Vector Tile Geospatial Data Protection Using Quantization-Based Watermarking

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Abstract: The paper proposes a watermarking method for protecting geodata presented in the Mapbox Vector Tile (MVT) format against theft. MVT is an open format that is gaining popularity in web mapping services due to efficient storage and fast rendering. However, the vector nature of the format makes it an easy target for attackers who want to steal data and use in their services. The method proposed in this paper protects MVT data with a digital watermark based on re-quantization of point coordinates of object geometry. The method can be adjusted using a number of parameters that allow finding a balance between the robustness of the digital watermark to map distortions and the error introduced when embedding. A series of experiments performed showed the robustness of this method to several distortions: removal of some objects and layers, reduction in the number of points of existing objects, addition of new objects, controlled shift of points in the tile geometry. With a proper choice of the watermark parameters, even with a moderate level of each of the listed distortions, which does not lead to a loss of significance of the protected geodata, the method can reach 100% watermark extraction accuracy of all bits of the built-in watermark.

1 INTRODUCTION

Geospatial web services are becoming more popular every year. Modern design standards for such systems require compliance with several requirements (Wallner, 2022). The most crucial of these requirements include efficient data storage, fast and high-quality rendering, and open format support.

Based on these requirements, the Mapbox Vector Tile (MVT) format is increasingly being used in modern web services. MVT is a relatively new open standard for storing and displaying geospatial data (Mapbox, 2023). Along with the traditional Web Map Tile Service (WMTS), it is a hierarchical tile format that is the most convenient for the web. However, unlike WMTS, it stores data not in raster but in vector form, reducing the amount of stored data and improving the rendering quality. The benefits of MVT have been demonstrated in several research papers (Netek, 2020), (Yu, 2017). However, using a vector format increases the risk of theft and manipulation of geospatial data. This is because vector tiles contain more detailed and accurate vector geospatial data, making them more valuable to potential attackers. Additionally, Mapbox Vector Tiles can be easily converted to geoJSON or KML, allowing an attacker to use stolen highprecision data in their GIS products without the permission of the copyright holder.

In contrast, raster tiles contain pre-rendered images of the geospatial data. They are also more challenging to extract and manipulate because they are composed of pixels rather than vectors. While it is possible to extract some information from raster tiles using image processing techniques, the quality of the recovered vector data is generally lower compared to vector tiles.

Despite the security risks associated with the use of vector tiles, the authors of this article could not find any publications in the scientific literature dedicated to the study of the security of MVT, nor the

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development of algorithms, protocols, or scenarios for using MVT sources that increase the security of MVT data. The only work that can be mentioned is (Zhang, 2022), which is dedicated to the MVT-based data collection and publishing technology based on Apache Sedona. The conclusion of this article states that the authors plan to improve the security of MVT data in further studies, but without specifying the methods they plan to use.

One of the most effective and widespread solutions for protecting data from theft and unauthorized manipulation is digital watermarking. This technology consists in a subtle managed change in the data representation or the way of its storage. This change makes it possible to embed some protective information, called a watermark, into the data. This approach was used for more than two decades to protect images, videos, audio, some kinds of medical and engineering data etc. (Cox, 2008). Watermarking methods were also used to protect raster tile maps (Ren, 2014).

Vector geospatial data do not have the same storage redundancy as raster geodata. Therefore, they are less suitable for digital watermarking. Despite this, there are many algorithms for protecting vector cartographic data with digital watermarks based on changing point coordinates, the order of vertex traversal, the index of the initial vertex in the polygon, etc. (Lee, 2013), (Cao, 2015), (Peng , 2018), (Vybornova, 2020). However, all of them are not designed to protect tile vector data, which have some specific features distinguishing them from digital maps.

This article proposes a method for protecting MVT from unauthorized use (theft) based on a robust watermarking method based on re-quantizing the points of polylines or polygons. At the time of writing, the authors could not find any paper describing methods for MVT watermarking.

The following section provides a summary of the MVT format that is important for understanding the proposed method. The method itself is described in Section 3. Section 4 presents the results of experimental studies, and Section 5 gives the main conclusions.

2 THE MAPBOX VECTOR TILE FORMAT

According to the specification (Mapbox, 2023), by default, MVT uses the Web Mercator projection and the Google Tile scheme, which determines the

coordinates and IDs of tiles at each zoom level. Tile data is stored using the Google Protocol Buffers serialization mechanism.

The internal structure of a tile is a collection of layers. Each layer must contain the *extent* property that describes the width and height of the tile in integer coordinates. In fact, the extent, together with the tile scale level, determines the discretization step of object point coordinates when converting them from a GIS vector map to MVT. The discretization step of the entire tile tree is determined by the maximum scale level for which the data is defined, as well as the extent value of the layers of this level. Thus, MVT provides the copyright holder with the opportunity to publish data at an arbitrarily high resolution.

A layer is a collection of objects (features) characterized by their geometry and semantics. Three types of geometry are available: POINT, LINESTRING (polylines), and POLYGON. In multipoint objects, points are connected only by line segments. Circular arcs, Bezier curves, etc. are not supported. Thus, only three commands are used to describe the geometry: MoveTo, LineTo, and ClosePath. The first two commands have parameters that are the number of repetitions and point coordinates. Point coordinates are specified as a pair of integers between 0 and the extent. It is allowed to use points that are not included in this interval. However, only those line fragments that lie in the range $[0, extent] \times [0, extent]$ will be displayed.

3 PROPOSED MVT WATERMARKING METHOD

The proposed method is based on the re-quantization of point coordinates and is suitable for all three types of MVT features. This makes it possible to classify this method as one of the methods based on Quantization Index Modulation (QIM) (Chen, 2001). Moreover, due to the two-dimensional nature of the protected data (plane coordinates), the proposed method is closely related to the geometric interpretation of the QIM method presented in the cited paper.

Each vector tile contains a fragment of a watermark with a length of $N_b \ge 1$ bits. To embed a watermark, a tile is represented as a set of $M \times M$ non-intersecting squares. If *E* is the extent of the tile (its width and height in integer coordinates), then the size of each square is $E/M \times E/M$. *M* is preferably a divisor of *E*. This makes it easier to translate a

specific coordinate in meters to a coordinate in the tile space.

Each of these squares is embedded with one watermark bit. Thus, if the embedding is done without repeats, $N_b = M^2$. However, in practice, one of the most effective ways to increase the robustness is to embed the same bit in several squares. Let r be the number of squares in which the same bit is embedded. Then we arrive at the following equation

$$N_b = \lfloor M^2 / r \rfloor, \tag{1}$$

linking N_b with two other parameters. It is desirable to choose the parameters so that M^2 is evenly divisible by r. In this case, the entire area of the tile will be used to protect it. Embedding in a square can be performed if at least one geometry point falls into this square. Thus, for each bit of the input digital watermark to be embedded, it is necessary that at least one of the r squares corresponding to it contains one point. Certainly, in practice, there should be many more such points to increase the robustness of the watermark.

Let W_{inx} be an $M \times M$ matrix determined for each tile ID separately based on the secret key, containing numbers from 0 to $N_b - 1$. Moreover, each of them must occur exactly r times. This matrix specifies the correspondence between the index of the embedded bit and the squares in which it is embedded.

Also, based on the secret key for each tile ID, an $E \times E$ matrix Map is generated containing the values $\{0,1\}$. The generation is carried out in such a way that for any element of the matrix, the 4-connected distance to the nearest element of the matrix with the opposite value does not exceed the specified value q. We will use small q values, not exceeding 3.

To increase the robustness of the embedded watermark, we also consider a version of the method with Map initially formed as a p times smaller matrix, and then it is resized to $E \times E$.

We will design a couple of related algorithms for embedding and extracting information according to the informed embedding scheme. In this scheme, the information extraction algorithm is designed first, followed by the information embedding algorithm corresponding to it. So let us start with extraction. Consider two versions of the information extraction algorithm that differ in the order of data aggregation from different squares containing a watermark bit with the same index. The first version is based on majority voting, while the second one is based on the formation of general statistics. Extraction algorithm (version 1):

- 1. Loop through all squares $i = 0..M^2 1$:
 - 1.1. Find all geometry points of all tile objects that fall into the i-th square.
 - 1.2. If their number is less than T_1 , then it is decided that this square is not taken into account in the watermark extraction procedure since it can introduce an error. Go to step 1.1 for the square i + 1.
 - 1.3. Each point in the geometry corresponds to a binary value in *Map*. Count the number of zeros s_0 and ones s_1 in the square.
 - 1.4. If $|s_0 s_1|/(s_0 + s_1) < T_2$, then this square is also not taken into account when extracting the watermark. Go to step 1.1 for square i + 1.
 - 1.5. If $s_1 > s_0$, then we decide that the i-th square contains 1, otherwise it contains 0.
- 2. Loop over watermark bits $j = 0..N_b 1$:
 - 2.1. Define the set of square indices $\{i\}$ containing bit index $W_{inx}(j)$.
 - 2.2. The j-th bit is determined by the majority voting method for those squares that are decided to be taken into account when extracting.

Extraction algorithm (version 2):

1.

- Loop through all squares $i = 0..M^2 1$, replenishing statistics $s_0(j)$ and $s_1(j)$ common to the entire tile, where $j = 0..N_b 1$ are watermark bit indices:
 - 1.1. All geometry points of all tile objects that fall into the i-th square are found.
 - 1.2. Each point of the geometry corresponds to a binary value in the matrix *Map*. Count the number of zeros $s_{0,i}$ and ones $s_{1,i}$ in the i-th square and add them to the statistics that refers to bit $W_{inx}(i)$:

 $s_0(W_{inx}(i)) \coloneqq s_0(W_{inx}(i)) + s_{0,i},$

 $s_1(W_{inx}(i)) := s_1(W_{inx}(i)) + s_{1,i}.$

- 2. Loop over indices $j = 0..N_b 1$:
 - 2.1. If $s_0(j) + s_1(j) < T_1$, then a decision is made that we do not have enough data to reliably extract the j-th bit of the digital watermark.
 - 2.2. If $|s_0(j) s_1(j)|/(s_0(j) + s_1(j)) < T_2$, then the j-th bit is also not extracted.
 - 2.3. If $s_1(j) > s_0(j)$, then the value of the j-th bit is 1, otherwise 0.

Embedding algorithm:

1. Find all points of the geometry of all objects, and calculate s_0 and s_1 for each square or bit index depending on the extraction method chosen.

0	1	1	0	0	1	1	1	0	1	1	0	0	1	1	1
1	0	0	0	0	1	0	0.	1	0	0	0	0	1	0	0
1	Ì	0	1	1	1	0	1	1	Ą	0	1	1	1	0	1
0	1	1	0	1	1	1	1	0	1	1	0	1	1	1	1
0	0	0	1	0	1	1	1	0	0	0	1	0	1	1	1
0	4	1	0	1	0	¥	0	0	1	1	0	1	0	Ą	0
1	0	1	1	0	0	1	0	1	0	1	1	0	0	1	0
0	1	0	0	0	1	0	1	0	1	0	0	0	1	0	1

Figure 1: Illustration of the watermarking process: polyline points move with respect to corresponding *Map* values.

- 2. If $s_0 + s_1 < T_1$, move on to the next square (or the next watermark bit).
- 3. Ensure that the ratio is fulfilled:

$$\frac{|s_0 - s_1|}{s_0 + s_1} \ge T_2(1 + k),\tag{2}$$

where $k \ge 0$ is a parameter that increases the watermark robustness to changes in the map contents. The sign of the difference $s_0 - s_1$ is determined by the value of the embedded bit. If the initial data does not meet these conditions, then the necessary number of points is shifted to the nearest coordinates with the opposite binary value in the *Map*. Figure 1 shows a very simplified illustration of the embedding approach: in this example, points located in cells with *Map* = 0 are moved to neighboring cells where *Map* = 1.

Thus, the proposed method is configured with the following parameters:

- Three values {N_b, M, r} related by (1). A high N_b value characterizes a large volume of the watermark, while an increase in r, in turn, increases the watermark robustness.
- The extraction algorithm version.
- T_1, T_2, q parameters that determine the efficiency of the method and the level of distortion when watermark embedding. They should be selected based on the balance between the quality of the extraction and the quality of the protected data (closeness to the original).
- k, p both parameters are more convenient for balancing the quality of extraction (robustness) and the level of distortion caused by embedding than the previous trio of parameters.

It should be noted that in practice it is expedient to embed a protective digital watermark into tiles of the most detailed levels, the theft of which is the most sensitive for the copyright holder and commercially justified for the attacker.

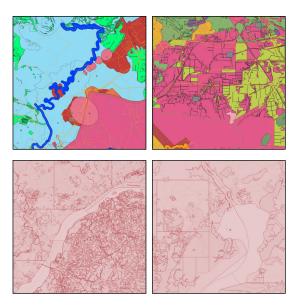


Figure 2: Test tile examples from STP (top) and Tegola (bottom). Different colors correspond to objects of different layers.

4 EXPERIMENTAL INVESTIGATION

4.1 Setting Up Experiments and Preliminary Study

To assess the performance of the proposed method, we first evaluated the effect of the method parameters on the quality of the digital watermark extraction and on the level of distortions associated with embedding. Next, a study was made of the robustness of the digital watermark, that is the effect of subsequent changes in protected tiles on the quality of watermark extraction. In our experiments, we used two data sources:

- The Samara Region Investment Map (STP);
- The Tegola demo map (https://tegola.io/).

The experiments utilized two STP tiles and two Tegola tiles. Each test tile contained more than 3000 line and polygon objects, including objects with complex geometry (see four examples in Figure 2).

During the research, the following quality metrics were calculated:

- Watermark extraction accuracy (percentage of correctly extracted bits).
- Hausdorff and Fréchet average distances as a measure of object geometry deviation when embedding a digital watermark (distance between the objects of the source map and the corresponding objects of the protected map).

 Hausdorff and Fréchet average distances as a measure of object geometry distortion in the study of watermark robustness (distance between protected map objects and the corresponding artificially distorted map objects).

Initially, we conducted a preliminary study. Watermark embedding was done with different parameter values, and the average geometry deviation was estimated. Then, the watermark was extracted, and the extraction accuracy was computed. The objective of this analysis was to identify the operational ranges of the parameters of our method that provide acceptable quality values. The results indicated that in a relatively broad range of parameter values, 100% accuracy of watermark extraction can be achieved on both data sources (STP and Tegola) with an acceptable geometry deviation (not less than 0.9 for both measures). This range includes the following values:

- T_2 : 0.05 to 0.95;
- k: 0.08 to 1;
- $T_1: 1$ to 75;
- q: 1 to 3;
- r: 1 to 200;
- *p*: 1 to 2 for STP and 1 to 8 for Tegola.

In this range of values, with p=1, the minimum metric values were recorded as follows:

- STP: 0.946 (Hausdorff) and 0.941 (Fréchet)
- Tegola: 0.980 (Hausdorff) and 0.941 (Fréchet)

Table 1 illustrates the effect of p on the geometry deviation when embedding a watermark. Other parameters were kept at average values based on the balance between the robustness and the level of distortion introduced. The watermark was extracted without errors for all values of p.

Table 1: Influence of p on the geometry deviation caused by watermark embedding

p	STI)	Tegola				
	Avg	Avg	Avg	Avg			
	Hausdorff	Fréchet	Hausdorff	Fréchet			
1	0.962	0.958	0.991	0.991			
2	0.931	0.922	0.980	0.979			
4	0.867	0.851	0.964	0.962			
8	0.788	0.765	0.940	0.937			
16	0.608	0.579	0.892	0.883			

Figure 3 illustrates the influence of the developed watermarking algorithm on the geometry of objects. Above, the original and protected tiles are shown in their entirety, and below are fragments of the overlay of the two tiles, showing the difference in shapes. The changes do not have a significant impact on the map data and are noticeable only with explicit comparison at a large scale. This embedding was performed with the following set of parameters: k = 0.6, $T_2 = 0.6$, $T_1 = 5$, p = 1, q = 3, $N_b = 20$, r = 10. The watermarking quality values are: accuracy = 1, hausdorff = 0.971, frechet = 0.969.

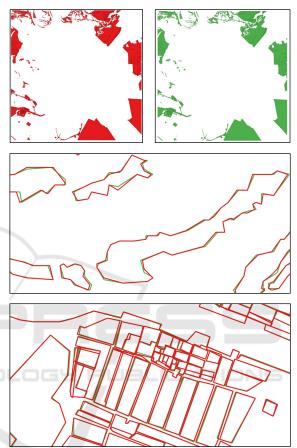


Figure 3: The effect of embedding a digital watermark: above is the original and the corresponding protected tile, in the middle and below are fragments of the overlay of two tiles (the red represents the original data, while the green represents the data with the embedded digital watermark).

4.2 Investigation of the Watermark Robustness in Case of Map Distortions

For reliable protection against theft, the built-in digital watermark must remain stable when the map changes. This should take into account changes that may occur naturally in the course of using the stolen data, as well as artificial changes that an attacker may make to destroy the embedded watermark.

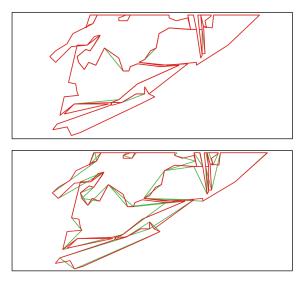


Figure 4: The effect of the ReducingNumberOfPoints attack (zoomed tile fragments) at *value* = 0.9 (upper, hausdorff: 0.990, Fréchet: 0.961) and *value* = 0.4 (lower, hausdorff: 0.726, Fréchet: 0.616). Watermark extraction accuracy is 1 in both cases.



Figure 5: The effect of the DeletingByArea attack (entire tiles) at value = 0.0001 (left) and value = 0.3 (right). Watermark extraction accuracy is 1 in both cases.

Of course, such distortions of the map should be considered within a range of parameter values for which the map retains its value as a source of reliable data.

The second stage of the study involved investigating the robustness of the embedded digital watermark against map distortions. The following distortions and their parameters were used:

1) ReducingNumberOfPoints:

This involved reducing the redundancy in the geometry by using the Largest Triangle Three Buckets (LTTB) algorithm (Alt, 2000), where the parameter is the proportion of points left. This reduction can be done naturally to decrease the size of the database, but it can also be maliciously used to remove the watermark.

2) DeletingByArea:

This involved removing objects with a small area, where the parameter is the proportion of the tile area used as a threshold for the area of objects. This removal can be done naturally or maliciously.

3) DeletingLayers

Removal of entire MVT layers (the parameter is the proportion of layers to be removed). It is a natural distortion that simulates the theft of part of the data related to some layers (if the rest of the layers are not of interest to the attacker).

4) SeparationByGeometryType

Separation by geometry type: keep only point, only line, or only polygonal features. It can also be natural. Has no parameters.

5) AddingNewObjects

Adding new objects to the map (the parameter is the number of new objects added in each layer). New objects have random geometry; the number of points is also random and depends on the geometry type (1 for points; from 2 to 100 for polylines; from 5 to 500 for polygons). It is a natural distortion simulating the addition of an attacker's objects to the map after its theft.

6) ShiftingPoints

Shift geometry points to one of the 4 adjacent cells in the *Map* matrix (the parameter is the proportion of all shifted points). Simulates an artificial action of an attacker to remove the watermark.

Figures 4-6 show examples of map distortions for different *values*, as well as the corresponding deviation metrics and the accuracy of watermark extraction. All distortions were tested for various combinations of the watermarking parameters.

Before presenting the main results of this investigation, it should be noted that version 2 of the information extraction algorithm proved to be less robust to all distortions, as shown in Tables 2-3.

Table 2 provides a summary of the watermark robustness investigation for version 1 of the extraction algorithm. Numbers 1-6 in column headers correspond to distortion indices, while data in rows differ in the strength of distortions. For the *SeparationByGeometryType* distortion (column 4), the rows correspond to using only polygons, only polylines, and only point features, respectively. Table 3 is similar to Table 2 but shows the results obtained for version 2. It is evident from the tables that version 1 is more robust. The following embedding parameters were used in this study: k = 0.6, $T_2 =$ 0.4, $T_1 = 5$, p = 1, q = 2, $N_b = 5$, r = 20, M =10.

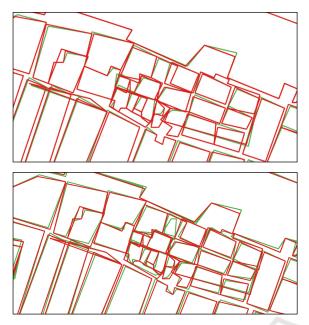


Figure 6: The effect of the ShiftingPoints attack (zoomed tile fragments) at *value* = 0.1 (upper, hausdorff: 0.957, Fréchet: 0.944) and *value* = 0.6 (lower, hausdorff: 0.949, Fréchet: 0.948). Watermark extraction accuracy: 1 and 0.2.

Table 2: Color map representing the watermark robustness against several distortions (columns) with several strength values (rows) (extraction algorithm version 1).

1	2	3	4	5	6
1	1	1	1	1	1
	1	1	1	1	1
1	1	1	0	1	1
1	1	1	-	1	1
1	0.75	1	-	1	1
1	0.75	1	-	1	0.5
1	0.75	1	-	1	0.5
1	0.75	1	-	0.95	0.35
1	0.45	0.8	-	0.9	0.5
1	0.25	0.35	-	0.85	0.45
1	0	0	-	0.7	0

In view of the large volume of studies conducted, we will first present general conclusions based on their results, and then we will delve in more detail into some individual studies. To simplify, all distortion parameters will be denoted as *value*, since it is clear from the name of the distortion what value we are talking about.

1) *ReducingNumberOfPoints* has almost no effect on extraction accuracy even with severe geometric distortions. Accuracy becomes less than 1 only when value < 0.01.

Table 3: Color map representing the watermark robustness against several distortions (columns) with several strength values (rows) (extraction algorithm version 2).

1	2	3	4	5	6
1	0.95	1	1	1	1
1	1	1	0.9	1	1
1	1	1	0	1	1
1	1	0.6	-	0.95	0.9
1	0.75	0.75	-	0.6	0
1	0.75	0.75	-	0.1	0
1	0.75	1	-	0	0
1	0.75	0.25	-	0	0
1	0.45	0.45	-	0	0
1	0.25	0.3	-	0	0
1	0	0	-	0	0

2) DeletingByArea. With value < 0.4, extraction accuracy equals 1 almost always, except for cases with large N_b and small r. But since value = 0.4 is the proportion of the area that cannot be considered insignificant, we can conclude that the watermark is sufficiently resistant to this distortion.

3) DeletingLayers. The quality values are not very good in the case of this distortion, because random layers are removed without taking into account the number of objects and points. However, almost always we have accuracy ≥ 0.9 at value ≤ 0.5 , which is a very good result. In practice, if necessary, it is possible to further increase the robustness against this distortion by embedding the watermark in separate layers independently.

4) *SeparationByGeometryType*. The watermark robustness to this distortion largely depends on the distribution of points across the different geometry types. Since there are relatively few point objects in the test tiles, removing all polyline and polygon objects leads to the destruction of the watermark. However, for other types of geometry, the watermark can be extracted with high accuracy. As with the the previous distortion, robustness to SeparationByGeometryType can be easily increased by embedding separate watermarks for different geometry types.

5) AddingNewObjects.

On average, when around 40 new objects are added to each layer, the extraction accuracy begins to decrease linearly. However, this drop can be significantly mitigated by using high values of k and r.

6) *ShiftingPoints*. This is a targeted attack designed to exploit weaknesses in the developed method, and therefore it has a greater effect compared to more natural attacks. However, the impact of this

distortion can be reduced by selecting appropriate watermark embedding parameters. For example, it was found that using k = 1 and $T_2 = 0.5$, makes it possible to achieve the absolute watermark extraction accuracy for *value* < 0.5.

Thus, we can conclude that the proposed watermark embedding method can be configured to be highly robust to distortions 1-4 for any *value* and to distortions 5-6 with an average level of *values*. Such results make the proposed method very attractive for practical use.

5 CONCLUSIONS

In this paper, we proposed a watermarking method to protect geodata in the Mapbox Vector Tile (MVT) format against theft. Despite its popularity in web mapping services due to its efficient storage and fast rendering, the vector nature of the MVT format makes it vulnerable to theft by attackers. The method proposed in the paper protected MVT data with a digital watermark that was based on the requantization of point coordinates of object geometry. The method could be adjusted using several parameters to balance the robustness of the digital watermark to map distortions and the error introduced when embedding.

A series of experiments were performed to test the robustness of the method against various distortions, including the removal of objects and layers, reduction in the number of points, adding new objects, and shifting some points in the tile geometry. We found that with a proper choice of watermark parameters, the proposed method could achieve a 100% watermark extraction accuracy for all bits of the built-in watermark, even with a reasonable level of the listed distortions that did not lead to a loss of significance of the protected geodata.

Planned areas for further work include further improvement and deeper investigation of the proposed method.

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