high for MRI, NMR (Fang Xiaorong, 2014), and normal electromagnetic calculations. The theory accuracy reaches 10-14 estimated by software's platform. Actual measured accuracy arrived above 10-11 on desktop computers. If running on large scale high performance workstation, the computational accuracy should get further improvement.

2 MATHEMATICAL METHODS

There are two methods for counting single circular current loop so far. They are separately series expansion method and elliptic integral method. They are both suitable for computer numerical calculation. Correspondingly to two version programs are developed according to the methods. The distinction between the two methods is present in their modularization core processing function, and different in input user interface. A brief overview summarized for these calculating methods are shown as follows.

2.1 Series Expansion Method

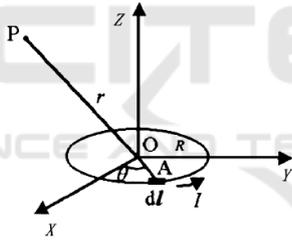


Figure 1: The calculating model based on circular current loop.

Suppose a circular current loop put in vacuum circumstance. Its radius is R . Current density is I . Establish a Cartesian coordinate system shown in Figure 1. By the symmetry, we can locate a point $P(x, z)$ by random. Select a current element Idl of the tiny part A on the circular loop (see Fig. 1).

So, by Biot-Savart Law, we can get the differential value of magnetic flux density at point P expressed as

$$d\mathbf{B} = \frac{\mu_0}{4\pi} \cdot \frac{Idl \times \mathbf{r}}{r^3}. \quad (1)$$

In which,

$$\begin{aligned} \mathbf{r} &= \overrightarrow{OP} - \overrightarrow{OA} \\ &= (X - R\cos\theta)\mathbf{i} - R\sin\theta\mathbf{j} + Z\mathbf{k} \\ r &= (R^2 + Z^2 + X^2 - 2RX\cos\theta)^{1/2} \end{aligned}$$

$$\begin{aligned} d\mathbf{l} &= d(R\cos\theta\mathbf{i} + R\sin\theta\mathbf{j}) \\ &= -Rd\theta\sin\theta\mathbf{i} + Rd\theta\cos\theta\mathbf{j} \end{aligned}$$

Put $d\mathbf{l}$ 、 \mathbf{r} 、 r into (1). By integral operation on angle θ in the interval $[0, 2\pi]$, we get the magnetic flux density generated by current element A at point P:

$$B_x = \frac{\mu_0 RI}{4\pi} \int_0^{2\pi} \frac{Z\cos\theta d\theta}{(R^2 + Z^2 + X^2 - 2RX\cos\theta)^{3/2}} \quad (2)$$

$$\begin{aligned} B_y &= \frac{\mu_0 RI}{4\pi} \int_0^{2\pi} \frac{Z\sin\theta d\theta}{(R^2 + Z^2 + X^2 - 2RX\cos\theta)^{3/2}} \\ &= \frac{\mu_0 RI}{4\pi} \left[\int_0^{\pi} \frac{Z\sin\theta d\theta}{(R^2 + Z^2 + X^2 - 2RX\cos\theta)^{3/2}} \right. \\ &\quad \left. + \int_{\pi}^{2\pi} \frac{Z\sin\theta d\theta}{(R^2 + Z^2 + X^2 - 2RX\cos\theta)^{3/2}} \right] \quad (3) \\ &= \frac{\mu_0 RI}{4\pi} \left[\int_0^{\pi} \frac{Z\sin\theta d\theta}{(R^2 + Z^2 + X^2 - 2RX\cos\theta)^{3/2}} \right. \\ &\quad \left. + \int_{-\pi}^0 \frac{Z\sin\theta d\theta}{(R^2 + Z^2 + X^2 - 2RX\cos\theta)^{3/2}} \right] \\ &= 0 \end{aligned}$$

Result zero is derived from the zero summation of odd function integral in the symmetric interval.

$$B_z = \frac{\mu_0 RI}{4\pi} \int_0^{2\pi} \frac{(R - X\cos\theta)d\theta}{(R^2 + Z^2 + X^2 - 2RX\cos\theta)^{3/2}} \quad (4)$$

Assuming dimensionless variables $z = Z/R$, $x = X/R$, substituting the magnetic flux density $B_0 = \mu_0 I / 2R$ at the center of the circle as the reference value, and writing $b_x = B_x / B_0$,

$b_z = B_z / B_0$, (2) and (4) can be reformed as:

$$b_x = \frac{\mu_0 RI}{2\pi} \int_0^{2\pi} \frac{z\cos\theta d\theta}{(1 + z^2 + x^2 - 2x\cos\theta)^{3/2}} \quad (5)$$

$$b_z = \frac{\mu_0 RI}{2\pi} \int_0^{2\pi} \frac{(1 - x\cos\theta)d\theta}{(1 + z^2 + x^2 - 2x\cos\theta)^{3/2}} \quad (6)$$

Substituting $\eta = 1 + z^2 + x^2$, $q = 2x/\eta$, we get

$$b_x = \frac{1}{2\pi} z\eta^{-3/2} \int_0^{2\pi} (1 - q\cos\theta)^{-3/2} \cos\theta d\theta \quad (7)$$

$$b_z = \frac{1}{2\pi} z\eta^{-3/2} \int_0^{2\pi} (1 - q\cos\theta)^{-3/2} (1 - x\cos\theta) d\theta \quad (8)$$

Operating $(1 - q\cos\theta)^{-3/2}$ with Taylor expansion, the finally results are got:

$$b_x = z\eta^{-3/2} \sum_{k=1}^{\infty} \left[\frac{3 \cdot 5 \cdots (4k-1)}{2 \cdot 4 \cdots (4k-2)} \cdot \frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdots (2k)} \cdot q^{2k-1} \right] \quad (9)$$

$$b_z = \eta^{-3/2} \left\{ 1 + \frac{1}{2} \sum_{k=1}^{\infty} \left[\frac{3 \cdot 5 \cdots (4k-1)}{2 \cdot 4 \cdots (4k-2)} \cdot \frac{1 \cdot 3 \cdot 5 \cdots (2k-1)}{2 \cdot 4 \cdot 6 \cdots (2k)} \cdot \left(1 + \frac{1}{2k} - z^2 - x^2 \right) \cdot q^{2k} \right] \right\} \quad (10)$$

Equations (3), (9) and (10) comprise the magnetic series solution of single circular current loop (Pawel Bienkowski, 2012). These kinds of gradual convergence expressions of series in last term are especially suitable for multiply accumulative calculation within computer solving.

By controlling the last term's accuracy of series solution, we can limit the finally resultant numerical precision in a needed range.

2.2 Elliptic Integral Method

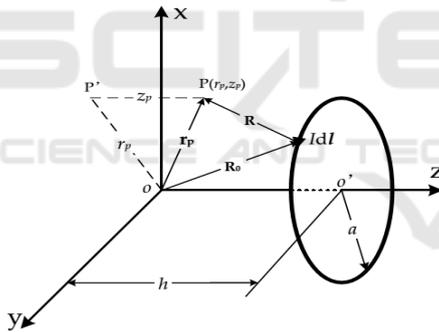


Figure 2: Calculate circular current loop by elliptic integral.

For a ideal current carrying ring of center coordinate O(0,0,h), radius a, current I (shown in Fig. 2), The magnetic flux density at arbitrary space point P(rp,zp) is expressed as (Reich Felix A., Martin N.):

$$\begin{cases} B_r(r_p, z_p) = \frac{\mu_0 I}{2\pi} \frac{h - z_p}{r_p \sqrt{(r_p + a)^2 + (z_p - h)^2}} \cdot \left[K(k) - \frac{r_p^2 + a^2 + (z_p - h)^2}{(r_p - a)^2 + (z_p - h)^2} E(k) \right] \\ B_z(r_p, z_p) = \frac{\mu_0 I}{2\pi} \frac{1}{\sqrt{(r_p + a)^2 + (z_p - h)^2}} \cdot \left[K(k) - \frac{r_p^2 - a^2 + (z_p - h)^2}{(r_p - a)^2 + (z_p - h)^2} E(k) \right] \end{cases} \quad (11)$$

Where, μ_0 is vacuum permeability,

$$k = \sqrt{4r_p a / [(r_p + a)^2 + (z_p - h)^2]}$$

$K(k)$ and $E(k)$ are separately elliptic integral of the first kind and the second kind, and expressed as:

$$\begin{cases} K(k) = \int_0^{\pi/2} \frac{d\alpha}{\sqrt{1 - k^2 \sin^2 \alpha}} \\ E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \alpha} d\alpha \end{cases} \quad (12)$$

The advantage of this kind of method is that we can directly utilize the integrated elliptic integral computing module integrated in Matlab to figure it out. So the computing efficiency is very high. Computational accuracy can be controlled by pass parameter Tol in the module. That is unnecessary to expand the elliptic integral for recalculating in program manually.

2.3 Other Methods

Besides the elliptic integral and series expansion method, some other methods (Vlasko-Vlasov V. K., Pathak Aritro.) also to be considered as choices for calculating the magnetic field intensity of solenoid with electric current are as follows: calculating with integral module integrated in Matlab software (numerical/character expression integral module), reckon in helical angle on solenoid (see Fig. 3), arithmetic combining helical angle with section shape (see Fig. 4). For separately defects, these methods are nullified finally. The flaws are described as follows:

Calculating with integral module integrated in Matlab software includes numerical and character expression integral module. The arithmetic speed is too slow despite higher accuracy. It may be spend 20 minutes in computing 10 current circles with a normal desktop computer. For certain coil, it has usually size of several hundreds of current circles. In this case, the operating time is too long to acceptable.

The efficiency of reckoning in helical angle is also very slow. Despite the module is more accurate, the calculating speed is unimaginable. Because the relative value of coil diameter to conductor diameter is very high (generally larger than 100), causing the lead angle less than $\text{atan2}(1, \pi * 100) * 180 / \pi = 0.1824^\circ = 10'57''$, facing such a little angle value, and for pursuing high efficient calculation, the lead angle should be negligible.

The computing speed is mostly unimaginable low if helical angle and section shape are concur-

rently put into account. For a desktop computer, the computing may lead to system halted, also even for small workstation. It is completely unnecessary when conductor diameter is far less than coil diameter because computational efficiency is more appreciative than tiny precision improvement.

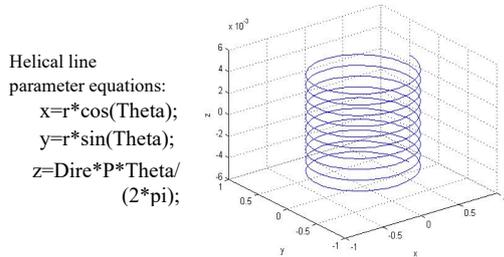


Figure 3: Coil model accounted helical angle.

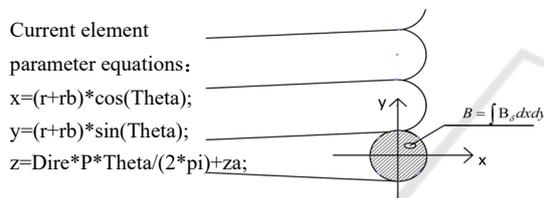


Figure 4: Coil model accounted conductor section shape.

3 PROGRAM IMPLEMENTATION

We chose Matlab software to apply the calculation method because of its high computational efficiency, nice precision, favorable visualization, running independently and good platform portability (maybe suitable for Windows, Linux, Unix, Ios, etc.).

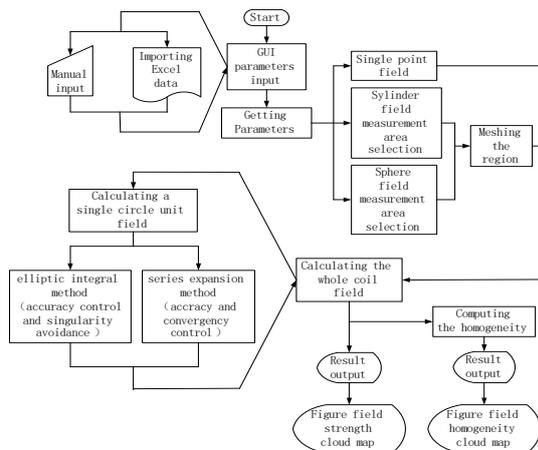


Figure 5: The basic principle flow chart of arithmetic procedure.

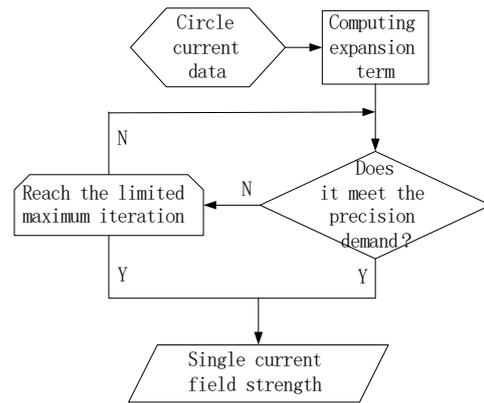


Figure 6: Internal series expansion count process flow chart.

The basic principle flow chart of arithmetic procedure (see Fig. 5) and internal series expansion count process are drawn in Figure 6.

4 RESULTS AND CONCLUSIONS

A superconducting magnet consisted of 4 coils in detail listed in Table 1 is tested. We found the results (see Fig. 7) are highly accordant between elliptic integral method and series expansion method. So the series expansion method is verified as an effective method well as traditional elliptic integral method. The former's accuracy can be easily controlled by total expanding term numbers.

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Table 1: Configurations of superconducting magnet coils.

Inner radius (mm)	Outer radius (mm)	Section coordinate Zmin(mm)	Section coordinate Zmax(mm)	Radial turns	Axial layers	Current (A)
49	100	35	145	52	100	114.62
49	100	-35	-145	52	100	114.62
113	165	30	150	62	92	114.62
113	165	-30	-150	62	92	114.62
165.5	200	30	150	50	100	114.62
165.5	200	-30	-150	50	100	114.62

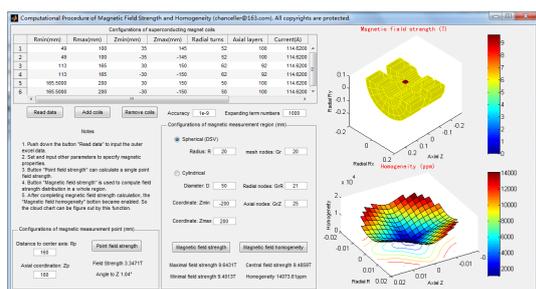


Figure 7: Calculation results of magnetic field strength and homogeneity.

REFERENCES

Ren Yong, Kuang Guangli, Chen Wenge. Analysis of the Experimental Results for the Nb3Sn Model Coil Being Built in China[J]. *Journal of Superconductivity and Novel Magnetism*. Oct. 2015, Vol. 28, No. 10, pp. 2941-2944. <https://doi.org/10.1007/s10948-015-3110-x>

Yong Ren, Guangli Kuang, Wenge Chen. Quench Simulation of the Hybrid Magnet Superconducting Outsert Being Built in China[J]. *Journal of Fusion Energy*. Oct. 2015, Vol. 34, No. 5, pp. 1146-1150. <https://doi.org/10.1007/s10894-015-9933-4>

Yong Ren, Guangli Kuang, Wenge Chen. Analysis of the Magnetic Forces Generated in the Hybrid Magnet Being Built in China[J]. *Journal of Fusion Energy*. Aug. 2015, Vol. 34, No. 4, pp. 733-738. <https://doi.org/10.1007/s10894-015-9876-9>

He P., Chen W. G., Chen Z. M. Pressure Drop Calculation of Gas Path for Heat Treatment System of Nb3Sn Superconducting Coils[J]. *Physica C-Superconductivity and Its Applications*. Jun. 2012, Vol. 476, pp. 73-76. <https://doi.org/10.1016/j.physc.2012.02.011>

Binder P-M, Grace Alyssa L., Hui Kaleonui J. Magnetic Field in the Plane of a Physical Dipole[J]. *European Journal of Physics*. Jul. 2016, Vol. 37, No. 4. <https://doi.org/10.1088/0143-0807/37/4/045203>

Du Di, He Peng, Zeng Yongchao. Dynamics of paramagnetic squares in uniform magnetic fields[J]. *Journal of Magnetism and Magnetic Materials*. Nov. 2016, Vol. 417, pp. 100-105. <https://doi.org/10.1016/j.jmmm.2016.05.032>

Arbab A.I., Is the Electromagnetic Field in a Medium a Fluid or a Wave?[J]. *Optik*, Feb. 2017, Vol. 130, pp. 154-161. <https://doi.org/10.1016/j.ijleo.2016.10.130>

Chovan Jaroslav, Slodicka Marian. Induction Hardening of Steel with Restrained Joule Heating and Nonlinear Law Formagnetic Induction Field: Solvability[J]. *Journal of Computational and Applied Mathematics*. Feb. 2017, Vol. 311, pp. 630-644. <https://doi.org/10.1016/j.cam.2016.08.016>

Zhilichev Y. Simulation of Critical Currents in Bulk Superconductors by Modified FEA and Hybrid Methods[J]. *IEEE Transactions on Applied Superconductivity*. Jun. 2017, Vol. 27, No. 4, pp. 5. <https://doi.org/10.1109/tasc.2016.2631475>

Takahashi Keita, Ainslie Mark D., Fujishiro Hiroyuki. Pulsed-Field Magnetizing Characteristics of Rectangular-Shaped Gd-Ba-Cu-O Bulk Using Split-and Solenoid-Type Coils[J]. *IEEE Transactions on Applied Superconductivity*. Jun. 2017, Vol. 27, No. 4. <https://doi.org/10.1109/TASC.2016.2625764>

Ramana Reddy J.V., Sugunamma V., Sandeep N. Effect of Frictional Heating on Radiative Ferrofluid Flow over a Slendering Stretching Sheet with Aligned Magnetic Field[J]. *European Physical Journal Plus*. Jan. 2017, Vol. 132, No. 1, pp. 7-20. <https://doi.org/10.1140/epjp/i2017-11287-1>

Evstafev A.M., Yakushev A.Ya., Sereda A.G. A Simulation Mathematical Model of a Traction Transformer with Tapped Secondary Windings[J]. *Russian Electrical Engineering*. May. 2016, Vol. 87, No. 5, pp. 275-281. <https://doi.org/10.3103/S1068371216050072>

Fang Xiaorong, Wang Nanfei, Zhang Jianfeng. Three-Dimensional Visualization of Maize Roots Based on Magnetic Resonance Imaging. Agriculture and Life Sciences[J]. *Journal of Zhejiang University*. 2014, Vol. 40, No. 2, pp. 181-187. <https://www.webofscience.com/wos/alldb/full-record/CSCD:5099879>

Pawel Bienkowski, Hubert Trzaska. Electromagnetic Measurements in the Near Field. *IET Digital Library; SciTech Publishing Inc*. 2012, pp. 89-111. <https://doi.org/10.1049/SBEW042E>

Reich Felix A., Stahn Oliver, Mueller Wolfgang H. The Magnetic Field of a Permanent Hollow Cylindrical Magnet[J]. *Continuum Mechanics and Thermodynamics*. Sep. 2016, Vol. 28, No. 5, pp. 1435-1444. <https://doi.org/10.1007/s00161-015-0485-7>

Martin N. Wilson. *Superconducting Magnets*. Clarendon Press Oxford. 1983, pp. 34-40.

Vlasko-Vlasov V. K., Glatz A., Koshelev A. E. Anisotropic Superconductors in Tilted Magnetic Fields[J]. *Physical Review B*. Jun. 2015, Vol. 91, No. 22. <https://doi.org/10.1103/PhysRevB.91.224505>

Pathak Aritro. An Elementary Argument for the Magnetic Field Outside a Solenoid[J]. *European Journal of Physics*. Jan. 2017, Vol. 38, No. 1. <https://doi.org/10.1088/0143-0807/38/1/015201>