# **Incremental Reliability Assessment of Large-Scale Software via Theoretical Structure Reduction**

Wenjing Liu<sup>1,2</sup>, Zhiwei Xu<sup>3</sup>, Limin Liu<sup>1</sup> and Yunzhan Gong<sup>2</sup>

<sup>1</sup>College of Data Science and Application, Inner Mongolia University of Technology, Hohhot 100080, China <sup>2</sup>State Key Laboratory of Networking and Switching Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China

<sup>3</sup>Institute of Computing Technology, Chinese Academy of Sciences, Beijing 100190, China

- Large-Scale Software, Incremental Reliability Assessment, Structure Sequentialization, Theoretical Keywords: Reduction, Importance Sampling Based Reliability Assessment.
- Problems of software quality assurance and behavior prediction of large-scale software systems have high Abstract: importance due to the fact that software systems are getting more prevalent in almost all areas of human activities, and always include an large number of modules. To continuously offer significant changes or major improvements over the existing system, software upgrading is inevitable. This involves additional difficulty to assess reliability and guarantee the quality assurance of the large-scale system. The existing reliability assessment methods cannot continuously vet effectively assess the software reliability because the program structure of the software is not taken into account to drive the assessment process. Thus, it is highly desired to estimate the software reliability in an incremental way. This paper incorporates theoretical sequentialization and reduction of the program structure into sampling-based software reliability evaluation. Specifically, we leverage importance sampling to evaluate reliability rates of sequence structures, branch structures and loop structures in the software, as well as transition probabilities among these structures. In addition, we sequentialize program structures to support the aggregation of reliability assessment results corresponding to different structures. Finally, a real-world case study is provided as a practical application of the proposed incremental assessment model.

#### **INTRODUCTION** 1

Large-scale software is a term used in fields including system engineering, computer science, data science and artificial intelligence to refer to software intensive systems with a large amounts of hardware, lines of source code, numbers of users, and volumes of data. The obstacles of large-scale systems in defect discovery and reliability assessment have become the crucial factors that affect the application of modern software systems in more fields. In October 2018 and March 2019, the Boeing 737 Max 8 crashed twice due to the design defects in its software system, causing 346 people dead (TRAVIS, 2019). Similar system failures caused by software defects always lead to serious accidents that result in death, injury, and large financial losses. To evaluate the risk of system failures, software reliability is introduced and represents the probability of software running

without failure in a given time and under given conditions (Committee et al., 1990). During the process of software development, reliability assessment helps project managers to evaluate the reliability level of the product, so as to make scientific management decisions on the software development. On the other hand, users can also have a quantitative understanding about the quality-related factors of software products. Therefore, the research of software reliability assessment has important theoretical and practical value. Without software reliability assessment, the increasingly pervasive use of software may bring about more frequent and more serious accidents. Measuring and predicting software reliability has become vital in software engineering (Bistouni and Jahanshahi, 2020).

Reliability assessment solutions can be roughly divided into black-box methods and white-box methods (Goševa-Popstojanova and Trivedi, 2001). Regard as a black-box, software is studied while only the interaction between the software and the external environment is considered. These methods rely heavily on the collection of test data and failure data. In some safety-critical application fields, such as aviation, aerospace, etc., failure data is scarce or even non-existent, and thus it is impossible to accurately estimate the reliability of such systems. On the other hand, to assess the reliability of the software in an accurate way, the white-box methods combine the dynamic information of the software structure with the failure behavior of the software. These methods quantitatively analyze the degree of dependence among modules and determine the crucial parts in the software. Considering the complex dependence among modules of a large-scale software. the existing white-box reliability assessment methods(Gokhale, 2007) still need to be improved to enhance assessment efficiency.

To speed up reliability assessment of large-scale software systems and discover the potential failure risk before it gets too late, an incremental assessment method is highly desired. The white-box evaluation process relies on the run-time sequence of modules or statement blocks, and this sequence is determined by both of the program structure and the input data of the software. In sight of this fundamental principle, we study the essential structures of a software to guide the reliability assessment process. In this way, the reliability rates of software modules or statement blocks are aggregated in terms of different run-time sequences. Since this aggregation process is performed according to a determined sequence, we can update each part of the reliability assessment result without any intervene of the others. The primary contributions of this work are listed as follows.

#### **Contributions**:

- Base on importance sampling, we propose a scheme to efficiently estimate reliability unit parameters (node reliability rate and transition probability). In this way, it becomes feasible to study software reliability in a modular way.
- We design a structural reduction and reliability assessment model to aggregate unit parameters with respect to three types of control structures. Moreover, an incremental assessment updating algorithm is developed to update the assessment result without model reconstruction.
- The proposed method can effectively assess software reliability in an incremental way (a real-world software module is provided as a use case in a technical report with the same title).

# 2 RELATED WORKS

Software reliability is an essential subject in the development stage, which is a combination of software engineering and reliability engineering, and can be roughly divided into black-box methods and white-box methods.

The black-box software reliability methods only concern the functionality of software without an attempt to understand its internal structure. Generally, reliability analysis is conducted mainly depending on failure data, assuming a parametric model of failure data. In this way, the statistical models (Pressman, 2005) and the software reliability growth models(SRGM) (Stringfellow and Andrews, 2002) are proposed. Kumar et al. proposed an Ideal solution for the selection of a suitable SRGM and applied it for optimal selection and ranking of SRGMs (Kumar et al., 2021). With considerations of the phenomenon of imperfect debugging, varieties of errors and change points during the testing period, Huang et.al extend the practicability of SRGMs (Huang et al., 2022). Black-box-based models need to continuously collect run-time failure data to estimate the software reliability in different scenarios. Actually, a failure occurs when the user perceives that the software has ceased to deliver the expected result with respect to the specific input Considering the complex structure of values. large-scale systems, the presence of errors in the large-scale systems does not always lead to system failures. We have little or no opportunity to observe such problems and collect failure data, and cannot achieve an accurate estimation of software reliability in practice.

White-box methods (Hsu and Huang, 2011; Bistouni and Jahanshahi, 2020) are proposed and leverage the hierarchical structure of modular software systems to access software reliability. The primary objective of white-box models has been applied to obtain the software reliability among modular interactions in large-scale systems. Compared with the black-box-based software reliability growth model, this type of white-box-based method has two advantages: (1) Considering the internal structure information of the software, it can evaluate the system more reasonably and accurately; (2) The system reliability can be evaluated in the early stage of software development, and errors can be found. Littlewood (Littlewood, 1979) constructed a software structure model comprised of a finite number of states developed by a semi-Markov process. Hsu (Hsu and Huang, 2011) proposed an adaptive framework to incorporate path

testing into reliability estimation for software systems. Though more accurate assessment has been achieved, lacking of the structural parameter reduction mechanism in terms of different basic program units, the existing white-box-based reliability methods need to reconstruct the model from the very beginning to update the reliability assessment result. To efficiently analyze and update the reliability evaluation results of modern large-scale software systems, an incremental estimation method is highly desired.

#### **3 PROBLEM FORMULATION**

The software system can be deconstructed with modules, and these modules may be further divided into sub-modules. Exchanging of control among modules is related to the hierarchical structure of the logical module dependencies. Assuming the modules and their sub-modules have their independent functions, this type of dependencies can be modeled as a Markov process (Cheung, 1980; Wang et al., 2006). Thus, we assume that the software system can be designed in a structural or modular way so that composition or decomposition into its constituent units is possible (Myers et al., 2011; Shooman, 1976; Jorgensen, 2013). Due to the complex characteristics of large-scale software systems, the reliability of software systems depend on not only the failure behavior of individual module but also the relations among different modules. To formulate the impact of these two facts, the related concepts are defined below. First of all, the control flow graph is used to model the dependencies of modules yet the overall software system.

**Definition 1** (Control flow graph). The control flow graph of a unit (a function or a statement block) can be represented as  $G = \langle N, E, N_1, N_n \rangle$ , where  $N = \{N_i | i = 1, 2, ..., n\}$  is a set of nodes, node  $N_i$ corresponds to a unit in the software system.  $E = \{E(i, j) | i, j = 1, 2, ..., n\}$  is a set of directed edges, and edge E(i, j) corresponds to the control flow from node  $N_i$  to node  $N_j$ , where  $N_1$  is the entry node of software system, and  $N_n$  is the exit node of the software system.

To enable reliability assessment based on a control flow graph, parametric estimations of node reliability and transition probability among nodes should be performed in advance. Among them, node reliability indicates the probability that a node provides the correct output. Meanwhile, the transition probability expresses the characteristics of different branches in the software structure (Yacoub et al., 2004).

**Definition 2** (Node reliability rate). *The reliability rate of node*  $N_i$  *is*  $R_i$ , *which represents the probability that node*  $N_i$  *performs the correct output, and transfers to the next node*  $N_i$ ,  $R_i \in (0, 1]$ .

**Definition 3** (Transition probability). *Edge*  $E(i, j) \in G$  corresponds to a transition probability  $P_{i,j}$  between node  $N_i$  and node  $N_j$ . If  $P_{i,j} = 0$ , it is impossible that node  $N_j$  is performed after node  $N_i$ . Otherwise,  $P_{i,j} \in (0,1]$ , and  $N_i$  is followed by node  $N_j$  by probability  $P_{i,j}$ . The sum of the transition probabilities regarding to predecessor node  $N_i$  is 1, that is,  $\sum_{i \neq j} P_{i,j} = 1$ .

In summary, our proposed method is based on the following assumptions:

- All program units are physically independent of each other. Modification of one unit has no impact on others. This assumption implies that we can independently design, implement, and test each unit and module in a complex system (Gokhale, 2007; Goševa-Popstojanova and Trivedi, 2001; Gokhale and Trivedi, 2002). Thus, the faults in each unit are also independent from each other. Moreover, each discovered defect can be easier to be isolated and fixed in a specified module during the processes of defect detection and correction.
- The transfer of control between units can be described by a Markov process. The Markov process depends on the structure of the software system so that it is helpful in modeling the execution between the functional units and branching characteristics (Gokhale, 2007). This assumption also indicates that the next unit to be executed depends only on the present unit and is independent of the past history. Without loss of generality, in this paper we only focus on the single-input and single-output program graph (Lo et al., 2002; Lo et al., 2003; Lo et al., 2005).

# 4 INCREMENTAL SOFTWARE RELIABILITY ASSESSMENT

To facilitate the incremental software reliability assessment, we analyze software structures and aggregate the reliability assessment results on different structures of the software. It has been proved by theory and practice that no matter how complicated a software system is, it can be deconstructed into three types of basic control

| CMMI level  | level 1&others  | level 2 | level 3  | level 4   | level 5  |
|---|---|---------|--|---|--|
| #Defects/KLOC   | 11.95   | 5.52    | 2.39   | 0.92  | 0.32   |
| P <sub>1,2</sub><br>P <sub>1,2</sub><br>P <sub>1,2</sub><br>P <sub>1,1</sub><br>P <sub>1,1</sub><br>P <sub>1,1</sub><br>P <sub>1,2</sub><br>P <sub>1,3</sub><br>P <sub>1,3</sub><br>P <sub>1,4</sub><br>P <sub></sub> | $P_{1,n-1}$<br>$N_{n-1}R_{n-1}$ Sequentializ<br>$P_{n-1,n}$ | ration  | Seq[1]<br>$\downarrow$<br>$P_{1,2}$<br>$P_{2,n}$<br>$P_{2,n}$<br>$N_n$ $R_n$ | $Seq[i]$ $\downarrow$ $P_{1,i}$ $P_{1,n}$ $P_{1,n}$ $R_{n}$ | $Seq[n-1]$ $\downarrow$ $P_{1,n-1}$ $P_{n-1,n}$ $(N_n)R_n$ |
| ¥   |   |         | $\bigvee$  | ¥   | $\downarrow$   |

Table 1: CMMI level and the corresponding defect number per thousand lines.

Figure 1: Sequentialization of the branch structure.

structures, *i.e.*, sequence structure, branch structure and loop structure. These structures regulate the order in which node (i.e., program statements) in a software are executed. Therefore, we analyze the software reliability in terms of different control structures. Specifically, we incorporate the control flow graph of the software into this structural analysis process to represent different structures. Through traversing the control flow graph, the parameters related to reliability assessment in terms of different structures are marked in the the control flow graph and aggregated to achieve the reliability assessment result of the entire software. Ultimately, if the statement block included in a node has been upgraded, the reliability rate corresponding to this node in a control structure should be updated. Therefore, an incremental assessment process is highly desired.

# 4.1 Node Reliability Rate and Transition Probability

Nodes and edges are the essential elements of a control flow graph, and correspondingly, the node reliability and transition probability between nodes are required for the reliability assessment process based on the control flow graph. The node reliability rate depends on the probability that this node can be executed correctly to obtain the intended output.

**Theorem 1** (Necessary Condition for Execution Failure).  $\forall s \text{ is a node (statement block)}, \mathbb{X} \text{ is the input set of s. When a failure occurs, } \exists x \in \mathbb{X}, \text{ the corresponding output of s, } s(x), \text{ is not the same as the intended output } \hat{s}(x), \text{ that is } s(x) \neq \hat{s}(x). \text{ This is a necessary condition for execution failure.}}$ 

The proof of Theorem 1 follows by inspection.

On the other hand, the transition probability between two adjacent program nodes indicates the probability that a successor node will be performed after a predecessor node. The probability is equal to "1" unless there exists a branch condition involved in the predecessor node. In this case, the input domain of the predecessor node should be taken into consideration to determine which successor node will be performed. It is in appearance that the estimation of node reliability rates or transition probabilities relies on the input domain of the predecessor node. Considering the input domain is always quite large and various, we leverage sampling technology on the path-based test suite to approach efficient estimation of node reliability rates and transition probabilities.

Though the existing theories, e.g, Interval arithmetic, etc., facilitate test data generation, all test data for a specific node on the test path cannot be completely determined. Therefore, their distribution is under-determined. Since the data that can accuse a failure is quite rare, most of sampling technology cannot cover all possible test data and evaluate their distribution comprehensively. If the focus is on a problem related to system reliability, the probability of rare events is better approximated with appropriate proposals. Even if a large number of samples have been drawn, it is possible that none of them will accuse a failure or a jump to the branch determined by a special condition. To tackle this problem, we collect test data of every node (statement block), and then leverage importance sampling to draw samples from the collected test data. As a widely applied system simulation technology, importance sampling (Tokdar and Kass, 2010) estimates



Figure 2: Sequentialization of the loop structure.

properties of a distribution, while only having less samples generated from a proposal distribution than the distribution of interest. Targeting to sampling different types of test data comprehensively, a proposal distribution is involved, with respect to which among the test data are drawn as samples. The density function of the proposal distribution is q(x). According to importance sampling theory, q(x)should dominate the density function of the distribution of test data, p(x), and consequently, we can cover the distribution of test data with less samples. That means,  $q(x) = 0 \Rightarrow p(x) = 0$ .

To cover rare events during the sampling process, we take a Poisson distribution,  $Pois(\lambda)$ , as the proposal distribution.  $\lambda$  is determined according to the CMMI level of the software designer. The CMMI model is used to assess the maturity of an designer's ability and to provide guidance on improving processes, with a goal of the advanced software systems. There is a relation between CMMI level land the corresponding defect number per thousand code, of namely, dKLOC lines (see Table 1) (CMMI-Institute, 2022). To dominate the density of the distribution of test data, we configure  $\lambda$ with a value lower than the corresponding dKLOC, equal to

$$\beta \times klDefects(l) \times \#statements$$
 (1)

where  $\beta$  is a constant less than 1, *klDefects()* is a function to look up the defect number per thousand lines of code, *l* is the CMMI level corresponding to the software designer, and *#statements* is the number of statements in this node.

The sampling process consists of four steps:

- Collect test data used to test a node (statement block), and insert these data into a test suite X, until their distribution converge to a stable density, p(X).
- Draw samples from X with respect to  $Pois(\lambda)$ , n samples and the density function values of these samples are obtained, *i.e.*,  $x_1, \ldots, x_n$ , and  $q(x_1), \ldots, q(x_n)$ .
- Weight the sample  $x_i$  with  $w(x_i) = \frac{p(x_i)}{q(x_i)}, i \in [1, n]$ .
- Using the samples and weights, the reliability rates and transition probabilities can be approximated by a self-normalized estimator as

$$\ell = \frac{1}{n\hat{Z}} \sum_{k=1}^{n} w_k f(x_k), \qquad (2)$$

where  $\hat{Z} = (1/n) \sum_{k=1}^{n} w_k$  is an unbiased estimator of  $Z = \int_X p(x) dx$  (Bugallo et al., 2017).

The detailed estimation process of node reliability is formulated as the follows:

$$R_{i} = 1 - \frac{1}{n\hat{Z}} \sum_{k=1}^{n} w_{k} I(x_{k}), \qquad (3)$$

where  $I(x_k)$  is an indicator function that takes the value 1 if the output of statement block *s* corresponding to test data  $x_k$  is not the same as the intended output  $\hat{s}(x_k)$ , and 0 otherwise. Statement block *s* constitute node  $N_i$ . That is

$$I(x_k) = \begin{cases} 1 & s(x_k) \neq \hat{s}(x_k) \\ 0 & s(x_k) = \hat{s}(x_k) \end{cases}$$

Similarly, the transition probabilities from the i-th node to the j-th node can be calculated as the

following:

$$P_{i,j} = \frac{1}{n\hat{Z}} \sum_{k=1}^{n} w_k I_{i,j}(x_k)$$
(4)

Another indicator function  $I_{i,j}(x_k)$  is used, which outputs 1 if the satisfied condition of *s* indicates a jump to the j-th node, and 0 otherwise.

#### 4.2 Sequentialization and Reliability Evaluation of Control Structures

To efficiently analyze the reliability of a large-scale software system, we need to simplify its complex structure. To achieve that, we traverse and sequentialize program units by using a hierarchical approach (Hsu and Huang, 2011). The sequence structure is the basic control structure for a program. The program units (statement blocks) are executed according to their sequence. In a modular way, a program can be written using the sequence structures, and thus, with respect of the concepts introduced in Section 3, its reliability rate can be computed as the following,

$$R = R_1 \times \prod_{i=2}^{n} R_i^{\prod_{j=2}^{i} P_{j-1,j}}$$
(5)

where  $R_i$  represents the reliability rate of the i - th node (statement block), and  $P_{i,j}$  is the transition probability between the i - th node and the j - th node.

The other two types of control structures, branch structures and loop structures. They also only have an entry node  $N_1$  and an exit node  $N_n$ . Among them, the branch structure determines the execution sequence of different branches according to a condition. The loop structure is an instruction that repeats until a specified condition is reached. In order to simplify these program structures, we sequentialize branch structures and loop structures to sequence structures, which is the preliminary work of structural reduction.

As shown in Fig. 1, each of these branch structures can be transformed into a sequence structure through sequentialization. Only if all branches are reliable, the entire structure is reliable. Therefore, the reliability of the branch structure is calculated by considering the reliability rates of all branches.

$$R_{Bran} = \prod_{i=1}^{n-2} R_{Seq[i]} \tag{6}$$

where  $R_{Bran}$  represents the reliability rate of the branch structure,  $R_{Seq[i]}$  represents the reliability rate of the i-th sequence structure.

Algorithm 1: Structural reduction and  
reliability assessment.  
Input : control flow graph G  
Output: reliability rate R  
1 iterate 
$$(G, P) /* P$$
 is the probability  
transiting to the module  
corresponding to G. \*/  
2 state = next(G); // A statement block  
3 sstate = next(G); // Next statement block  
4 while sstate belongs to a sequence structure  
do  
5  $R = R \times nr(state)^P$ ; /\* Function nr is  
used to estimate node reliability  
rates \*/  
6  $P = P \times tp(state, sstate); /* Function tp$   
is used to estimate transition  
probabilities \*/  
7 mark(state,P); /\* Mark node  
reliability rates & transition  
probabilities \*/  
8 state = next(G);  
10  $R = R \times nr(state)^P$ ;  
11 pstate = state;  
12 state = next(G);  
14 if state belongs to a branch structure then  
15 foreach G' in seq(subGraph(G, state)) do  
// Iterate sequentialization  
results of the rest part of G  
R = R × iterate(G', P);  
17  $Mark(state,P);$   
18 if state belongs to a loop structure then  
19 mark(state,P);  
10 mark(state,P);  
11 pstate = then  
12 foreach G in seq(subGraph(G, state)) do  
14 if state belongs to a loop structure then  
15 mark(state,P);  
16 mark(state,P);  
17 mark(state,P);  
18 if state belongs to a loop structure then  
19 mark(state,P);  
10 mark(state,P);  
11 mark(state,P);  
12 mark(state,P);  
13 state = next(G',P);  
14 mark(state,P);  
15 mark(state,P);  
16 mark(state,P);  
17 mark(state,P);  
18 mark(state,P);  
18 mark(state,P);  
19 mark(state,P);  
10 mark(state,P);  
10 mark(state,P);  
11 mark(state,P);  
12 mark(state,P);  
13 mark(state,P);  
14 mark(state,P);  
14 mark(state,P);  
15 mark(state,P);  
15 mark(state,P);  
16 mark(state,P);  
17 mark(state,P);  
18 mark(state,P);  
18 mark(state,P);  
19 mark(state,P);  
10 mark(state,P);  
10 mark(state,P);  
10 mark(state,P);  
11 mark(state,P);  
12 mark(state,P);  
13 mark(state,P);  
14 mark(state,P);  
14 mark(state,P);  
15 mark(state,P);  
15 mark(state,P);  
16 mark(state,P);  
17 mark(state,P);  
18 mark(state,P);  
18 mark(state,P);  
19 mark(state,P);  
19 mark(state,P);  
10 mark(state,P);  
10 mark(state,P);  
1

20 
$$P_{1} = \frac{tp(pstate,state)}{1-tp(state,sstate)tp(ssstate,state)};$$
21 
$$P_{2} = \frac{tp(pstate,sstate) \times tp(sstate,sstate)}{1-tp(ssstate,state)tp(state,sstate)};$$
22 
$$R = R \times nr(state)^{P_{1}} \times iterate(subGraph(G,state), P)^{P_{2}};$$
23 return R

A loop structure executes iterative statements or procedures, according to a condition or an iteration. Whilst the condition is true or the iteration has not stopped, the loop body will be carried out repetitively. In sight of the run-time process of the loop structure, a loop structure can be expanded into a branch structure. For each branch of this structure, the loop body is executed different number of times. In this way, this loop structure is equivalent to one branch structure. As illustrated in Fig. 2.b, when the

$$\begin{split} R_{Loop} \\ &= (R_1 \times R_2^{1 \times P_{1,2}} \times R_3^{1 \times P_{1,2} \times P_{2,3}} \times R_n^{1 \times P_{1,2} \times P_{2,3} \times P_{3,n}}) \\ &\times (R_2^{1 \times P_{1,2} \times P_{2,3} \times P_{3,2}} \times R_3^{1 \times P_{1,2} \times P_{2,3} \times P_{3,2} \times P_{2,3}} \times R_n^{1 \times P_{1,2} \times P_{2,3} \times P_{3,2} \times P_{3,n}}) \\ &\times (R_2^{1 \times P_{1,2} \times (P_{2,3} \times P_{3,2})^2} \times R_3^{1 \times P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^2} \times R_n^{1 \times P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^2 \times P_{3,n}}) \\ &\times \dots \\ &\times (R_2^{1 \times P_{1,2} \times (P_{2,3} \times P_{3,2})^m} \times R_3^{1 \times P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^m} \times R_n^{1 \times P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^m \times P_{3,n}}) \\ &= R_1 \times R_2 \sum_{i=0}^m P_{1,2} \times (P_{2,3} \times P_{3,2})^i \times R_3 \sum_{i=0}^m P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^i \times R_n \sum_{i=0}^m P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^i \times R_n \sum_{i=0}^m P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^i \times R_n \sum_{i=0}^m P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^i \times R_n \sum_{i=0}^m P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^i \times P_{3,n}) \\ &= R_1 \times R_2 \sum_{i=0}^m P_{1,2} \times (P_{2,3} \times P_{3,2})^i \times R_3 \sum_{i=0}^m P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^i \times R_n \sum_{i=0}^m P_{1,2} \times P_{2,3} \times (P_{3,2} \times P_{2,3})^i \times (P_{3,2} \times P_{3,3})^i \times (P_{3,2} \times P_{3,3} \times (P_{3,2} \times P_{3,3})^i \times (P_{3,2} \times P_{3,3})^i \times (P_{$$

Algorithm 2: Incremental updating of the assessment result.

|   | <b>Input</b> : G, R, state, <i>state</i>          |
|---|---|
|   | Output: R   |
| 1 | < NR, NTP >= search(G, state); // Search          |
|   | statement blocks in G                             |
| 2 | NR' = nr(state'); // Get the updated              |
|   | reliability rate                                  |
| 3 | $R = R \times \left(\frac{NR'}{NR}\right)^{NTP};$ |
| 4 | return R  |

condition in node  $N_3$  becomes false, the exit node  $N_n$  of the loop structure is executed, and the loop body will not be executed again. Otherwise, loop body  $N_2$  will be executed repeatedly. The blue box and green box indicate the node sequences for the first and second time when the loop body is executed. Ultimately, distinct sequence structures are obtained. When the number of loop times approaches infinity, the reliability of the loop structure can be executed as follows:

#### 4.3 Incremental Reliability Evaluation with Structural Reduction

Based on the above sequentialization results, we can incorporate the control flow graph of the software into an incremental structural reduction and reliability analysis process. Through traversing the control flow graph, three basic program control structures are identified and sequentialized. Meanwhile, their reliability rates are evaluated (see Section 4.2). The propsed incremental reliability evaluation process includes 6 steps:

• The parameters (node reliability rates and transition probabilities) related to reliability assessment in terms of different structures are estimated and marked on the control flow graph.

- Since the control flow graph accurately represents the flow inside of nodes (statement blocks), we traverse the control flow graph of the software to look up control structures included in the software.
- With respect to statement blocks included every structure, their node reliability rates and transition probabilities to the successors are estimated through importance sampling and marked in the the control flow graph.
- Sequentialize the statement blocks in every structure in terms of the structure type (*i.e.*, sequence, branch or loop), and aggregate the node reliability rates of these blocks.
- Reduce different structures and obtain the reliability assessment result of the software (detailed in Algorithm 1).
- If the reliability rate of a node has changed, taking advantage of the node parameters marked in the control flow graph, the corresponding assessment result can be updated according to Algorithm 2.

# **5** CONCLUSION

In this paper, a structure-based reliability assessment model is proposed. We first take importance sampling to evaluate reliability rates of three types of control structures in a software, as well as transition probabilities among these structures. Then, we reduce these structures while aggregating their reliability rates to the overall assessment result of the software. With a case study on a elevator system, it is shown that the proposed model can give a promising estimation of software reliability. In summary, we can conclude that in an incremental way, the proposed incremental assessment model is viable for estimating reliability of a large-scale software system. There exist more open theoretical issues about Structure Reduction for future research. For instance, dependent faults may occur if the data exchanges among distributed sub-systems, this increase complexity of structure-based reliability assessment.

#### ACKNOWLEDGEMENTS

This work was supported by the Basic Scientific Research Expenses Program of Universities directly under Inner Mongolia Autonomous Region, Grant/Award Number: JY20220273.

#### REFERENCES

- Bistouni, F. and Jahanshahi, M. (2020). Evaluation of reliability in component-based system using architecture topology. *Journal of the Institute of Electronics and Computer*, 2(1):57–71.
- Bugallo, M. F., Elvira, V., Martino, L., Luengo, D., Miguez, J., and Djuric, P. M. (2017). Adaptive importance sampling: The past, the present, and the future. *IEEE Signal Processing Magazine*, 34(4):60–79.
- Cheung, R. C. (1980). A user-oriented software reliability model. *IEEE transactions on Software Engineering*, (2):118–125.
- CMMI-Institute (2022). White papers. Last accessed 21 January 2022.
- Committee, I. S. C. et al. (1990). Ieee standard glossary of software engineering terminology (ieee std 610.12-1990). los alamitos. CA: IEEE Computer Society, 169:132.
- Gokhale, S. S. (2007). Architecture-based software reliability analysis: Overview and limitations. *IEEE Transactions on dependable and secure computing*, 4(1):32–40.
- Gokhale, S. S. and Trivedi, K. S. (2002). Reliability prediction and sensitivity analysis based on software architecture. In 13th International Symposium on Software Reliability Engineering, 2002. Proceedings., pages 64–75. IEEE.
- Goševa-Popstojanova, K. and Trivedi, K. S. (2001). Architecture-based approach to reliability assessment of software systems. *Performance Evaluation*, 45(2-3):179–204.
- Hsu, C.-J. and Huang, C.-Y. (2011). An adaptive reliability analysis using path testing for complex componentbased software systems. *IEEE Transactions on Reliability*, 60(1):158–170.
- Huang, Y.-S., Chiu, K.-C., and Chen, W.-M. (2022). A software reliability growth model for imperfect debugging. *Journal of Systems and Software*, 188:111267.
- Jorgensen, P. C. (2013). Software testing: a craftsman's approach. Auerbach Publications.

- Kumar, V., Saxena, P., and Garg, H. (2021). Selection of optimal software reliability growth models using an integrated entropy-technique for order preference by similarity to an ideal solution (topsis) approach. *Mathematical Methods in the Applied Sciences*.
- Littlewood, B. (1979). Software reliability model for modular program structure. *IEEE Transactions on Reliability*, 28(3):241–246.
- Lo, J.-H., Huang, C.-Y., Chen, Y., Kuo, S.-Y., and Lyu, M. R. (2005). Reliability assessment and sensitivity analysis of software reliability growth modeling based on software module structure. *Journal of systems and software*, 76(1):3–13.
- Lo, J.-H., Huang, C.-Y., Kuo, S.-Y., and Lyu, M. R. (2003). Sensitivity analysis of software reliability for component-based software applications. In *Proceedings 27th Annual International Computer Software and Applications Conference. COMPAC* 2003, pages 500–505. IEEE.
- Lo, J.-H., Kuo, S.-Y., Lyu, M. R., and Huang, C.-Y. (2002). Optimal resource allocation and reliability analysis for component-based software applications. In *Proceedings 26th Annual International Computer Software and Applications*, pages 7–12. IEEE.
- Myers, G. J., Sandler, C., and Badgett, T. (2011). *The art of software testing*. John Wiley & Sons.
- Pressman, R. S. (2005). Software engineering: a practitioner's approach. Palgrave macmillan.
- Shooman, M. L. (1976). Structural models for software reliability prediction. In *Proceedings of the 2nd international conference on Software engineering*, pages 268–280.
- Stringfellow, C. and Andrews, A. A. (2002). An empirical method for selecting software reliability growth models. *Empirical Software Engineering*, 7(4):319–343.
- Tokdar, S. T. and Kass, R. E. (2010). Importance sampling: a review. *Wiley Interdisciplinary Reviews: Computational Statistics*, 2(1):54–60.
- TRAVIS, G. (2019). How the boeing 737 max disaster looks to a software developer. Last accessed 21 January 2022.
- Wang, W.-L., Pan, D., and Chen, M.-H. (2006). Architecture-based software reliability modeling. *Journal of Systems and Software*, 79(1):132–146.
- Yacoub, S., Cukic, B., and Ammar, H. H. (2004). A scenario-based reliability analysis approach for component-based software. *IEEE transactions on reliability*, 53(4):465–480.