

# Simulation of Curvature Ductility of Reinforced Concrete Cantilever Beam Under Variety of Section Ratio

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**Abstract:** The purpose of this research is to look into the effects that the ductility of reinforced concrete cantilever beams has on their curvature. A numerical program was simulated to estimate moment-curvature and available curvature ductility of reinforced concrete cantilever beams with or without axial loads. The study evaluate five cantilever beams with various factors were examined. Concrete strength, the quantity of longitudinal reinforcement, and the spacing of transverse reinforcement are the factors that are measured. Beam geometry, material characteristics, and weight make up the input. properties of the retrofitted material by using a variety of different approaches. A confined stress-strain curve was generated for concrete using SAP2000's adopted methods, in the same way that a steel stress-strain model was generated. From the evaluation, the curvature ductility increases with the longitudinal reinforcement and concrete strength with it representing by the distance length of cantilever beam. However, there is no discernible relationship between the curve ductility and the spacing of the transverse reinforcement.


## 1 INTRODUCTION


When structural integrity depends on resistance to brittle failure during flexure, reinforced structures' ductility is a desirable characteristic. Plastic joints placed strategically throughout the structural frame can be used to create a ductile behavior in a structure (Marta, 2014). These are made to be sufficiently ductile to withstand structural failure once the material's yield strength has been reached. Based on the configuration of the moment-curvature relations,


the available ductility of plastic hinges in reinforced concrete is found (Arslan and Cihanli, 2011).


Ductility is defined as the ability to endure deformations without significantly reducing the member's flexural capacity (Szczeszen, Szwed and Li, 2007). According to the findings of previous study, this deformability is affected by factors such as the tensile reinforcement ratio, the quantity of longitudinal compressive reinforcement, the degree of lateral tie, and the strength of the concrete (Park and Paulay, 1975). The ductility of a reinforced


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concrete section could be represented as curvature ductility, see Equation (1).

$$\mu_\phi = \frac{\phi_u}{\phi_y} \tag{1}$$

$$M_y = A_s f_y d'' \tag{2}$$

$$\phi_y = \frac{\varepsilon_{sy}}{(1-k)d} = \frac{f_y}{E_s(1-k)d} \tag{3}$$

$$M_u = 0.85 f'_c ab \left( \frac{d-a}{2} \right) + A_s' f_y (d-d') \tag{4}$$

$$\phi_u = \frac{\varepsilon_c}{c} = \frac{\varepsilon_c \beta_1}{a} \tag{5}$$

Where

$$a = \frac{A_s f_y - A_s' f_y}{0.85 f'_c a} \tag{6}$$

$$k = \sqrt{\frac{(\rho + \rho')^2 n^2 + 2 \left( \rho + \frac{\rho' d}{d} \right) n}{-(\rho + \rho') n}} \tag{7}$$

$$\rho = \frac{A_s}{bd} \tag{8}$$

Equation (1) to (8) presenting the curvature parameter in accordance with (Park and Paulay, 1975), where  $\mu_\phi$  is curvature ductility and respectively,  $\phi_u$  and  $\phi_y$ , are curvature ultimate and curvature point when the reached yield strength. Normally,  $\phi_u$  defined as the effect of the ultimate strain from concrete compression and  $\phi_y$  defined as the influence of the yield strength of reinforcement steel on the calculation of  $\phi_u$ . The vast majority of the regulations on the design curvature analysis stated that the yield curvature of a reinforced concrete beam should be taken when the tension steel first yields. This condition, which may be derived from Equation (2), and (3). Where  $k$ ,  $\rho$  is the tensile reinforcement ratio and  $\rho'$  is the compression steel ratio. In addition,  $n$  is the modular ratio were taken from the comparison of modulus elasticity of steel over modulus elasticity of concrete.  $d''$  is centroid distance over compressive force in steel and centroid of tension over concrete.

Furthermore, to understand the ultimate curvature, Equation (4) – (5) are used. In this stage, evaluate the capacity of reinforced concrete where the crushing of RC section occurred.  $f'_c$  denoted as compressive strength,  $f_y$  denoted as yield strength, where  $\beta_1$  is the depth of equivalent rectangular stress block. The design parameters in accordance with the SNI 2847-2019 (Badan Standardisasi Nasional,

2019a) and ACI 318-71 (American concrete Institute, 2014) conservatively recommend stress block 0.003. Some researchers identified the nominal value of ultimate concrete strain ( $\varepsilon_{cu}$ ) required to compute Conventional Curvature Ductility Factor for unconfined concrete is 0.0035, while it is implicitly greater for confined concrete.

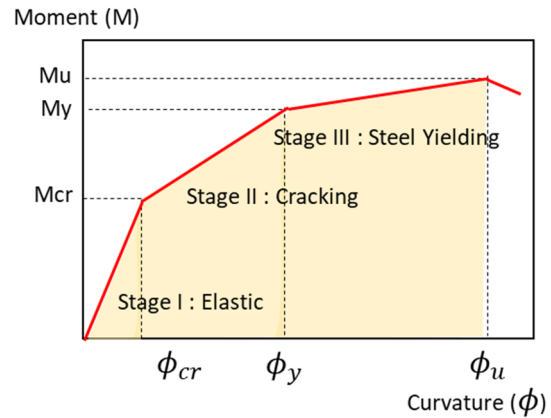


Figure 1: A moment curvature relationship under three different stage condition (American concrete Institute, 2014).

$$EI = MR = \frac{M}{\phi} \tag{9}$$

Where  $EI$  is the section flexural rigidity.

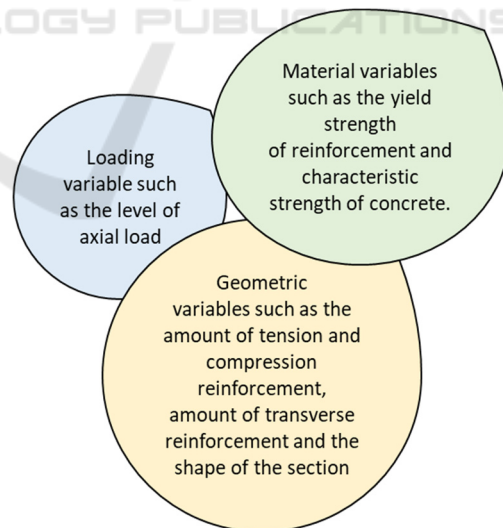


Figure 2: Curvature behaviour parameter (Olivia, Riau and Mandal, 2005).

Theoretical moment-curvature analysis for reinforced concrete structural components can be performed to determine the available flexural strength and ductility, provided the stress-strain relations for

both concrete and steel are known (Arslan and Cihanli, 2011; Zhou, He and Liu, 2014). Curvature and the bending moment of the section for a given load raised to failure can be used to calculate the moment-curvature relationship (Fischer and Li, 2003, 2003). The illustration of trilinear moment-curvature in accordance with Park and Paulay can be seen in Figure 1 (Park and Paulay, 1975). From the illustration, the curvature classified by three different straight stages line represent elastic conditions, after first cracking and yielding. It is also illustrated by classic elastic Eq. (9), relationship between moment and curvature. Some past works also investigated based on variety of concrete materials, which is not included in the research. The concrete material will also represent a different behaviour (Komara *et al.*, no date; Kartiko *et al.*, 2021; Pertiwi *et al.*, 2023)

The study evaluates five different cantilever beams with the variable of section ratio as the demand of section dimension which is normally used in the midrise building. The empirical model was developed to consent the ductility analysis. Some identifications also included into this model to corroborate findings.

## 2 RESEARCH METHODS

The study was carried out by determining the range of parameters used in the cantilever beam elements. These parameters came in the sectional beam property. The length of beam implied as 3 m length and this length placed as the current case study from office building located in East Sumatera. The simulation appears in the same condition, elevate to 14.4 m with four lever stories with 825 m<sup>2</sup>. The section model is in accordance with the Figure 3 where the variation presented in Table 1. The  $f'c$  used in this simulation is factually the same for all beam section, 30 MPa. The phase of evaluation illustrates in Figure 4.

To identify loading mechanism, SNI 1726-2019 (Badan Standardisasi Nasional, 2019b) is used, followed by earthquake regulation SNI 1726-2019 and concrete SNI 2847-2019 (Badan Standardisasi Nasional, 2019a). After clarifying all preliminary analysis, structural analysis program using SAP2000 is conducted (2000, 2008; Interface, Implemented and Implemented, 2013). This study relates to the implementation of load & resistance factors of cantilever beam can be seen in Figure 4 and the 3D model in SAP200 can be accessed on Figure 5. Modification load factor is assigned with the value as  $\frac{l_e}{R} \times g = \frac{1}{3} \times 9.81 = 3.27$ .

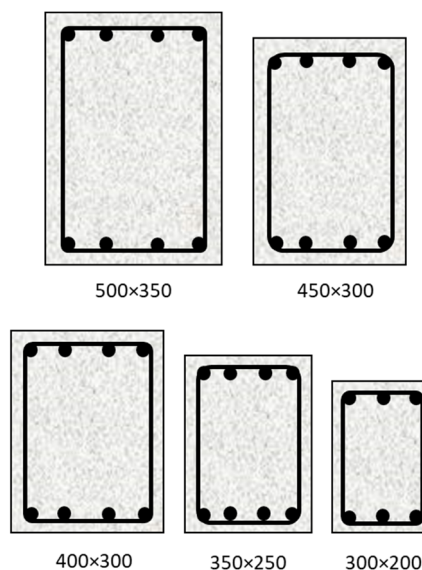


Figure 3: Cross section cantilever beam [unit: mm].

Table 1: Reinforcement proportion varied by length of cantilever beam.

Section ratio (length)	Section dimension (mm)	Rein. ratio	Reinforcement section area (mm)
L/6	500×350	0.012	1860
L/7	450×300	0.012	1412
L/8	400×300	0.012	1231
L/9	350×250	0.012	874
L/10	300×200	0.012	578

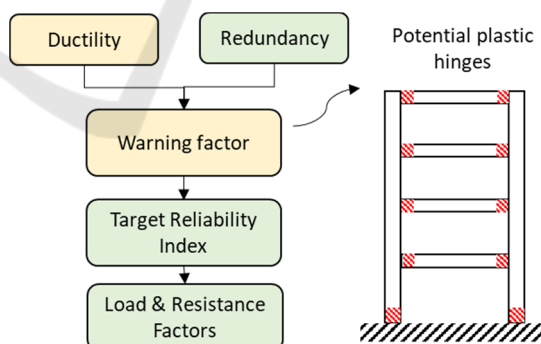


Figure 4: structure reliability concept (Frangopol, Lin and Estes, 1997).

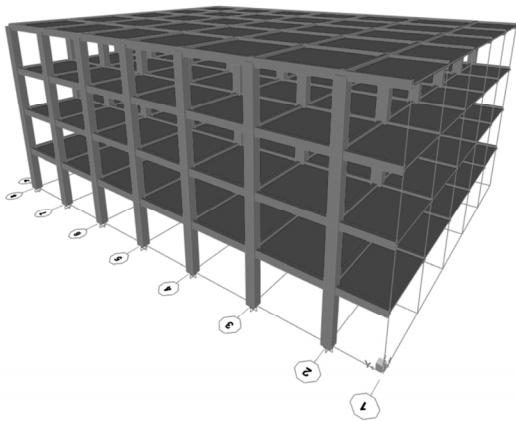


Figure 5: Three-dimensional illustration building modelling using SAP2000.

Table 1 inform the distribution of reinforcement ratio designed into cantilever beam. From that evaluation, the reinforcement bar used for mid and start point, respectively 4D15, 150-Ø12 and 4D15, 180-Ø12.

The total dead load (DL) according to the office building summarized with the total 1.5 kN/m<sup>2</sup> and DL for roof particularly given less, with 0.65 kN/m<sup>2</sup>. The life load (LL) is given higher than DL, count as 2.5 kN/m<sup>2</sup> and for roof, LL 1.0 kN/m<sup>2</sup>. The fundamental period parameter considering the location, with the detail data are  $S_s = 0.29$  with  $F_a = 0.9$  and  $S_T = 0.25$  and  $F_v = 0.8$ , is 0.7593.

The computational approach for deriving the curvature ductility from the moment curvature behavior of the cross section is as follows, see Eq. 10, assess the ultimate axial load, and then derive the curvature ductility (Badan Standardisasi Nasional, 2013).

$$P_o = (A_c - A_s)(max. stress) + A_s f_y \quad (10)$$

$A_c$  is the gross area of confined concrete,  $A_s$  is the area of longitudinal steel,  $f_y$  is the yield strength.

The strain at the extreme compressive fibre is then analysed as if the section were applied with a single axial load without any moment (Mihashi and Leite, 2004; Yu *et al.*, 2017). For the value of fibre strain, the strain profile is created. It is presumable that strain varies linearly with beam cross section. As shown in Figure 4, the section is cut into rectangular strips to estimate compressive forces in concrete. The relevant stress-strain models are used to compute the corresponding stresses in concrete and steel. Calculations are made for the internal forces

supporting steel (Gagg, 2014; Jensen, Kovler and Belie, 2016).

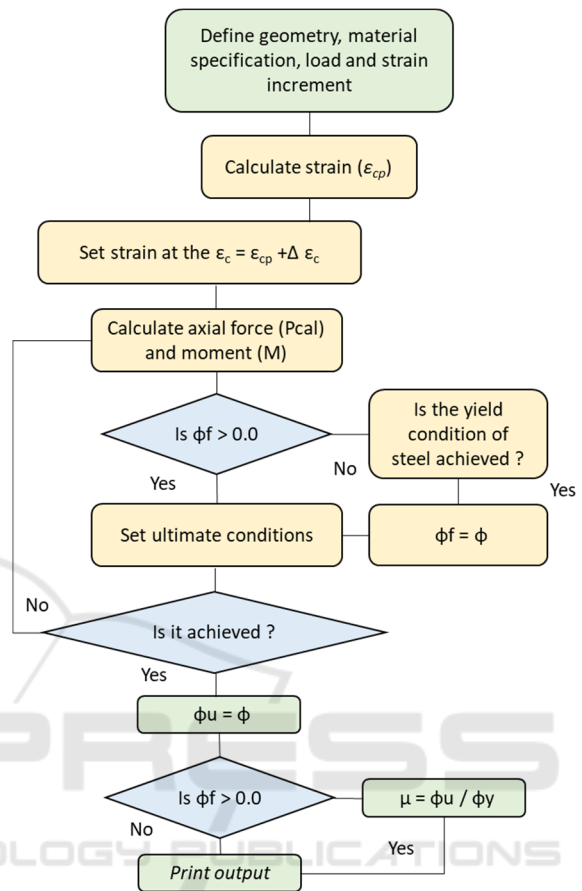


Figure 6: Stage of evaluation moment curvature (Zhou, He and Liu, 2014).

### 3 RESULTS AND DISCUSSION

In order to conduct an analysis of the 5 beams, the numerical model was utilized. The output of the program comprises of numerical results as well as values for the curvature ductility. The ductility calculation took into account the parameters that were taken into consideration. In this section, it will discuss the effects that the key factors have on the moment-curvature curves. The stage of the evaluation can be seen in Figure 6. A comparison of the moment-curvature relationship for five beams is provided in Figure 7. These beams have the same concrete strength and longitudinal reinforcement but differing confinement reinforcement spacing. When a segment is constricted, both the ultimate compressive strain and the ductility of the material are increased. The

yield and maximum moment capacity of the section are unaffected by the transverse steel because the stress-strain model used in the numerical analysis assumes that the shape of the initial ascending segment is unaffected by the amount of transverse steel. Transverse steel reinforcement does not change yield curvature. However, compressive strain increases curvature.

It is also determined that closer spacing delays the buckling of the compressive reinforcement but has no effect on the material's ductility because failure occurs in tension steel. In light of this, the researchers conducting the present study have concluded that a tighter confinement spacing is ineffective. Figure 7 also shows that parameter of  $\rho'/\rho$  classifying from 1/4 to 1.0 which come to be one of the important aspects. Cantilever beam L/10 shows the lowest moment curvature ductility, relating to the parameter of longitudinal reinforcement. The reason is considerate by the increase amount of tension steel as well as the depth of the neutral axis. When yield is reached in longitudinal steel, the stress remains fixed, and the depth of the neutral axis increases with curvature. When the strain at the maximum compressive fiber of concrete is fixed at ultimate condition, the curvature at ultimate condition reduces.

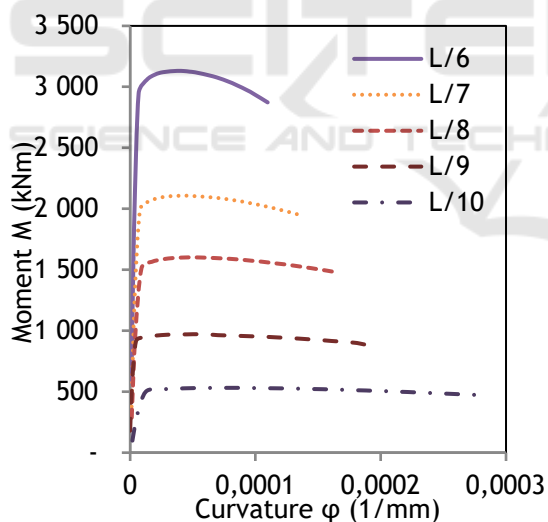


Figure 7: Moment curvature curves for cantilever beam with different length.

On the other hand, cantilever beam L/6 illustrates to have the highest moment curvature ductility. It is informed that having the very low amount of tension steel. The breaking of tension steel is a possibility that could lead to the ultimate situation. In this scenario, the strain on the tension steel is held constant at the ultimate condition; hence, the curvature at the final condition grows. As a consequence of this, the

ductility of the curvature improves in proportion to the reduction in the amount of tension steel. When yield is reached in longitudinal steel, the stress remains fixed, and the depth of the neutral axis increases with curvature. When the strain at the maximum compressive fiber of concrete is fixed at ultimate condition, the curvature at ultimate condition diminishes.

Curvature ductility decrease gradually as the variable length is extended, L/6 – L/10. It needs to improve the confined parameter and also the relation of various steel reinforcement ratio to accommodate the difference.

#### 4 CONCLUSIONS

This research is evaluated on verifying the model of cantilever reinforced concrete beam using SAP2000, which indicate the value of moment curvature. This such evaluation to anticipate the fracture condition to the structure. The modelling process include 5 different cantilever beams classified by length different. According to the findings, the effect of materials properties and its geometric on the curvature ductility of cantilever reinforced concrete under this case study is very low. As expected, it was found that length variable determines the value of moment curvature. The availability of curvature decreased by the length of span of cantilever beam which is composed of longitudinal reinforcement. One that should be noted, there is no significant affect increase on the confined reinforcement while the model and the numerical analysis having a good agreement.

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