Design of Double-Loop Trajectory Tracking Control System for Mobile Robot

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Abstract: A mobile robot means that it can autonomously perform real-time motion in a designated location, integrating functions such as autonomous decision-making, path planning, information collection, and motion control. For mobile robots used in various fields, motion control is the premise to achieve various tasks, and trajectory tracking control is one of its main technologies. This paper mainly aims at the trajectory tracking of wheeled mobile robots based on the kinematics model. Taking the position control subsystem as the outer loop and the attitude control subsystem as the inner loop, a double-loop trajectory tracking system for mobile robots is proposed, which is proved by Lyapunov stability theory. The stability of the system and the convergence of tracking error are improved. The designed controller can effectively overcome the influence of unknown disturbance and better realize the trajectory tracking of mobile robots. The simulation results verify the validity and correctness of the control law.

1 INTRODUCTION

With the continuous development and progress of science and technology, robot technology has also developed rapidly. Robots have been widely used in military, manufacturing, agriculture, science and technology industries due to their high mobility, high autonomy, and high environmental adaptability, which is also an important symbol of human society moving towards technological civilization (Qu, 2015). At the same time, the robot itself integrates many high-tech technologies, including mechanical processing, automatic control, information fusion of various sensors, information engineering, programming technology, artificial intelligence and other interdisciplinary subjects (Tan, 2013). This not only promotes the progress of the robot itself, but also promotes the improvement and progress of various interdisciplinary disciplines. The rapid progress of interdisciplinary technology has made the once difficult technical problems solved.

A mobile robot means that it can autonomously perform real-time motion in a designated location, integrating functions such as autonomous decision-making, path planning, information collection, and motion control. For mobile robots used in various fields, motion control is the premise to achieve various tasks, and trajectory tracking control is one of its main technologies (Hu, 2016). Trajectory tracking control of mobile robots means that at a certain initial position, the robot tracks the desired trajectory with respect to time under the action of the controller, and stably runs along the desired trajectory. The trajectory tracking problem of mobile robots can generally be divided into two types: trajectory tracking based on kinematic model, trajectory tracking based on kinematic model and dynamic model (Liu, 2020). For the trajectory tracking problem of mobile robots, scholars at home and abroad have proposed many control methods. These trajectory tracking control methods mainly include PID control (Feng, 2017), inversion control (Zhao, 2020), nonlinear state feedback control (Chang, 2015), fuzzy control Logic control (Zheng, 2017), control based on extended state observer (Zhang, 2019), etc. Sliding mode control has the advantages of robustness and strong anti-interference ability, so sliding mode control can be used to deal with the trajectory tracking problem of mobile robots.

Therefore, for the trajectory tracking of mobile robots based on the kinematic model, this paper takes the position control subsystem as the outer loop and the attitude control subsystem as the inner loop, and proposes a dual-loop trajectory tracking system for mobile robots, which is proved by the Lyapunov stability theory. System stability and tracking error convergence. The designed controller can effectively

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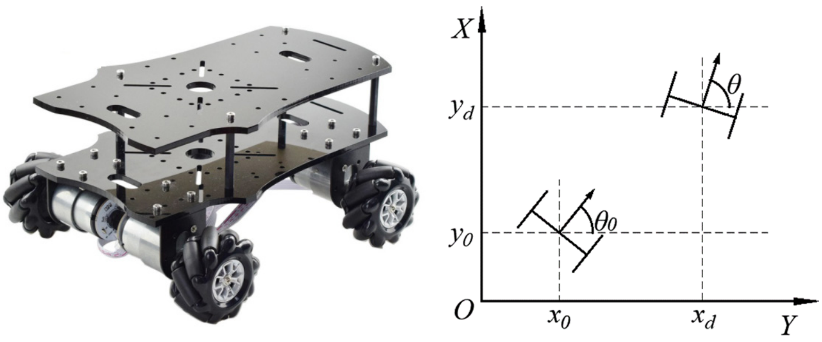


Figure 1: The mobile robot base and the schematic diagram of the motion.

overcome the influence of unknown disturbance and better realize the trajectory tracking of mobile robots. The simulation results verify the validity and correctness of the control law.

2 MOBILE ROBOT KINEMATICS MODEL

Taking wheeled mobile robots as an example, most of these robots have two larger rear wheels, which are driving wheels, and two smaller front wheels, which are driven wheels. The left and right rear wheels are each driven by a motor. If the rotational speeds of the two motors are different, the left and right rear wheels will generate a "differential motion", thereby enabling cornering (Tsuchida, 2009).

A simplified model of a wheeled mobile robot base moving in the *X-Y* plane is shown in Figure 1. Define the midpoint M of the line connecting the centers of the two driving wheels as the reference point of the robot, then the pose P of the wheeled mobile robot can be represented by the position coordinates and heading angle of the reference point M in the inertial coordinate system, where $[x, y]$ is the position of the mobile robot, and θ is the angle between the forward direction of the mobile robot and the x-axis. The control law q of a wheeled mobile robot is represented by the linear velocity v and angular velocity ω of the mobile robot, where v is the position control in the control law, ω is the attitude control in the control law, and the control input in the kinematic model (Liu, 2020).

We might assume

$$P = [x \quad y \quad \theta]^{\top} \tag{1}$$

$$q = [v \quad \omega]^{\top} \tag{2}$$

Assuming that there is no slippage between the wheel and the motion plane during the movement of the wheeled mobile robot, the kinematics equation of the wheeled mobile robot can be expressed as:

$$\dot{p} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} q \tag{3}$$

It can be seen from the kinematics equation that the robot model system has 2 degrees of freedom, and the model output is 3 variables, so the model is an underactuated system, which can only achieve active tracking of 2 variables, and the remaining variables are in the follow-up or steady state. This control is a trajectory tracking problem, that is, the tracking of the position $[x, y]$ of the mobile robot is realized by designing the control law, and the follow-up of the included angle θ is realized.

From formula (3), the kinematic model of the mobile robot can be obtained as:

$$\begin{cases} \dot{x} = v\cos(\theta) \\ \dot{y} = v\sin(\theta) \\ \dot{\theta} = \omega \end{cases} \tag{4}$$

3 TRAJECTORY TRACKING CONTROL

Trajectory tracking control includes two parts: position control law design and attitude control law design. The process is to formulate a set of ideal motion trajectories $[x_d \quad y_d \quad \theta_d]$ in advance, by designing the position control law v, to realize the actual trajectory $[x \quad y]$ tracking the ideal trajectory $[x_d \quad y_d]$, and then design the attitude control law ω to achieve the actual attitude θ tracking the ideal attitude θ_d. The following is the design of the corresponding control law for the above content.

3.1 Design of Position Control Law

First, the position control law v is designed to realize the actual position tracking the ideal position.

Define the error trajectory tracking equation as:

$$\begin{cases}\dot{x}_e = v\cos\theta - \dot{x}_d \\ \dot{y}_e = v\sin\theta - \dot{y}_d\end{cases} \quad (5)$$

where x_e and y_e represent the position error of the x-axis direction and the y-axis direction, respectively:

$$\begin{cases}x_e = x - x_d \\ y_e = y - y_d\end{cases} \quad (6)$$

Assume

$$\begin{cases}v\cos\theta = u_1 \\ v\sin\theta = u_2\end{cases} \quad (7)$$

For $\dot{x}_e = v\cos\theta - \dot{x}_d$, take the sliding mode function as $s_1 = x_e$, then

$$\dot{s}_1 = \dot{x}_e = u_1 - \dot{x}_d \quad (8)$$

The design control law is

$$u_1 = \dot{x}_d - k_1 s_1 \quad (9)$$

where $k_1>0$, so $\dot{s}_1 = -k_1 s_1$.

Take the Lyapunov function:

$$V_x = \frac{1}{2}s_1^2 \quad (10)$$

So there is

$$\dot{V}_x = s_1\dot{s}_1 = -k_1 s_1^2 = -2k_1 V_x \quad (11)$$

So that the x_e index converges to zero.

For $\dot{y}_e = v\sin\theta - \dot{y}_d$, take the sliding mode function as $s_2 = y_e$, then

$$\dot{s}_2 = \dot{y}_e = u_2 - \dot{y}_d \quad (12)$$

The design control law is

$$u_2 = \dot{y}_d - k_2 s_2 \quad (13)$$

where $k_2>0$, so $\dot{s}_2 = -k_2 s_2$.

Take the Lyapunov function:

$$V_y = \frac{1}{2}s_2^2 \quad (14)$$

So there is

$$\dot{V}_y = s_2\dot{s}_2 = -k_2 s_2^2 = -2k_2 V_y \quad (15)$$

So that the y_e index converges to zero.

From formula (7), $\frac{u_2}{u_1} = \tan\theta$ can be obtained. If the value range of θ is (π/2, π/2), the θ that satisfies the ideal trajectory tracking can be obtained as

$$\theta = \arctan\frac{u_2}{u_1} \quad (16)$$

θ obtained by formula (16) is the angle required by position control law formula (9) and formula (13). If θ is equal to θ_d, the ideal trajectory control law formula (9) and equation (13) can be realized, but θ and θ_d in the actual model equation (4) cannot be completely consistent, especially in the initial stage of control, which will cause the closed-loop tracking system equation (1) unstable.

To this end, the angle θ obtained by equation (16) needs to be regarded as an ideal value, that is, take

$$\theta_d = \arctan\frac{u_2}{u_1} \quad (17)$$

When designing an ideal pose instruction $[x_d \quad y_d]$, we must pay attention to the need to make the value range of θ_d satisfy (π/2, π/2).

The difference between the actual θ and θ_d will cause the position control laws (9) and (13) to be unable to be accurately realized, resulting in the instability of the closed-loop system. A simpler solution is to make θ track θ_d as quickly as possible by designing an attitude control algorithm that converges faster than the position control law.

From equation (7), the actual position control law can be obtained as:

$$v = \frac{u_1}{\cos\theta_d} \quad (18)$$

3.2 Design of Attitude Control Law

Next, the attitude control law ω is designed to make the actual attitude θ track the ideal attitude θ_d.

For $\theta_e = \theta - \theta_d$, take the sliding mode function as $s_3 = \theta_e$, then

$$\dot{s}_3 = \dot{\theta}_e = \omega - \dot{\theta}_d \quad (19)$$

The design control law is

$$\omega = \dot{\theta}_d - k_3 s_3 - \eta_3\, sgn\, s_3 \quad (20)$$

where $k_3>0$, $\eta_3>0$, so $\dot{s}_3 = -k_3 s_3 - \eta_3\, sgn\, s_3$.

Take the Lyapunov function:

$$V_\theta = \frac{1}{2}s_3^2 \quad (21)$$

So there is

$$\dot{V}_\theta = s_3\dot{s}_3 = -k_3 s_3^2 - \eta_3|s_3| \le -k_3 s_3^2 \quad (22)$$

That is $\dot{V}_\theta \le -2k_3 V_\theta$, so that the angle θ exponentially converges to θ_d.

4 THE KEY TO THE DESIGN OF THE DOUBLE LOOP SYSTEM

The above double-loop system belongs to a closed-loop control system composed of inner and outer loops. The position subsystem is the outer loop, and the attitude subsystem is the inner loop. The outer loop generates an intermediate command signal θ_d and transmits it to the inner loop system. The inner loop passes the sliding mode control law. Realize the tracking of this intermediate command signal. The structure of a closed-loop system with double loops is shown in Figure 2.

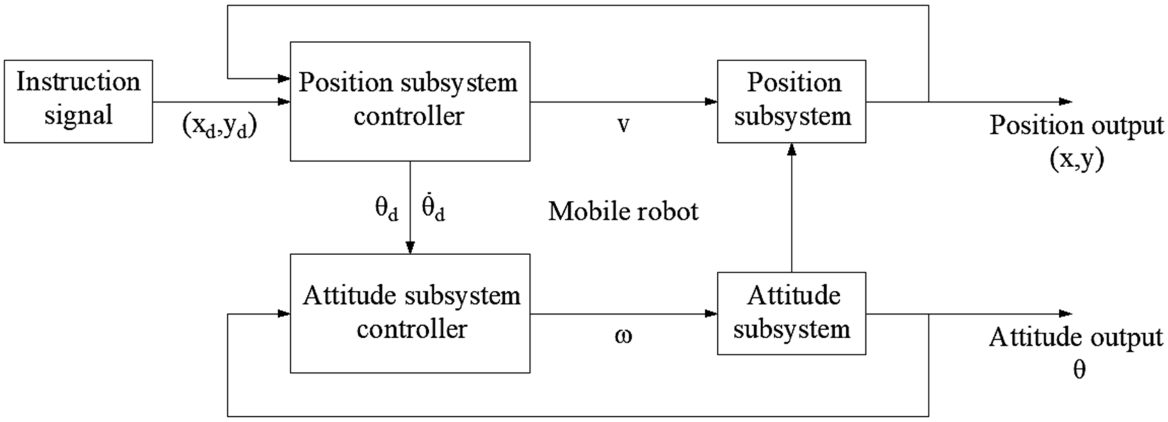


Figure 2: Structure of a closed-loop system with dual loops.

It needs to be explained as follows:

(1) Due to the need to obtain $\dot{\theta}_d$ when designing the inner loop controller, this requires θ_d to be a continuous value, thus requiring the control laws u_1 and u_2 to be continuous. Therefore switching functions should not be included in u_1 and u_2.

(2) In the control law (20), the intermediate command signal θ_d generated by the outer loop needs to be derived. However, the derivation is too complicated. For convenience, the following linear second-order differentiator can be used to obtain $\dot{\theta}_d$ [11]:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -2R^2\big(x_1 - n(t)\big) - Rx_2 \\ y = x_2 \end{cases} \tag{23}$$

Among them, the input signal to be differentiated is n(t), x_1 is to track the signal, x_2 is the estimation of the first-order derivative of the signal, and the initial value of the differentiator is $x_1(0)=0$, $x_2(0)=0$. Since the differentiator has an integral chain structure, when derivation of a signal containing noise in engineering, the noise is only contained in the last layer of the differentiator, and the noise in the first derivative of the signal can be more fully suppressed by integrating.

(3) In the inner and outer loop control, the dynamic performance of θ tracking θ_d in the actual model will affect the stability of the outer loop, which will affect the stability of the entire closed-loop control system. For this problem, the literature (Bertrand, 2011; Jankovic, 1996; Amit, 2010; Amit, 2010) gives a strict solution is proposed, in which the literature (Bertrand, 2011) deduces the relationship between the control gains between the inner and outer loops, thus guaranteeing strict closed-loop system stability.

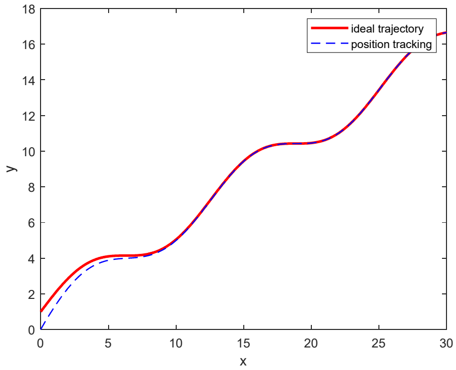
In order to achieve stable inner loop sliding mode control, this section introduces the method generally used in engineering, that is, the method that the inner loop convergence speed is greater than the outer loop convergence speed, and the stability of the closed-loop system is ensured by θ fast tracking θ_d. In this algorithm, by adjusting the control gain coefficients of the inner and outer loops, the convergence speed of the inner loop is guaranteed to be much faster than that of the outer loop, but this method is only an empirical method, which cannot theoretically guarantee the stability of the closed-loop system.

5 SIMULATION EXAMPLE AND CONCLUSION

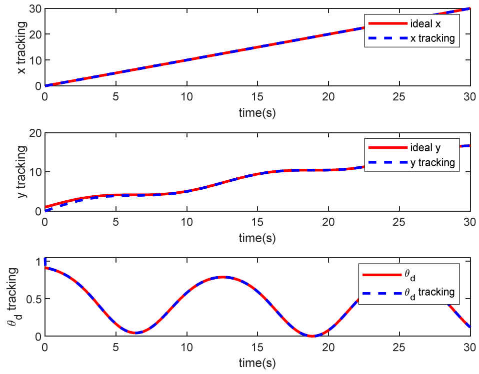
5.1 Aperiodic Trajectory

The controlled object is equation (20), and the pose instruction $[x_d \quad y_d]$ is $x_d=t$, $y_d=sin(0.5x)+0.5x+1$. Take $k_1=k_2=0.30$, $k_3=3.0$, $\eta_3=0.50$, the initial value of the pose is $[0 \quad 0 \quad 0]$, adopt the control law formula (18) and formula (20), for the switching of the attitude control law formula (20) term, the saturation function is used instead of the switching function, the thickness of the boundary layer is set to 0.10, and the differentiator parameter is set to R=100. The simulation results are shown in Figures 3(a)~(d).

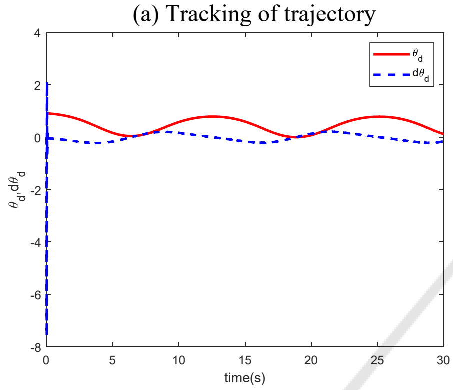
It can be seen from the simulation that the maximum value of θ_d is 0.9526rad, which belongs to the interval (-π/2, π/2) and meets the requirements of formula (17). It can be seen that there is a good tracking effect for the values in this range.



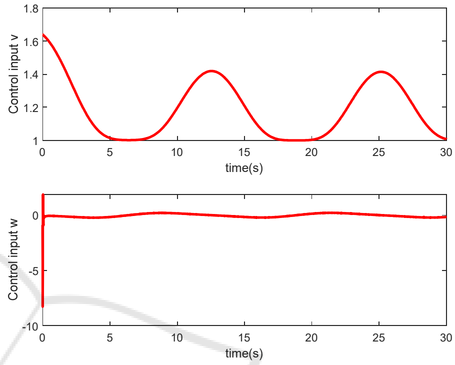
(a) Tracking of trajectory



(b) Tracking of position and angle

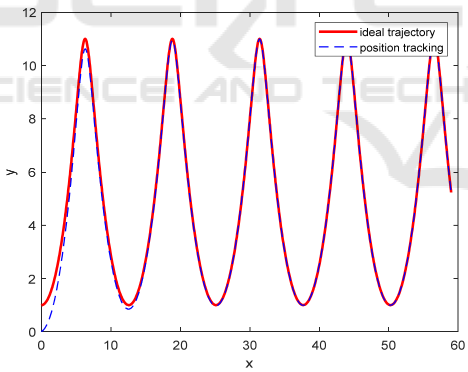


(c) Input and output of the differentiator

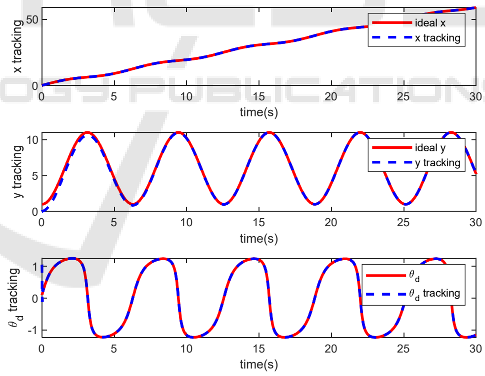


(d) Control input signals v and ω.

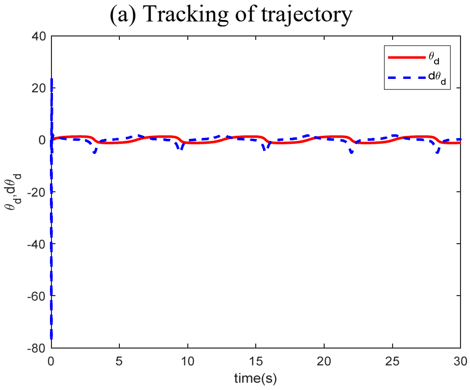
Figure 3: The simulation results of aperiodic trajectory.



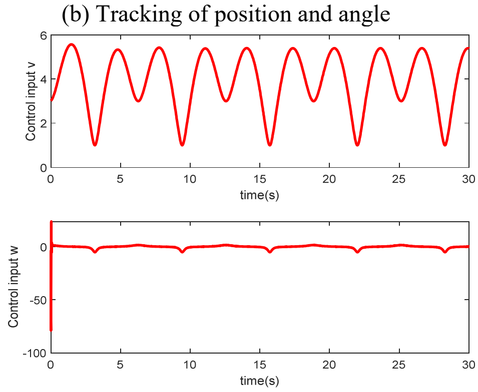
(a) Tracking of trajectory



(b) Tracking of position and angle



(c) Input and output of the differentiator



(d) Control input signals v and ω.

Figure 4: The simulation results of periodic trajectory.

5.2 Periodic Trajectory

The periodic trajectory mentioned here is generally more commonly used in trajectory planning, and a more complex periodic trajectory function is defined here to verify the practicability of the algorithm. The pose instruction $[x_d \quad y_d]$ is x_d=2*t*-*sin*(*t*), y_d=6-5*cos*(*t*). Take k_1=k_2=0.30, k_3=3.0, η_3=0.50, the initial value of the pose is $[0 \quad 0 \quad 0]$, the thickness of the boundary layer is set to 0.10, and the differentiator parameter is set to *R*=100. The simulation results are shown in Figures 4(a)~(d). Finally obtained θ_d is 1.2476rad and still meet the requirements.

If it exceeds this range, given a continuous trajectory function, it can be seen from equation (17) that there must be a point in the tracking trajectory that makes $\dot{\theta}_d = \infty$, so that the robot The tracking curve of *v* and *ω* appears faulty, and the movement of the robot will be stuck and incoherent. Therefore, special attention should be paid to the design.

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