# Creating a Math Expression Filter to Extract Concise Math Expression Images

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Abstract: Even though the web is an effective resource to search for math expressions, finding appropriate ones among the obtained documents is time-consuming. Therefore, we propose a math expression filter that presents appropriate images for such searches. We call an appropriate math expression a *concise math expression*, such as  $\hat{f}_h(x) = \frac{1}{Nh} \sum_{i=1}^n K(\frac{x-x_i}{h})$ , written in a compact form whose content can be interpreted by the math expression itself. We determined the conditions satisfied by a concise math expression and developed classifiers that discriminate the images of concise math expressions from web images using supervised machine learning methods based on these conditions. We performed two experiments: Experiment 1 used methods other than deep learning, and Experiment 2 used deep learning. A convolutional neural network (CNN) with transfer learning and fine tuning by VGG16 shows high performance with an obtained F-measure of 0.819. We applied this filter to a task that presents math expression images by entering mathematical terms into a web search engine as queries. All of the evaluation metrics outperformed the previous study, including F-measure, MAP, and MRR.

# **1** INTRODUCTION

In recent years, the importance of mathematical information retrieval (MIR) has increased (Zanibbi and Blostein, 2012). Since mathematics is a tool for expressing various concepts in science fields, understanding math expressions is imperative in such fields. When someone wants to clearly understand an expression, the web provides a natural searching resource. However, web search for math expressions has its problems. After web documents are obtained that contain math expressions, finding the appropriate math expressions in the located documents is timeconsuming. Such documents contain many various types of math expression images. For example, such variables as "x" have no intrinsic meaning, and such fragments as "log" and " $\leq$ " are not math expressions, and some long expressions simply show the process for deriving a formula. Among expressions and fragments of expressions, we extracted math expressions, including  $\hat{f}_h(x) = \frac{1}{Nh} \sum_{i=1}^n K(\frac{x-x_i}{h})$ , which are long enough for human interpretation without supplementary explanation, although shorter than expressions from which a formula is derived. We call these concise math expressions. First, we collected candidate expressions, analyzed their features, and determined the conditions of a concise math expression. We then produced a math expression filter by developing classifiers to perform binary classification and compared the performance of each classifier with the others. Finally, we applied our filter to a task that presents math expression images on web searches and discussed the results.

### 2 RELATED WORK

Various types of research have been pursued in MIR. The first is math formula similarity searches, such as adopting three different similarity measures (Ohashi et al., 2016), using two different tree structures (Davila and Zanibbi, 2017), and focusing on the similarity of substructures (Zhong et al., 2020). Other types of MIR include a math document classification method based on text combined with the structures of math expressions (Suzuki and Fujii, 2017), extracting identifier-definiens pairs to improve performance in MIR tasks (Schubotz et al., 2017), and focusing on partial equations within equations to analyze the frequency distributions of math expressions in large

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scientific datasets (Greiner-Petter et al., 2020). For math image recognition, there are conversions such as image to LaTeX (Peng et al., 2021; Wang and Liu, 2016) and image to markup (Deng et al., 2017). Other work summarizes content by generating headlines with math equations (Yuan et al., 2020). However, none of these are intended for the web, and the datasets are also specific. For the web, a formula search available from a browser is "Approach Zero" (Zhong, 2022), which allows users to search for formulae in specific databases. For PDFs, there is research on analyzing PDFs using OCR software and presenting math expression images in response to a query (Yamada and Murakami, 2020) and research on detecting math formula regions as bounding boxes around formulae in PDFs using a CNN (Dey and Zanibbi, 2021). Our research aims to extract concise math expression images from images in HTML documents on the Web by binary classification without directly analyzing the contents of the images. To our knowledge, no similar studies were found.

# **3 EXPERIMENTS**

After setting a dataset, we applied preprocessing including elimination of duplicate images, and determined the concise math expression conditions. We then checked the correct images of the dataset based on the conditions and conducted two experiments for evaluating the performance of the created classifiers. Experiment 1 used machine learning methods other than deep learning. Experiment 2 used CNNs. Finally we compared all of the classifiers and selected the best one.

#### 3.1 Dataset

Table 1: Dataset. These raw data include duplicate images and errors in preprocessing. "Other than html" includes PDFs, slides, Google Books and so on.

Dataset	Image		Acquired webpage breakdown			
	Total	Correct	Other than html	Error	Html	
D <sub>0_trn</sub>	19,470	442	1,091	80	1,829	
$D_{0_val}$	15,427	351	1,269	60	1.671	
D <sub>0_tst</sub>	23,988	929	1,662	72	2,266	
Total	58,885	1,722	4,022	212	5,766	

We use the same dataset as studied in a prior work (Yamada et al., 2018). We randomly selected 100 keywords from the index of Bishop's "Pattern Recognition and Machine Learning" (Bishop, 2006) and performed a web search using these keywords as queries to obtain the top 100 web pages. We created a dataset by extracting all the images from those pages. In Table 1,  $D_{0.tm}$  is the keywords from 31 to 60 as the training dataset,  $D_{0.val}$  is the dataset from keywords 1 to 30 as validation, and  $D_{0.tst}$  is the keywords from 61 to 100 as the testing dataset. The first author manually judged images to determine whether they were related to the keywords. When unclear cases surfaced, judgments were made in consultation with another person (the same person throughout all judgments). Keyword examples are softmax function, SVM, kernel density estimation method, Heaviside step function, Gaussian kernel, convex function, Probit function, Boltzmann distribution, functional derivative, and leastmean-squares algorithm.

#### 3.2 Preprocessing

Because of the method used to create  $D_{0,trn}$  and  $D_{0,val}$ , they included the same image registered with different IDs. Therefore, we deleted the ones with overlapping features. In Experiment 1, the basic features (file size, width, and height) were used, so images with these values overlapping were deleted. In Experiment 2, the images were used directly, so the images with the same features and the same appearance were deleted. In addition, unnecessary icons such as buttons and logos were removed from the dataset for Experiment 1. We extracted the common strings from the image names of the unnecessary icons in  $D_{0,trn}$ , and images with these strings in their image names were deleted in advance.

## 3.3 Determining Concise Math Expression Conditions

After preprocessing, we obtained 314 of the original 442 keyword-related correct images in D<sub>0\_trn</sub> (Table 1) and analyzed them to identify the conditions of a concise math expression. As a result of a web search using the above keywords, many of the correct math expressions have proper names such as Gaussian kernel. Therefore, they are written in an organized form and are interpretable by the expressions themselves. That means these images are considered suitable candidates for concise math expression images. We examined the following by directly viewing the images: "Number of horizontal characters (including symbols)," "number of vertical characters (including symbols)," "number of lines," "number of expressions<sup>1</sup>," and "number of concatenations" (=,<,and so on). Because the fonts used in web math ex-

<sup>&</sup>lt;sup>1</sup>The number of expressions in () is 1. Nested expressions in an expression are not counted.

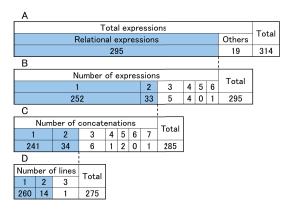


Figure 1: Threshold determination. The third row of each figure represents the frequency. We determined the thresholds in order from the easiest one (from top to bottom in the table). After determining a threshold, we searched for the next one in the images that were already within the threshold.

pression images are quite different and the layouts of expressions are also different, the number of horizontal characters is counted regardless of the size of the font. For the number of vertical characters, an exponent, a fractional line, and others are set to 0.5. For example, if an image is  $\frac{x-1}{x^2+x-1}$ , the number of horizontal characters is 6 and the number of vertical characters is 3.

The thresholds were determined in order from the easiest one in Fig. 1. (1) From A, a relational expression is first adopted. A relational expression is one in which the left and right sides are connected by "=," "<," and so on<sup>2</sup>. (2) From B, i = 1 or 2, i: number of expressions. (3) From C, j = 1 or 2, j: number of concatenations. (4) From D, k = 1 or 2, k: number of lines. Then (5) from Fig. 2, the frequency between 4 and 35 is 2 or more, so  $4 \le l \le 35, l$ : number of horizontal characters. (6) From Fig. 3, the frequency of 6 or less is 2 or more, so  $1 \le m \le 6, m$ : number of vertical characters.

Therefore, a concise expression satisfies (1) a relational expression, (2) i = 1 or 2, i: number of expressions, (3) j = 1 or 2, j: number of concatenations, (4) k = 1 or 2, k: number of lines, (5)  $4 \le l \le 35$ , l: number of horizontal characters, and (6)  $1 \le m \le 6$ , m: number of vertical characters.

We manually checked the correct images of the total dataset based on the above conditions. Since our dataset is very imbalanced, we randomly undersampled the data with "\*" in Table 2. In Experiment 2, we adjusted the numbers of datasets (other than test

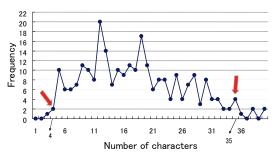


Figure 2: Number of horizontal characters. This figure shows the number of characters from 1 to 40 (total frequency 257). The remaining 41 to 91 are omitted because they are sparsely scattered (total frequency 18).

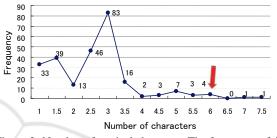


Figure 3: Number of vertical characters. The frequency of 4 to 35 characters in Fig. 2 is 251, and this figure shows the results using those images.

data  $D_{2.tst}$ ) for batch processing. Figure 4 shows examples of correct and incorrect images. Correct image (a) is an expression for the complementary error function and correct image (b) is an expression for the *t*-distribution. Incorrect image (c) is the coordinates of a point, and (d) is the output of the math software for a cell decomposition of an annulus.

#### **3.4 Evaluation Metric for Classifiers**

In Experiment1 and 2, our dataset is very imbalanced, so we use the F-measure (Eq. (1)) to compare a large number of classifiers.

$$F\text{-measure} = \frac{2 \cdot \text{precision} \cdot \text{recall}}{\text{precision} + \text{recall}}, \qquad (1)$$

where

$$precision = \frac{true positives}{true positives + false positives}$$

and

$$recall = \frac{true positives}{true positives + false negatives}$$

<sup>&</sup>lt;sup>2</sup>"=" is 299, "inequality sign" is 14, " $\approx$ " is 4, " $\simeq$ " is 2, " $\equiv$ " is 2, ":=" is 2, " $\sim$ " is 2, and " $\gg$ " is 1 (including multiple concatenations in a single image).

Table 2: Data for experiments. After preprocessing dataset in Table 1, we randomly undersampled the data with "\*" in the table (The criteria for "Correct" in Table 1 and in this Table are different ).

	Experiment 1			Experiment 2				
Dataset	$D_{1\_trn}$	$D_{1\_val}$	D <sub>1_tst</sub>	$D_{2_{\text{trn}}}$	$D_{2\_val\_bal}$	$D_{2\_val\_imbal}$	$D_{2\_tst}$	
Correct	2,410	1,868	4,336	2,400	1,900	1,900	4,462	
Incorrect	2,562*	6,076	12,689	2,400*	1,900*	8,180	18,678	
Total	4,972	7,944	17,025	4,800	3,800	10,080	23,140	

(a)

$$y = \frac{2}{\sqrt{m}} \int_{x}^{+\infty} e^{-t^{2}} \cdot dt \qquad f(t) = \frac{\Gamma((\nu+1)/2)}{\sqrt{\nu\pi} \Gamma(\nu/2)} (1 + t^{2}/\nu)^{-(\nu+1)/2}$$

(b)

(d)

(c)

$$(-1/\sqrt{2}, -1/\sqrt{2}) \qquad \qquad \begin{pmatrix} (-\sqrt{2} < x \le 1 - \delta \delta - \sqrt{2 - x^2} < y < \sqrt{2 - x^2}) \| \\ (-1 < x < 1 \delta \delta - \sqrt{2 - x^2} < y \le -\sqrt{1 - x^2} \| \sqrt{1 - x^2} \le y < \sqrt{2 - x^2}) \| \\ (1 \le x < \sqrt{2} \delta \delta - \sqrt{2 - x^2} < y \le \sqrt{2 - x^2}) \end{pmatrix} \|$$

Figure 4: Examples of correct and incorrect images. Images labeled (a) and (b) are correct, while (c) and (d) are incorrect.

## 3.5 Experiment 1: Machine Learning Methods Other Than Deep Learning

In order to create classifiers, we used Weka (Frank et al., 2022), which has an easy-to-understand interface that allows us to combine many classifiers and easily change hyperparameters. It also outputs various evaluation metrics, such as confusion matrices, making it easy to compare classifiers. We created classifiers to discriminate concise math expressions using image features. The basic features were "file size in bytes," "number of file width pixels," and "number of file height pixels." Furthermore, "density (Eq. (2))," "aspect ratio (Eq. (3))," and "gray value (Eq. (4))" were used.

density = 
$$\frac{\text{filesize}}{\text{width} \cdot \text{height}}$$
, (2)

aspect ratio = 
$$\frac{\text{width}}{\text{height}}$$
, (3)

where width is the number of width pixels and height is the number of height pixels.

gray = 
$$|R - G| + |G - B| + |B - R|$$
, (4)

where R is the red component of the image, G is green, and B is blue. Eq. (4) is used because some math expression images may appear to be gray but actually have a color component. If the gray value is

0, then the image is gray. The "+gray value" in Table 3 is calculated again as a correct image when the predicted value of the classifier created with features other than the gray value is correct and the gray value is 0. We performed 10-fold cross-validation on every classifier and searched for hyperparameters. For a multilayer perceptron, we considered the number of hidden layers, and for a random subspace (Ho, 1998), which is an ensemble learning method, we considered several classifiers for use. For an SVM (Chang and Lin, 2011), we used a nonlinear SVM with an RBF kernel and searched for the values of c and  $\gamma$ .

In Table 3, methods B and J were the best, so we conducted an evaluation experiment using test data  $D_{1.tst}$  in Table 2. As a result, both had an F-measure of 0.628 using SVM with c = 7000 and  $\gamma = 6$ . Comparing both, true positives and true negatives were almost the same number, but B had less false positives and more false negatives, and J had exactly the opposite.

# 3.6 Experiment 2: CNN Methods

We created CNNs to conduct binary classification that discriminated between concise math expression images and other images, using Keras (Chollet et al., 2022) with a TensorFlow backend. Previously, a large amount of image data was required for deep learning, but recently, trained models for transfer learning have been widely released and are generally available. We used "data augmentation," "transfer learning," and "fine tuning" methods and examined the models provided by Keras. Data augmentation is a common data increasing operation to avoid overfitting caused by having too few data. Random rotation, translation, equal volume deformation, zooming, and horizontal inversion of half of the image were performed. The models of transfer learning were pre-trained by the ImageNet. The number of epochs for fine tuning was set at 100, but only NASNetMobile was stopped early to avoid overfitting. Table 4 shows the results of the final test data using Table 2's data. First, as a baseline, we constructed method A with four Conv2D layers and four MaxPooling2D layers. The activation function is the sigmoid function, and the loss function

Method	Feature	Classifier	F-measure	Rank
А	basic features	SVM	0.654	6
В	basic features, density	SVM	0.677	1
С	basic features, aspect ratio	SVM	0.661	4
D	basic features, density, aspect ratio	SVM	0.669	3
E	basic features, gray value	MultilayerPerceptron	0.638	9
F	basic features, density, gray value	RandomSubSpace	0.649	8
G	basic features, aspect ratio, gray value	MultilayerPerceptron	0.635	10
Н	basic features, density, aspect ratio, gray value	RandomSubSpace	0.653	7
Ι	basic features, +gray value	SVM	0.658	5
J	basic features, density, +gray value	SVM	0.677	1
K	basic features, aspect ratio, +gray value	SVM	0.623	12
L	basic features, density, aspect ratio, +gray value	SVM	0.626	11

Table 3: Creating classifiers using machine learning method other than deep learning. Top results by each feature.

is binary cross entropy. We examined eight different patterns for each model, which were whether to use  $D_{2\_val\_bal}$  (balanced data) or  $D_{2\_val\_imbal}$  (imbalanced data) as validation data, whether to use data augmentation or not, and whether to use fine tuning or not. As a result, method N was found to be the best.

#### 3.7 Experimental Results

Despite various performances in Experiment 1, method N (Table 4) was the best throughout Experiment 1 and 2. The conditions were (1) use VGG16 (Simonyan and Zisserman, 2014) as transfer learning, (2) use balanced data as validation data, (3) do not use data augmentation, and (4) use fine tuning. Thus, the design of our math expression filter was completed.

### 4 APPLICATION

We applied the experimental results to the previous study (Yamada et al., 2018). The task performs web searches using a math term as a query and presents the top ten math expressions related to the term with its surrounding information. Score( $i_k$ ) is given to each expression  $i_k$  by Eq (5):

$$\operatorname{score}(i_k) = x_{\operatorname{line}} + x_{\operatorname{key}} + x_{\operatorname{svm}} + x_{\operatorname{bonus}},$$
 (5)

where  $x_{\text{line}} = 1$  if  $i_k$  is in a separate line, 0 otherwise;  $x_{\text{key}} = 1$  if  $i_k$  has a keyword within a window size set from -200 to +200 characters, 0 otherwise;  $x_{\text{svm}} = 1$ if  $i_k$  is discriminated as positive by the SVM, 0 otherwise; and  $x_{\text{bonus}} = 1$  if  $i_k$  has a perfect score so far and appears first in the same web document, 0 otherwise. The classifier is a nonlinear SVM with an RBF kernel. The correct image is a math expression image related to the keyword, which is different from this study. Evaluation metrics are the F-measure (Eq. (1)) where precision  $= \frac{r}{n}$  and recall  $= \frac{r}{c}$ , Mean Reciprocal Rank (MRR) (Eq. (6)), and Mean Average Precision (MAP). MAP is the macro mean of the Average Precision(AP) (Eq. (7)):

MRR = 
$$\frac{1}{n} \sum_{k=1}^{n} \frac{1}{r'_k}$$
, (6)

$$AP = \frac{1}{\min(n,c)} \sum_{s} I(s) \operatorname{Prec}(s), \qquad (7)$$

where r is the number of correct images of top n, c is the total number of correct images,  $r'_k$  is the rank of the correct images at the top of the k-th keyword, I(s) is a flag indicating whether the image at s-th is correct, and Prec(s) is the precision at s-th.

We reranked them by replacing the  $x_{svm}$  values in Eq. (5) with our filter's values. Table 5 shows that our filter worked well.

#### 5 DISCUSSION

Concerning our data, using data augmentation should be avoided because none of the math web images are tilted or upside down. However, since the sizes of the image margins vary, and some of the images are unclear after preprocessing ( $\alpha$  channel removal and so on), data augmentation might be considered in these cases. We plan to study data augmentation in such cases in our future work. For the validation data when dealing with imbalanced data, Table 4 does not show which choice was better, balanced or imbalanced data. For the application, since the original classifier discriminated whether an image was related to a given keyword, the same image may be correct or incorrect, depending on its context. The ability to identify correct images regardless of context led to improvement.

Method	Transfer learning model	Validation data	Data	Fine	F-measure	Rank
			augmentation	tuning		
М	baseline	$D_{2\_val\_imbal}$	_		0.758	10
N	VGG16	D <sub>2_val_bal</sub>	_	+	0.819	1
0	DenseNet169	$D_{2\_val\_imbal}$	_	+	0.807	2
Р	VGG19	$D_{2\_val\_imbal}$	_	+	0.804	3
Q	DenseNet201	$D_{2\_val\_bal}$	_	+	0.803	4
R	DenseNet121	$D_{2\_val\_bal}$	_	+	0.801	5
S	ResNet50	$D_{2\_val\_imbal}$	+	+	0.801	5
Т	InceptionResNetV2	$D_{2\_val\_bal}$	+	+	0.795	7
U	MobileNet	$D_{2\_val\_imbal}$	+	+	0.794	8
V	MobileNetV2	$D_{2\_val\_bal}$	+	+	0.782	9
W	Xception	$D_{2\_val\_imbal}$	+	+	0.753	11
Х	InceptionV3	D <sub>2_val_imbal</sub>	_	+	0.747	12
Y	NASNetMobile	$D_{2\_val\_imbal}$	_	+	0.741	13

Table 4: Creating classifiers using CNNs. Top results by each model, where "+" is used and "-" is not used.

Table 5: Comparison of evaluation metrics@10.

	F-measure	MRR	MAP	
Previous study	0.40	0.84	0.42	
Using filter	0.55	0.86	0.45	

# 6 CONCLUSION

We developed a math expression filter to extract concise math expression images. We determined the conditions satisfied by a concise math expression and developed classifiers that discriminate the images of concise math expression images from web images using supervised machine learning methods based on these conditions. To investigate our filter's performance, we applied it to a task that presents math expression images and obtained good results.

#### REFERENCES

- Bishop, C. M. (2006). Pattern Recognition and Machine Learning. Springer, NY, New York.
- Chang, C.-C. and Lin, C.-J. (2011). Libsvm: A library for support vector machines. ACM Transactions on Intelligent Systems and Technology, 2(3):27:1–27:27.
- Chollet, F. et al. (2022). Keras. https://github.com/fchollet/ keras.
- Davila, K. and Zanibbi, R. (2017). Layout and semantics: Combining representations for mathematical formula search. In Proceedings of the 40th International ACM SIGIR Conference on Research and Development in Information Retrieval, pages 1165–1168.
- Deng, Y., Kanervisto, A., Ling, J., and Rush, A. M. (2017). Image-to-markup generation with coarse-to-fine attention. In *Proceedings of the 34th International Con-*

ference on Machine Learning (ICML 2017), pages 980–989.

- Dey, A. and Zanibbi, R. (2021). Scanssd-xyc: Faster detection for math formulas. In *Document Analysis and Recognition, (ICDAR 2021)*, volume 12916 of *LNCS*, pages 91–96. Springer.
- Frank, E., Hall, M. A., and Witten, I. H. (2022). The weka workbench. online appendix for "data mining: Practical machine learning tools and techniques". https://www.cs.waikato.ac.nz/ml/weka/ Witten\_et\_al\_2016\_appendix.pdf.
- Greiner-Petter, A., Schubotz, M., Müller, F., Breitinger, C., Cohl, H. S., Aizawa, A., and Gipp, B. (2020). Discovering mathematical objects of interest - a study of mathematical notations. In *Proceedings of the Web Conference 2020 (WWW '20)*, pages 1445–1456.
- Ho, T. K. (1998). The random subspace method for constructing decision forests. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 20(8):832– 844.
- Ohashi, S., Kristianto, G. Y., Topić, G., and Aizawa, A. (2016). Efficient algorithm for math formula semantic search. *IEICE Transactions on Information and Systems*, 99(4):979–988.
- Peng, S., Gao, L., Yuan, K., and Tang, Z. (2021). Image to latex with graph neural network for mathematical formula recognition. In *Proceedings of 16th International Conference on Document Analysis and Recognition (ICDAR 2021)*, volume 12822 of *Lecture Notes in Computer Science*, pages 648–663. Springer.
- Schubotz, M., Krämer, L., Meuschke, N., Hamborg, F., and Gipp, B. (2017). Evaluating and improving the extraction of mathematical identifier definitions. In Proceedings of Experimental IR Meets Multilinguality, Multimodality, and Interaction- 8th International Conference of the CLEF Association (CLEF 2017), volume 10456 of Lecture Notes in Computer Science, page 82–94. Springer, Cham.
- Simonyan, K. and Zisserman, A. (2014). Very deep convolutional networks for large-scale image recognition.

In Proceedings of 3rd International Conference on Learning Representations, (ICLR 2015). https://arxiv. org/abs/1409.1556.

- Suzuki, T. and Fujii, A. (2017). Mathematical document categorization with structure of mathematical expressions. In *Proceedings of ACM/IEEE Joint Conference* on Digital Libraries (JCDL 2017), pages 119–128.
- Wang, Z. and Liu, J.-C. (2016). Translating math formula images to latex sequences using deep neural networks with sequence-level training. *International Journal on Document Analysis and Recognition*, 24(1):63–75.
- Yamada, K. and Murakami, H. (2020). Mathematical expression retrieval in pdfs from the web using mathematical term queries. In Proceedings of Trends in Artificial Intelligence Theory and Applications. Artificial Intelligence Practices 33rd International Conference on Industrial, Engineering and Other Applications of Applied Intelligent Systems (IEA/AIE 2020), volume 12144 of Lecture Notes in Computer Science, pages 155–161. Springer, Cham.
- Yamada, K., Ueda, H., Murakami, H., and Oka, I. (2018). Mathematical expression image retrieval on web using mathematical terms as queries. *Transactions of the Japanese Society for Artificial Intelligence*, 33(4):A– H91\_1–13.
- Yuan, K., He, D., Jiang, Z., Gao, L., Tang, Z., and Giles, C. L. (2020). Automatic generation of headlines for online math questions. In *Proceedings of the Thirty-Fourth AAAI Conference on Artificial Intelligence (AAAI-20)*, pages 9490–9497.
- Zanibbi, R. and Blostein, D. (2012). Recognition and retrieval of mathematical expressions. *International Journal on Document Analysis and Recognition*, 45(4):331–357.
- Zhong, W. (2022). Approach zero. https://approach0.xyz/ search/.
- Zhong, W., Rohatgi, S., Wu, J., Giles, C. L., and Zanibbi, R. (2020). Accelerating substructure similarity search for formula retrieval. In *Proceedings of Advances in Information Retrieval - 42nd European Conference on IR Research (ECIR 2020)*, volume 12035 of *Lecture Notes in Computer Science*, pages 714–727. Springer.