AIM-RL: A New Framework Supporting Reinforcement Learning Experiments

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Abstract: This paper describes a new framework developed to facilitate implementing new problems and associated models and use reinforcement learning (RL) to perform experiments by employing these models to find solutions for those problems. This framework is designed as being as transparent and flexible as possible, optimising and streamlining the RL core implementation and allowing users to describe problems, provide models and customise the execution. In order to show how AIM-RL can help with the implementation and testing of new models we selected three classic problems: 8-puzzle, Frozen Lake and Mountain Car. The objective results of these experiments, as well as some subjective observations, are included in the latter part of this paper. Considerations are made with regards to using these frameworks both as didactic support as well as tools adding RL support to new systems.

1 INTRODUCTION

A very popular open-source framework built to develop, test and showcase Reinforcement Learning (RL) capabilities is Gym/Gymnasium (Brockman et al., 2016). Studies have shown that RL within Gym/Gymnasium can be very helpful in both solving and testing solutions for various AI problems (He et al., 2021) and (Yu et al., 2020), as well as benchmarking RL based solutions such as continuous control games (Duan et al., 2016) or new policy control methods (Schulman et al., 2017).

The potential of RL is enhanced by the ability to work both as model-free (Chen et al., 2019) and (Yarats et al., 2021) as well as allowing users to employ and test various models to boost problem-solving tasks (Moerland et al., 2023) and (Kaiser et al., 2019). Due to its flexibility and power, RL has been proven useful also in education (Nelson and Hoover, 2020), (Lai et al., 2020) and (Paduraru. et al., 2022).

As part of a larger system being built to describe, test and employ various model based solutions for AI problems, a need was identified for an alternative RL framework adapted to a more flexible and involved approach as opposed to the most prominent solution available, Gym/Gymnasium. This approach should offer our framework advantages in streamlining applying RL to new problems and varied models as well as being a platform to support students building and testing RL solutions.

Contributions. The main contribution of this paper is the introduction of a novel framework that simplifies the implementation of new AI problems and associated models, using reinforcement learning to find solutions for those problems through experiments. The framework is designed to be as transparent and flexible as possible, streamlining the core implementation of RL. We demonstrate the usefulness of this framework by modeling several classic problems, highlighting its ease of use and potential applications.

Paper Organisation. Section 2 briefly describes the challenges to adding Reinforcement Learning support to solving AI problems. Section 3 presents our framework and three example toy problems and corresponding models within our framework. Section 4 includes some experimental results, and we conclude in Section 5.

2 RL AND GYMNASIUM

2.1 Reinforcement Learning

Reinforcement Learning (RL) is a generalisation of Q-Learning (Watkins, 1989) introduced as a method of combining the older "trial-and-error" learning as

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well as delayed and probabilistic learning with the training data independence provided by Monte Carlo algorithms. The basic idea is that an agent explores a state-based problem space initially at random then guided by rewards provided according to a model (usually a heuristic). The reward associated with each step adjusts the score associated with the original state and the step required to reach the current state in a matrix of associations called a Q-table. This exploration, repeated in enough epochs (from the initial state up to a goal/fail state) should produce a Q-table which will guide the agent so that it chooses the better rewarded action, to a shorter path to the goal.

2.2 Gymnasium Approach

According to its authors (Brockman et al., 2016), the Gym/Gymnasium framework was developed following the principles:

- Environments, not agents: Separation of the problem environment, defined as everything particular to a specific problem, from the agent itself.
- Emphasise sample complexity, not just final performance: Metrics provided to measure both final performance as well as performance at each epoch and even each decision.
- Encourage peer review, not competition: Allowing users to easily contribute their own "agents" (models) to the framework and compare them with similar contributions.
- Strict versioning for environments: Each environment is associated with a version which would allow only compatible agents to be used in it.
- Monitoring by default: keeping a record of all actions taken while running an agent in an environment. Since it's initial release, Gym/Gymnasium has emerged as the most prominent open source environment used by RL researchers and students, with over 5000 references in scientific papers.

3 USING AIM-RL TO IMPLEMENT EXPERIMENTS

In this section, we describe in more detail the AIM-RL framework, and we illustrate its flexibility by modelling several problems within this framework. We aim to show that our approach allows users to easily customise and run their RL experiments.

3.1 The AIM-RL Design Principles

Developing AIM-RL we considered the main benefits of RL as being its flexibility provided both by training data independence and the ability to employ a great variety of models to solve a wide range of problems. Our goal is to provide a framework suitable for both students to learn and test, and for researchers to quickly experiment with new problems and models.

Considering this, we viewed transparency as the main objective, by allowing users full visibility and control over all aspects of a RL environment, and flexibility with regards to what types of models can be employed to solve AI problems. Both complex models and policy functions as well as model-free solutions can be implemented, minimising code redundancy for various experiments. We also aimed to improve reproductibility and analysis of the results by making persistent profiles available for input parameters, trained Q-tables and experimental results.

3.2 The AIM-RL Framework

AIM-RL was implemented as a Python package¹ which can be compiled and installed via pip. The main objective of our package was to provide a parametric implementation for RL that can be easily instantiated on various problems and models.

The main component is the qlearning module which includes the following function:



This method takes as input the description of a problem (i.e., a concrete implementation of the abstract Model class described below in Section 3.2) and the usual parameters for RL. When set, the verbose flag prints the current epoch and the number of steps until a solution is reached. The discount_optimisation flag enables a linear decrease adjustment for the discount parameter, and it is active by default. The function returns a tuple (Q, results, solutions, percentage). Q is the Q table as associations between a state, an action and a value. The solutions

¹The source code, together with the experiments and a README.md file which contains installation and running instructions is available here: https://tinyurl.com/4mhx83yd

dictionary assigns to each epoch the number of steps until a solution is found or the epoch is ended otherwise, while the results dictionary stores all the transitions made in every epoch. The rate value represents the ratio between the number of successful epochs (when a solution has been reached) over the total number of epochs.

The user can either call the above method directly with the required parameters or he can define them in a JSON file. In that configuration json file the user can provide the following parameters:

- epsilon: the ε value(s) used to determine initial random choice chance for the epsilon-greedy approach. If not needed, ε can be set to 0.
- decay: the decay factor value(s) for ε after each epoch. Set the value to 1 if no decay is needed.
- alpha: the learning rate α value(s) used to weight impact of new rewards on q-table updates.
- discount: the γ value(s) used to weight impact of older updates of q-table values.
- epochs: the number of epochs tried for each run.
- limit: the maximum number of states visited in each epoch. If reached, the current epoch ends.
- runs: the number of repeated runs for each configuration of parameters.
- model: a list of references to models used in the experiments. For each model the user has to implement at least a reward function which will be used to update the Q-table.
- generate_graph: either *true* or *false* depending whether the user wants the framework to generate the graphical representations of each experiment.
- instances: a list of instances for the problem. For every item in the list, every configuration is executed. The format of an instance is up to the user to establish and parse.

In the same configuration file multiple values can be provided, as a list, for the *epsilon*, *decay*, *alpha*, *discount*, *model* and *instances* parameters. In this case the framework will perform a run for all possible combination of these values. This can facilitate easy experimentation, the results can then be compared using the outputted graphs and csv file (cf. Section 4).

The glearning module also provides a function for generating graphs using the matplotlib library. The graphical representation includes for each epoch the length of the solution found or the number of steps made up to the end of the current epoch. It also includes the ratio of successful epochs.

A brief description of the steps required by the user in order to employ this framework for a new

problem and new models is given below. In order to run a new RL experiment two abstract classes have to be implemented: State and Model.

The State abstract class has four abstract methods: get_possible_actions() which returns the next possible actions from the current state; is_final() which returns true if the current state is final; get_next (action) which, given an action, returns the next state; and get_id(), which returns an unique identifier for the current state.

The Model abstract class has three abstract methods: get_no_of_actions() which returns the total number of actions for the current problem instance; get_initial_state() which returns the initial state for the current problem instance; and get_reward(state, next_state, action) which returns the reward for the transition state to next_state via action.

As explained above, our framework provides the entire machinery for Reinforcement Learning at an abstract level. Users are expected to plug in some concrete implementations for the State and Model abstract classes. Then, the only thing left is to write a main.py program that uses these concrete implementation according to users' needs. Here is a template that we suggest for main.py:

```
from qlearning import aimrl as QL
from puzzleModel import PuzzleModel
from puzzleState import PuzzleState
```

```
def run(folder, fname, instance, no_of_epochs,
  epsilon, alpha, discount, decay, limit):
   Q, results, solutions, percentage =
   QL.qlearning(instance, no_of_epochs,
       epsilon, alpha, discount, decay,
       limit, True, True)
   QL.save_as_graph(results, solutions,
     percentage, no_of_epochs, folder, fname)
   return Q, results, solutions, percentage
#
 Steps:
 1) create instances of PuzzleState
#
#
     and PuzzleModel
#
  2) initialise parameters or read them
     from the configuration file
#
  3) describe at least one problem
#
     instance or read them
#
     from the configuration file
# 4) call run on the desired inputs
```

The QL.qlearning(...) call inside the run function calls our Reinforcement Learning implementation. We use the same template for all the problems that we approach in this paper. While our primary goal was to develop a user-friendly framework for experiments, we also considered performance. To improve the efficiency, we utilised various Python tricks such as substituting for loops for while loops and using dictionaries instead of matrices, which was also applied to the Q table. Additionally, we meticulously streamlined the number of methods required for users to implement, further optimising the framework's overall performance.

The components of the tuple returned by QL.qlearning(...) can be used to generate a graph using the QL.save_as_graph(...) function. Another output provided for all experiments ran using AIM-RL is a csv file including values for²:

- graph: the random and unique generated name for the graph representation of this run;
- model: the model used in this run;
- instance: the input used in this run;
- epsilon, decay, alpha and discount: RL parameters used in this run;
- run: the run number identifier;
- time: the run duration, in milliseconds;
- percentage: the fraction of epochs ending in a goal state versus the total number of epochs;
- Q size: the size of the final Q table (the total number of states explored in all epochs).

3.3 Implementation of 8-Puzzle

The 8-puzzle problem (Ratner and Warmuth, 1986) and (Piltaver et al., 2012) is one of the classical AI puzzles used to experiment new technologies and methods developed. It consists of a 3×3 grid with eight numbered tiles and one blank space. The objective is to rearrange the tiles from their initial state into a target state by sliding them one at a time into the blank space. A more general formulation of this problem consists in working with an $m \times n$ grid.

Implementing this problem using our Reinforcement Learning package presented in Section 3.2 is straightforward. First, we create a PuzzleState class which inherits the State abstract class. In PuzzleState we use the fields m and n as dimensions of our grid, and a $m \times n$ -sized list A of values from 0 to $m \cdot n - 1$, where 0 stands for the blank space. The abstract methods of the State class are implemented as explained below:

• get_possible_actions() returns the next possible actions from the current state, that is, a list of values in the set {up, down, left, right}, where each action is encoded as an integer. Note that the returned list does not always include all the actions, because certain boundary conditions need to be fulfilled.

- is_final() returns true if the list A is ordered, no matter what is the position of the blank space.
- get_next(action) returns an instance of PuzzleState which is the next state obtained when sliding a tile as specified by action.
- the get_id() function returns a number which uniquely identifies each state.

The abstract methods of the Model class are implemented inside the PuzzleInstance class as follows:

- get_initial_state() returns the initial state as an instance of PuzzleState, where *m*, *n*, and *A* are provided by the user.
- get_no_of_actions() returns 4 because there is a total of | {up, down, left, right} | actions.
- get_reward(state, next_state, action)

determines the rewards to be used to update the Q table for the action applied to state which produces next_state. Two different versions were tried here, without changing any other details about the implementation. One implemented a basic model version (no reward except for the goal state which rewards 100). The second uses the manhattan distance as reward, except for the goal state which rewards 100. Some details about the results are given in Section 4.

3.4 Implementation of Frozen Lake

The Frozen Lake problem (Brockman et al., 2016) is a simple grid-world game where the agent has to navigate a frozen lake represented as a two-dimensional grid starting from an initial tile with the goal of reaching a destination tile. Some tiles are thin ice (holes in the overall ice cover) and when stepped one casues the failure of the search. The player can only move one tile at a time with no diagonal movement allowed.

The Frozen Lake state is an implementation of the State abstract class. We use *m* and *n* as grid dimensions, and we represent the grid itself as a $m \times n$ -sized list *A* of labels in the set { 'S', 'F', 'H', 'G' }, where:

- 'S' stands for the start position;
- 'G' stands for the goal position;
- 'F' represents a frozen tile; and
- 'H' represents a hole (thin ice).

In addition, we also keep a *poz* field which holds the current position of the player in *A*. The abstract methods of the State class are implemented as below:

• get_possible_actions() returns the next possible actions of the player, that is, a list of values in the set {up, down, left, right}, where each action is encoded as an integer. The returned list

²Examples for both graphs and csv files generated are provided in Section 4.1.

does not always include all the actions, because certain boundary conditions need to be true.

- is_final() checks if the current state determines the end of the epoch and returns an integer value. The possible values returned are 0 for fail (the current state is not final), 1 for success (a goal has been reached) or greater than 1 for additional end states. An additional end state is determined when stepping on a hole in the ice (H tile).
- get_next (action) returns an instance of FLState which is the next state obtained when player moves on a tile specified by action.
- the get_id() function returns *poz* which uniquely identifies each state.

The abstract methods of the Model class are implemented inside the FrozenLakeModel class. The get_initial_state() returns the initial state as an instance of FrozenLakeState, where *m*, *n*, *A* are user provided. The get_no_of_actions() returns 4 because there are only four actions. The get_reward(state, next_state, action) was tried in two versions, one model-free (maxReward for goal, 0 otherwise), and one a model using the reward heuristic get_reward(state, next_state, action):

 $\begin{cases} mR, & \text{if } isFinal(next_state) = 1 \\ -100, & \text{if } isFinal(next_state) > 1 \\ poz(state) - goal, & \text{otherwise.} \end{cases}$ (1)

The results are discussed in Section 4.

3.5 Implementation of Mountain Car

Mountain Car (Sutton, 1995) and (Heidrich-Meisner and Igel, 2008) is yet another classic problem often used in reinforcement learning experiments. The objective is to train an agent to move a car from the bottom of a valley up a steep hill to a specific goal position located at the top of another hill. The car is subject to gravity and friction, so it cannot simply drive up the hill. Instead, it must first gain momentum by moving back and forth in the valley, building up enough speed to eventually make it up the hill to the goal position. The agent must learn how to balance the need to move back and forth to build up momentum with the need to move up the hill towards the goal position. This problem is challenging for RL agents because of the sparse rewards and the long-term dependencies between actions and rewards.

Unlike 8-Puzzle and Frozen Lake, this problem has a completely different notion of state, and it is a good example for illustrating how versatile our framework is. A state in MountainState includes:

- *spot* represents the position of the car on a curved (sinusoidal) line, the initial spot value being -0.5;
- *velocity* represents the velocity of the car (positive for movement to the right, negative for left), the initial velocity being 0;
- *force* the force with which the car accelerates;
- gravity the gravitational acceleration;
- *current_step* the step in the current epoch.

The MountainState class implements the following:

- get_possible_actions() returns the next possible actions of the player, that is, a list of values in the set {push_left, push_right, dont_push;
- get_next(action) returns an instance of MountainState where the velocity is updated according to the formula

v = v + (a - 1) * f - cos(3 * spot) * g (2)

and the *spot* is updated as spot = spot + v;

• the get_id() function returns a pair (*spot*, *v*) which uniquely identifies each state.

In MountainCarInstance two different rewards were implemented: (1) a basic model version which assigns -1 reward for each step if not reaching the goal spot which rewards maxReward; and (2) a more complicated reward heuristic *get_rewards*(s, s', a):

$$\begin{cases} -(spot-0.5+v), & \text{if } a = left\\ spot+v & \text{if } a = dont_push, \quad (3)\\ (spot-v-1) & \text{otherwise.} \end{cases}$$

The results are included in Section 4.

4 EXPERIMENTAL RESULTS

We performed a series of experiments for the three implemented problems described previously. For simplicity of explanation we used similar parameters configuration files for all three, which were:

```
"epsilon" : [1.0],
"decay": [0.9],
"alpha" : [0.1, 0.5, 0.9],
"discount" : [0.1, 0.5, 0.9],
"no_of_epochs" : 200,
"limit": 100000,
"runs" : 2,
"generate-graph" : true,
```

}

The *model* and *instances* parameters were specific for each problem and are indicated below. Multiple values were tested for both *alpha* and *discount* since

graph	model	instance	epsilon	decay	alpha	discount	run	time	percentage	Qsize
fxttgdplljwpodu	0	[2, 5, 3, 1, 0, 6, 4, 7, 8]	1	0.9	0.5	0.9	1	60534.84	88.5	181431
vapbgsvfikmmcny	1	[2, 5, 3, 1, 0, 6, 4, 7, 8]	1	0.9	0.1	0.1	0	2363.86	100	63999
oglcuwowsaxikcg	0	[2, 7, 5, 0, 8, 4, 3, 1, 6]	1	0.9	0.5	0.5	1	72751.35	84	181431
wbqtvnpdmuezgur	1	[2, 7, 5, 0, 8, 4, 3, 1, 6]	1	0.9	0.1	0.1	1	180027.93	28	124819
njodxxsxttnwfae	0	[8, 6, 7, 2, 5, 4, 0, 3, 1]	1	0.9	0.5	0.9	1	73698.61	84.5	181431
eklikltolpownxu	1	[8, 6, 7, 2, 5, 4, 0, 3, 1]	1	0.9	0.1	0.5	1	166346.21	35	136110

Table 1: Sample of the csv file for the 8-puzzle experiments.

these are the two most frequently adjusted parameters in RL experiments. The results for each of the three problems are briefly discussed below. The examples provided are not the extreme cases. Since the purpose of this paper in not to evaluate certain models or reward functions, they are provided just to indicate potential benefits to evaluating and adjusting the experiment's parameters and associated models.

4.1 8-Puzzle

For the classic 8-puzzle problem we ran a series of experiments using the configurations described in Section 3.3 with a limit of 100 000 steps for each epoch. A sample of the generated csv file including the results for three instances (with short, medium and long optimal solutions) are shown in Table 1. The complete file is included in the archive provided³.

Even in the sample results you can easily make some relevant observations: the first model (model 0) works reasonably well for all instances and is virtually unaffected by the length of the solution with regards to the number of epochs required to reach a reasonable performance, while also being the most consistent over different runs with the same parameters. Also, this model generates the largest Q-table, exploring close to all 181 440 states which can reach a solution for the 8-puzzle problem (Johnson et al., 1879).



Figure 1: Sample representation of a run for 8-puzzle.

The second model, using the manhattan distance, performs the best overall for the shortest-solution instance, but would probably require more epochs to reach the same performance for the other instances. The number of explored states is significantly less than for the previous model. This indicates that the reason for the poorer performance for the more difficult instances is a tendency to reward too greatly approaching the goal state which impedes the exploration of possible paths to the goal. A sample of a graphical representation of the second run in Table1 is shown in Figure 1.

Various data could be extracted from these results with regards to the impact of the *epsilon*, *decay*, *alpha* and *discount* parameters. For example, using the data provided by the AIM-RL output, we can observe a correlation between the training rate, the discount rate and the average size of the Q-tables built in various runs. In Table 2 we can see that a higher training rate seems to contribute to a significant reduction in the number of explored states. Similar statistics can be produced for particular models or instances, which could lead to potential adjustments of these parameters or of the reward function.

4.2 Frozen Lake

The experiments used the same instance as exemplified in Gym/Gymnasium in the discrete (8) example⁴, which is a 8x8 size square matrix with 7 hidden areas. The run in Figure 2 was generated for the first model with the parameters *epsilon* = 1, *decay* = 0.9, *alpha* = 0.9 and *discount* = 0.5. The association between the other details of a run, included in the csv file, and the generated graphical representation is made by using the name of the generated file, as indicated in Sec-

Table 2: The impact of α and γ over the average number of explored states.

	$\alpha = 0.1$	α = 0.5	α = 0.9
$\gamma = 0.1$	151029.75	130502.5	125424.75
$\gamma = 0.5$	145733.1667	139352.1667	126577.4167
$\gamma = 0.9$	151429.0833	137933.75	128731.5833

⁴https://gymnasium.farama.org/environments/toy_text/ frozen_lake/

³https://tinyurl.com/4mhx83yd



Figure 2: Sample representation of a run for FrozenLake.

tion 3.2. Using just this graph we can make various observations about this experiment:

- Very good performance after the initial 35 epochs: a solution (generally the shortest path) is found in 96.5% of epochs.
- This model with these parameters requires about 30 epochs from reaching a solution to stabilising to the shortest one. An anomalous epoch is still apparent after the stabilisation, probably due to a catastrophic decision made early on by random chance given by the always non-zero *epsilon*.

Looking at the entire set of generated graphs can also provide significant insights into the way various models and parameters influence the training outcome.

4.3 Mountain Car

The mountain car experiments have made apparent the difficulty in handling control problems where the association between actions and reaching the goal state is less significant. The instance parameters were selected to be identical to those used in the Gym/Gymnasium discrete(3) example⁵. Due to the fact that an epoch can end if the car exists the limits of the (-1, 1) range, and the car starts at -0.5, the initial epoch tend to fail by the agent passing the left side limit as seen in Figure 3. After it learns that the goal is to the right (after about 50 epochs), it always finds a solution even if it is of varied length.

From the csv file (included in the mentioned archive) the user can also make a series of interesting observations. One observation is that the second more complex model explores significantly less states (which is of especially significance for this problem



Figure 3: Sample representation of a run for MountainCar.

with an extreme amount of states), even if it successfully completes just slightly more epochs than the basic model. Another interesting observation is that the training rate has a more prominent influence over the average Q-table size than the discount rate, for both models, with the better value from the three tried being 0.9 with an approximate of 86% reduction of the Q-table size as opposed to the next best value of 0.5.

5 CONCLUSIONS

Gym/Gymnasium is the most feature rich framework for implementing and executing RL tasks, with a wide range of implemented problems and models (environments). As an alternative to it, AIM-RL aims to cover particular use cases when the requirements are focused on transparency and flexibility, allowing the user to both implement RL experiments for any problem and any model with minimum code overhead, as well as observing the results and applying corrections to the experiments parameters. In section 3.2 we have shown the steps required within AIM-RL in order to implement a new problem and a new model from scratch. For the three problems described the effort is similar and proportional to the complexity of the problem and the model employed. Efforts have been made to optimise the speed of execution of the actual RL, while also providing to the user metrics to evaluate the performance of his experiments. The results are presented in both numerical and graphical representations, as shown in 4.1.

5.1 Future Work

Our framework is already available as a Python package, with immediate plans to be used in several di-

⁵https://gymnasium.farama.org/environments/ classic_control/mountain_car/

dactic and research RL applications. The very near future development plans for AIM-RL include the extension to allow as an option Deep Q-Learning (François-Lavet et al., 2018) as well as alternatives to the ε -greedy selection such as Randomised Probability Matching (Scott, 2010). As a later development we plan to facilitate the usage of AIM-RL as a support framework for e-learning by adding a graphical UI and a validator for implemented models.

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